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BILINEAR INTERPOLATION OF PROGRAM CONTROL IN APPROACH PROBLEM

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Abstract. We consider a controlled system involving a constant two-dimensional vector parameter, the approximate value of which is reported to the controlling person only at the moment of the start of movement. Apriori only the set of possible values of these unknown parameter is given. For this controlled system we pose the problem on approaching the target set at a given time. At the same time, we suppose that the controlling person has no the ability to carry out cumbersome calculations in real time associated with the construction of such resolving structures as reachability sets and integral funnels. Therefore, to solve this problem, it is proposed to calculate in advance several "node" resolving controls for parameter values, which are nodes of a grid covering a set of possible parameter values. If at the moment of the beginning of the movement, the parameter value turns out not coincide with any of the grid nodes, it is proposed to calculate the software control by using linear interpolation formulas. However, this procedure can be effective only if a linear combination of controls corresponding to the same "guide" is used in the terminology of the N.N. Krasovsky extreme aiming method. For the possibility of effective use of linear interpolation, it is proposed to build four "node" resolving controls for each grid node and, in addition, to use the method of dividing the control into the main and compensating ones. Due to the application of the latter method, the computed solvability set turns out to be somewhat less than the actual one, but the accuracy of translating the state of the system to the target set increases. A nonlinear generalization of the Zermelo navigation problem is considered as an example.

Keywords: controlled system, approach problem, unknown constant parameter, bilinear interpolation.

Mathematics Subject Classification: 93C41

1. Introduction

One of the most significant problems of control theory [1] is that the calculation of a resolving program control (or positional strategy in differential games [2], [3]) is often associated with cumbersome computational procedures that cannot be performed in real time. In the case when the conditions of a problem are completely defined and known in advance, the long-term calculation of resolving structures is not a problem, since resolving control can be built before the controlled system starts to move. The situation changes if there are some uncertainties in the conditions of the control problem that cannot be found out before the initial time [4]– [7]. For example, according to [8], solving a control problem with an incompletely known initial condition consists of three stages:

1) collecting information about the system,
2) applying this information to eliminate uncertainty,
3) passage to active management.

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In this scheme, it is worth to pay an attention to the passage between the second and third stages: after the elimination of uncertainty it is hardly possible to carry out an instantaneous construction of resolving control during the movement of some dynamic system that has already begun.

One can also consider, for example, a completely natural problem, when the location of the target set for a dynamical system is not known in advance, but the situation requires an immediate response as soon as the target set is found in the observed phase space.

Generalizing the data of the problem, by a change of variables all uncertainties in the initial position, parameter, or location of the target set can be reduced to an indefinite multidimensional parameter. In addition, in this paper, we do not dwell on the process of identifying an uncertain parameter, but we focus on a quick response in the form of program control once this uncertain parameter has been reported to us.

As a solution we propose to pre-build resolving controls corresponding to several values of a constant vector parameter, and for intermediate values of the parameter, it is proposed to use simple linear interpolation formulas. The problem is that, in the general case, a linear combination of controls corresponding to different "guides" (according to the terminology of N.N. Krasovsky's extreme aiming method [9], [10]), does not give nice results. This is why we apply a more complicated scheme on the base of splitting the control into main and compensating ones.

In the present work we consider a bilinear interpolation in a two-dimensional vector parameter; the case of the scalar parameter was considered before in [11].

2. Formulation of problem

On a time interval $[t_0, \vartheta]$, $(t_0 < \vartheta < \infty)$, we consider a controlled system

$$\begin{cases} \dfrac{dx}{dt} = f(t, x(t), u(t), \alpha), \quad t \in (t_0, \vartheta), \\ x(t_0) = x^{(0)}, \end{cases} \tag{2.1}$$

where $x^{(0)} \in \mathbb{R}^n$ is the initial state, t is the time, $x(t) \in \mathbb{R}^n$ is a phase vector of the system, $u(t)$ is a Lebesgue measurable vector function (a vector of controlling actions) with values in a compact set $P \subset \mathbb{R}^p$, n and p are natural numbers, $\alpha \in \mathscr{L}$ is a constant parameter, $\mathscr{L}$ is a compact set in $\mathbb{R}^2$.

We assume the following conditions.

A. The vector function $f(t, x, u, \alpha)$ is well-defined, continuous on $[t_0, \vartheta] \times \mathbb{R}^n \times P \times \mathscr{L}$ and for each bounded and closed domain $\Omega \subset [t_0, \vartheta] \times \mathbb{R}^n$ there exists a constant $L = L(\Omega) \in (0, \infty)$ such that

$$\|f(t, x^{(1)}, u, \alpha) - f(t, x^{(2)}, u, \alpha)\| \leqslant L\|x^{(1)} - x^{(2)}\|, \qquad (t, x^{(i)}, u, \alpha) \in \Omega \times P \times \mathscr{L}, \qquad i = 1, 2;$$

here $\|\cdot\|$ is the Euclidean norm of a vector in $\mathbb{R}^n$.

Remark 2.1. *Taking into consideration Condition* A, *we obtain that the modules of continuity*

$$\omega^{(3)}(\delta) = \max\big\{\|f(t, x, u_*, \alpha) - f(t, x, u^*, \alpha)\| : (t, x, u_*, \alpha), (t, x, u^*, \alpha) \in D \times P \times \mathscr{L},\ \|u_* - u^*\| \leqslant \delta\big\}, \qquad \delta \in (0, \infty),$$

$$\omega^{(4)}(\delta) = \max\big\{\|f(t, x, u, \alpha_*) - f(t, x, u, \alpha^*)\| : (t, x, u, \alpha_*), (t, x, u, \alpha^*) \in D \times P \times \mathscr{L},\ |\alpha_* - \alpha^*| \leqslant \delta\big\}, \qquad \delta \in (0, \infty),$$

satisfy limiting relations $\omega^{(k)}(\delta) \downarrow 0$ *as* $\delta \downarrow 0$, $k = 3, 4$.

B. There exists $\gamma \in (0, \infty)$ such that

$$\|f(t, x, u, \alpha)\| \leqslant \gamma(1 + \|x\|), \qquad (t, x, u, \alpha) \in [t_0, \infty) \times \mathbb{R}^n \times P \times \mathscr{L}.$$

Remark 2.2. *By an admissible control $u(t)$ we mean a Lebesgue measurable on $[t_0, \vartheta]$ vector function with the values in P. Conditions* A *and* B *ensure that for each admissible control $u(t)$, there exists a corresponding motion $x(t)$ being a solution of system (2.1) in the class of absolutely continuous functions [12, Sect. 2.1]. Here the derivative $\dot{x}(t)$ is treated in the generalized sense and it obeys the Newton-Leibnitz formula, see, for instance, [13, Ch. 2, Sect. 4].*

Remark 2.3. *By Condition* B *there exists some sufficiently large domain $\Omega \subset [t_0, \vartheta] \times \mathbb{R}^n$, which apriori contains all possible motions of system (2.1) together with all auxiliary constructions for constructing resolving control. In what follows we employ exactly this domain Ω and a corresponding Lipschitz constant $L = L(\Omega)$.*

C. A vectorgram of velocities $F(t, x, \alpha) = f(t, x, P, \alpha) = \{f(t, x, u, \alpha) : \ u \in P\}$ is a convex set in $\mathbb{R}^n$ for all $(t, x, \alpha) \in [t_0, \theta] \times \mathbb{R}^n \times \mathscr{L}$.

We denote by $B^p(u, \rho) = \{\xi \in \mathbb{R}^p : \|\xi - u\| \leqslant \rho\}$ a closed ball in $\mathbb{R}^p$,

$$\check{P} = \check{P}(\rho) = P \dot{-} B^p(\mathbf{0}, \rho) = \{u \in \mathbb{R}^p \ : \ u + B^p(\mathbf{0}, \rho) \subset P\}$$

is the restriction of the set of values of the control.

D. For all $(t, x, \alpha) \in [t_0, \vartheta] \times \mathbb{R}^n \times \mathscr{L}$ and each non-empty restriction $\check{P}$ the set

$$\check{F}(t, x, \alpha) = f(t, x, \check{P}, \alpha) = \big\{f(t, x, u, \alpha) : \ u \in \check{P}\big\}$$

is convex.

E. Let points (t_*, x^*) and (x^*, t^*) belong to the domain Ω and

$$t^* = t_* + \Delta, \qquad x^* = x_* + \Delta \cdot f(t_*, x^*, \overline{u}, \alpha), \qquad \Delta > 0, \qquad \overline{u} \in \check{P}(\rho(\Delta)), \qquad \alpha \in \mathscr{L}.$$

Moreover, let not a too large number $\Delta_\alpha > 0$ be given. Then we can defined a function $\rho(\Delta)$ so that the problem

$$\begin{cases} \dot{x}(t) = f(t, x(t), \overline{u} + w, \widetilde{\alpha}), & t \in (t_*, t^*), \\ x(t_*) = x_*, \qquad x(t^*) = x^* \end{cases}$$

is solvable with respect to the compensator vector w in $B^p(\mathbf{0}, \rho(\Delta))$ and an absolutely continuous $x(\cdot)$ for each value $\widetilde{\alpha} \in B^2(\alpha, \Delta_\alpha \sqrt{2})$. At the same time, the dependence $w = w(\widetilde{\alpha})$ should of class $C^2(B^2(\alpha, \Delta_\alpha \sqrt{2}))$ and for all $\widetilde{\alpha} \in B^2(\alpha, \Delta_\alpha \sqrt{2})$ it should satisfy the inequalities

$$\left\|\frac{\partial^2 w}{\partial \widetilde{\alpha}_1^2}\right\| \leqslant M_2, \qquad \left\|\frac{\partial^2 w}{\partial \widetilde{\alpha}_2^2}\right\| \leqslant M_2, \qquad \left\|\frac{\partial^2 w}{\partial \widetilde{\alpha}_1 \partial \widetilde{\alpha}_2}\right\| \leqslant M_2,$$

where a constant $M_2 \geqslant 0$ is determined by the function $f(\cdot, \cdot, \cdot, \cdot)$, the domain Ω, and the values Δ and Δ_α.

Remark 2.4. *The optimal numerical method for calculating the function $w = w(\widetilde{\alpha})$ and sufficient conditions for its convergence are unresolved issues and can be the subject of a separate theoretical study. For the practical calculation of the compensator vector w, the following algorithm can be proposed.*

1) As an initial approximation of the compensator vector w we choose $w_0 = \mathbf{0}$.

2) With each approximation w_k, $k = 0, 1, 2, \ldots$, we associated an error $\|x_k(t^*) - x^*\|$, where a grid function $x_k(t)$ is a numerical solution of the Cauchy problem

$$\begin{cases} \dot{x}_k(t) = f(t, x_k(t), \overline{u} + w_k, \widetilde{\alpha}), & t \in (t_*, t^*), \\ x_k(t_*) = x_*. \end{cases}$$

The solution of the Cauchy problem can be found by the Runge-Kutta method [14, Ch. 8, Sect. 2], an optimal order of which directly depends on the smoothness order of the function $f(\cdot,\cdot,\cdot)$ in the first and second variables.

3) The choice of next approximations w_k, $k = 1, 2\ldots$, can be made by the coordinate descent algorithm [14, Ch. 7, §3] by the corresponding error (or by simple enumeration from $B^p(\mathbf{0}, \rho(\Delta))$).

4) The stopping condition is a sufficiently small error $\|x_k(t^*) - x^*\|$.

However, since the stability of w with respect to a small change in x^* has not been studied, then, in fact, an addition to Condition E *is the assumption that we can calculate the function $w = w(\widetilde{\alpha})$ with a negligible error.*

Apart of Conditions A, B, C, D, E, we specify the information conditions under which the control of system (2.1) is made.

We assume that at the initial time t_0 the person making the choice of program control $u(t)$ is informed of some approximate value $\alpha^* \in \mathscr{L}$ of the parameter $\alpha \in \mathscr{L}$ with an error not exceeding

$$\|\alpha^* - \alpha\| < \delta_\alpha. \tag{2.2}$$

In addition, long before the initial moment t_0 of the motion, the control person knows the constraint itself, the compact set $\mathscr{L}$ and an approximate location $x^*(t_0)$ of the initial point $x(t_0)$ with an error

$$\|x^*(t_0) - x(t_0)\| < \delta_x. \tag{2.3}$$

An additional restriction is that the control person cannot perform "heavy" calculations after the initial moment t_0 of the motion, it is necessary to build and store in a limited amount of memory enabling program controls for all possible values of the indefinite constant parameter α in advance by having only information about $\mathscr{L}$ and $x^*(t_0)$.

Thus, we have stipulated the information conditions.

Let M be some compact set in $\mathbb{R}^n$, which is the target set for system (2.1). Let us formulate a problem of approaching M for system (2.1).

Problem 1. *To determine the existence of an admissible program control $u(t)$ that transfers the motion $x(t)$ of system (2.1) at time ϑ to a small neighborhood M, and, in the case of a positive answer, to construct it.*

3. Algorithm of solving problem on approaching 1

We denote by $\Omega^{(\delta)}(\cdot)$ a mapping, which "decimates" the set, that is, with each bounded set $A \subset \mathbb{R}^k$, $k \in \mathbb{N}$, it associates a finite set $\widetilde{A} = \Omega^{(\delta)}(A)$ consisting possibly from a smaller number of its points and having the property:

$$d(A, \widetilde{A}) \leqslant \delta,$$

where $d(A, \widetilde{A})$ is the Hausdorff distance between A and $\widetilde{A}$. The methods for constructing such "decimated" set $\widetilde{A}$ are given in [15].

We denote $\widetilde{P} = \Omega^{(\Delta_u)}(\check{P})$, where $\Delta_u > 0$ is a sufficiently small constant chosen by the condition of optimality between the accuracy and computational performance, $\check{P}$ is restriction of the control from Condition E.

We introduce a mapping $X^{(\Delta)} : \mathbb{R} \times \mathbb{R} \times 2^{\Omega} \times \mathscr{L} \mapsto 2^{\Omega}$ acting by rule:

$$X^{(\Delta)}(t^*, t_*, \widetilde{X}_*, \alpha) = \bigcup_{x \in \widetilde{X}_*} \{x + (t^* - t_*) f(t_*, x, \widetilde{P}, \alpha)\}$$
$$= \bigcup_{x \in \widetilde{X}_*} \bigcup_{u \in \widetilde{P}} \{x + (t^* - t_*) f(t_*, x, u, \alpha)\}.$$

After introducing the necessary notation, we formulate a numerical method for solving problem 1 in the form of two algorithms. The first algorithm is for computing produced before the system starts moving, and the second algorithm is applied directly during the movement.

Algorithm 3.1.

1) We choose a sufficiently large natural number N and introduce a uniform partition $\Gamma = \{t_0, t_1, t_2, \ldots, t_i, \ldots, t_N = \vartheta\}$ time interval $[t_0, \vartheta]$ with a diameter $\Delta = \Delta(\Gamma)$, which satisfies the relations

$$\Delta = t_{i+1} - t_i = N^{-1} \cdot (\vartheta - t_0), \qquad i = \overline{0, N-1}.$$

2) Denote by $\Delta_\alpha > 0$ a sufficiently small constant satisfying Condition E for $\Delta = \Delta(\Gamma)$. In addition, from Condition E we determine the function $\rho(\Delta)$ and the control restriction $\check{P} = \check{P}(\rho(\Delta))$.

3) As a finite subset of $\widetilde{\mathscr{L}} \subset \mathscr{L}$, we choose a set of vectors $\{\alpha^{(j)}\}_{j=1}^{N_\alpha}$ such that each $\alpha \in \mathscr{L}$ is inside "its" square $\alpha^{(j,-,-)}\alpha^{(j,-,+)}\alpha^{(j,+,-)}\alpha^{(j,+,+)}$ with four vertices $\alpha^{(j,\pm,\pm)} = (\alpha_1^{(j)} \pm \Delta_\alpha/2, \alpha_2^{(j)} \pm \Delta_\alpha/2)$.

4) We choose a sufficiently small constant $\Delta_x > 0$ and for all $j = \overline{1, N_\alpha}$ we define the sets

$$\widetilde{X}_0 = \{x^{(0)}\}, \qquad \widetilde{X}_k(\alpha^{(j)}) = \Omega^{(\Delta_x)}(X^{(\Delta)}(t_k, t_{k-1}, \widetilde{X}_{k-1}, \alpha^{(j)}), \qquad k = \overline{1, N}.$$

While constructing finite sets $\widetilde{X}_k(\alpha^{(j)})$, $k = \overline{1, N}$, $j = \overline{1, N_\alpha}$, for each point $\overline{x}^{(k,j)} \in \widetilde{X}_k(\alpha^{(j)})$ we remember a "parent" point $\overline{x}^{(k-1,j)} \in \widetilde{X}_{k-1}(\alpha^{(j)})$ and a control $\overline{u}^{(k,j)} = \mathrm{const}$, for which the relation holds:

$$\overline{x}^{(k,j)} = \overline{x}^{(k-1,j)} + \Delta \cdot f(t_{k-1}, \overline{x}^{(k-1,j)}, \overline{u}^{(k,j)}, \alpha^{(j)}).$$

5) If for all $\alpha^{(j)} \in \widetilde{\mathscr{L}}$ the distance

$$\rho(M, \widetilde{X}_N(\alpha^{(j)})) = \min\{\|x - y\| \,:\, x \in M,\ y \in \widetilde{X}_N(\alpha^{(j)})\} > \Delta_x,$$

we then conclude that it is impossible to construct the resolving program control mapping the motion of system (2.1) to the target set M at a moment ϑ with an appropriate accuracy by our method and we complete the solving the approach problem.

If for all $\alpha^{(j)} \in \widetilde{\mathscr{L}}$ the distance

$$\rho(M, \widetilde{X}_N(\alpha^{(j)})) = \min\{\|x - y\| \,:\, x \in M,\ y \in \widetilde{X}_N(\alpha^{(j)})\} \leqslant \Delta_x,$$

we then conclude that problem 1 is solvable for each $\alpha \in \mathscr{L}$ realized in the system.

If for some $\alpha^{(j)} \in \widetilde{\mathscr{L}}$

$$\rho(M, \widetilde{X}_N(\alpha^{(j)})) = \min\{\|x - y\| \,:\, x \in M,\ y \in \widetilde{X}_N(\alpha^{(j)})\} \leqslant \Delta_x,$$

while for other $\alpha^{(j)} \in \widetilde{\mathscr{L}}$ this inequality is not satisfied, then we can not provide a solution to problem 1 with an appropriate accuracy for the provided value of the parameter α.

6) For each $j = \overline{1, N_\alpha}$ we choose one point $\overline{x}^{(N,j)} \in \widetilde{X}_N(\alpha^{(j)})$ among the nearests to M. We can assume that if our algorithm is not over at Step 5), then $\rho(\overline{x}^{(N,j)}, M) \leqslant \Delta_x$, $j = \overline{1, N_\alpha}$. For each $j = \overline{1, N_\alpha}$ we denote by $\overline{x}^{(k,j)}$ and $\overline{u}^{(k,j)}$ exactly the points and constant controlling vectors, which led us to $\overline{x}^{(N,j)}$.

7) For each $j = \overline{1, N_\alpha}$ and $k = \overline{1, N}$, in accordance with Remark 2.4, we solve four boundary value problems for constant compensator vectors $w^{(k,j,\pm,\pm)} \in B^p(\mathbf{0}, \rho(\Delta))$ and absolutely continuous functions $x^{(k,j,\pm,\pm)}(t)$:

$$\begin{cases} \dot{x}^{(k,j,\pm,\pm)}(t) = f(t, x^{(k,j,\pm,\pm)}(t), \overline{u}^{(k,j)} + w^{(k,j,\pm,\pm)}, \alpha^{(j,\pm,\pm)}), & t \in (t_{k-1}, t_k), \\ x^{(k,j,\pm,\pm)}(t_{k-1}) = \overline{x}^{(k-1,j)},\ x^{(k,j,\pm,\pm)}(t_k) = \overline{x}^{(k,j)}. \end{cases}$$

8) With each $\alpha^{(j)} \in \widetilde{\mathscr{L}}$ *we associated four piece-wise constant "node" controls*

$$u^{(j,\pm,\pm)}(t) = \begin{cases} \overline{u}^{(1,j)} + \overline{w}^{(1,j,\pm,\pm)}, & t \in [t_0, t_1), \\ \dots & \\ \overline{u}^{(k,j)} + \overline{w}^{(k,j,\pm,\pm)}, & t \in [t_{k-1}, t_k), \\ \dots & \\ \overline{u}^{(N,j)} + \overline{w}^{(N,j,\pm,\pm)}, & t \in [t_{N-1}, t_N]. \end{cases} \tag{3.1}$$

Remark 3.1. *At Step 3) of algorithm 3.1, one can choose a slightly smaller set* $\widetilde{\mathscr{L}}$*, allowing moreover some* $\alpha \in \mathscr{L}$ *points to be located slightly outside "their" squares with vertices* $\alpha^{(j,-,-)}$, $\alpha^{(j,-,+)}$, $\alpha^{(j,+,-)}$ *and* $\alpha^{(j,+,+)}$. *However, in doing so, we must require that, first, Condition* E *be satisfied in the same neighborhood of the square* $\alpha^{(j,-,-)}\alpha^{(j,-,+)}\alpha^{(j,+,-)}\alpha^{(j,+,+)}$, *and secondly, for such* $\alpha = c_1c_2\alpha^{(j,-,-,)} + (1-c_1)c_2\alpha^{(j,+,-,)} + c_1(1-c_2)\alpha^{(j,-,+,)} + (1-c_1)(1-c_2)\alpha^{(j,+,+,)}$ *all inclusions*

$$c_1c_2w^{(k,j,-,-,)} + (1-c_1)c_2w^{(k,j,+,-,)} + c_1(1-c_2)w^{(k,j,-,+,)} + (1-c_1)(1-c_2)w^{(k,j,+,+,)} \in B^p(\mathbf{0}, \rho(\Delta))$$

to be true for $k = \overline{1, N}$.

Algorithm 3.2.

1) By the approximate value α^* *obtained at the moment* t_0*, we determine the corresponding* $\alpha^{(j)} \in \widetilde{\mathscr{L}}$ *according to the distribution that was defined at Step 3) of Algorithm 3.1.*

2) We represent the vector α^* *as a linear combination of the vectors* $\alpha^{(j,-,-,)}$, $\alpha^{(j,-,+,)}$, $\alpha^{(j,+,-,)}$, $\alpha^{(j,+,+,)}$ *as follows:*

$$\alpha^* = c_1c_2\alpha^{(j,-,-,)} + (1-c_1)c_2\alpha^{(j,+,-,)} + c_1(1-c_2)\alpha^{(j,-,+,)} + (1-c_1)(1-c_2)\alpha^{(j,+,+,)},$$

where $0 \leqslant c_1 \leqslant 1$, $0 \leqslant c_2 \leqslant 1$, *except for the case described in Remark 3.1.*

3) As the initial resolving program control we use the function

$$\hat{u}(t) = c_1c_2u^{(j,-,-,)}(t) + (1-c_1)c_2u^{(j,+,-,)}(t) + c_1(1-c_2)u^{(j,-,+,)}(t) + (1-c_1)(1-c_2)u^{(j,+,+,)}(t). \tag{3.2}$$

4. Estimate for error

Lemma 4.1. *Assume that we are given constants* $c_1, c_2 \in \mathbb{R}$, *points* $P_1 = (x_1, y_1)$ *and* $P_2 = (x_2, y_2)$ *and* $\mathbb{R}^2$, *and a function* $f : \mathbb{R}^2 \to \mathbb{R}^n$, $f \in C^2(\mathbb{R}^2)$, *all second derivatives of which are bounded by some constant* $m_2 > 0$, *that is,*

$$\left\|\frac{\partial^2 f(x,y)}{\partial x^2}\right\| \leqslant m_2, \qquad \left\|\frac{\partial^2 f(x,y)}{\partial y^2}\right\| \leqslant m_2, \qquad \left\|\frac{\partial^2 f(x,y)}{\partial x \partial y}\right\| \leqslant m_2.$$

Then

$$\begin{aligned} \big\| f(c_1x_1 + (1-c_1)x_2, c_2y_1 + (1-c_2)y_2) - c_1c_2f(x_1,y_1) - (1-c_1)c_2f(x_2,y_1) \\ - c_1(1-c_2)f(x_1,y_2) - (1-c_1)(1-c_2)f(x_2,y_2)\big\| \\ \leqslant \frac{3}{2}\big|c_2(1-c_2)\big|m_2(y_2-y_1)^2 \\ + \frac{3}{2}\big|c_1(1-c_1)c_2\big|m_2(x_2-x_1)^2 \\ + \frac{3}{2}\big|c_1(1-c_1)(1-c_2)\big|m_2(x_2-x_1)^2. \end{aligned}$$

Proof. We expand the function $f(\xi, \eta)$ into the Taylor series with respect to the first variable at the points x_1 and x_2 with the remainder in an integral form and substitute $\xi = c_1x_1 + (1-c_1)x_2$ into these expansions. This gives the identities

$$f(c_1x_1 + (1-c_1)x_2, \eta) = f(x_1 + (1-c_1)(x_2-x_1), \eta)$$

$$
\begin{aligned}
&= f(x_1,\eta) + (1-c_1)(x_2-x_1)\cdot\frac{\partial f(x_1,\eta)}{\partial x_1} \\
&\quad + \int\limits_{x_1}^{c_1x_1+(1-c_1)x_2} (c_1x_1+(1-c_1)x_2-t)\frac{\partial^2 f(t,\eta)}{\partial t^2}\,dt, \\
f(c_1x_1+(1-c_1)x_2,\eta) &= f(x_2+c_1(x_1-x_2),\eta) \\
&= f(x_2,\eta)+c_1(x_1-x_2)\frac{\partial f(x_2,\eta)}{\partial x_2} \\
&\quad + \int\limits_{x_2}^{c_1x_1+(1-c_1)x_2} (c_1x_1+(1-c_1)x_2-t)\frac{\partial^2 f(t,\eta)}{\partial t^2}\,dt,
\end{aligned}
$$

which imply

$$
\begin{aligned}
&\big|f(c_1x_1+(1-c_1)x_2,\eta)-c_1f(x_1,\eta)-(1-c_1)f(x_2,\eta)\big| \\
&\quad = \big|c_1\big(f(x_1+(1-c_1)(x_2-x_1),\eta)-f(x_1,\eta)\big)+(1-c_1)\big(f(x_2+c_1(x_1-x_2))f(x_2,\eta)\big)\big| \\
&\quad = \Bigg|c_1(1-c_1)(x_2-x_1)\int\limits_{x_1}^{x_2}\frac{\partial^2 f(t,\eta)}{\partial t^2}dt + c_1\int\limits_{x_1}^{c_1x_1+(1-c_1)x_2}(c_1x_1+(1-c_1)x_2-t)\frac{\partial^2 f(t,\eta)}{\partial t^2}dt \\
&\qquad + (1-c_1)\int\limits_{x_2}^{c_1x_1+(1-c_1)x_2}(c_1x_1+(1-c_1)x_2-t)\frac{\partial^2 f(t,\eta)}{\partial t^2}dt\Bigg| \\
&\quad \leqslant \left|c_1(1-c_1)m_2(x_2-x_1)^2+c_1m_2\frac{(1-c_1)^2(x_2-x_1)^2}{2}+(1-c_1)m_2\frac{c_1^2(x_2-x_1)^2}{2}\right| \\
&\quad = \frac{3}{2}|c_1(1-c_1)|m_2(x_2-x_1)^2.
\end{aligned}
$$

Substituting $\eta = y_1$ and $\eta = y_2$ into this inequality, we get:

$$
\big\|f(c_1x_1+(1-c_1)x_2,y_1)-c_1f(x_1,y_1)-(1-c_1)f(x_2,y_1)\big\| \leqslant \frac{3}{2}|c_1(1-c_1)|m_2(x_2-x_1)^2, \tag{4.1}
$$

$$
\big\|f(c_1x_1+(1-c_1)x_2,y_2)-c_1f(x_1,y_2)-(1-c_1)f(x_2,y_2)\big\| \leqslant \frac{3}{2}|c_1(1-c_1)|m_2(x_2-x_1)^2. \tag{4.2}
$$

Similarly, expanding the function $f(c_1x_1+(1-c_1)x_2,\eta)$ with respect to the second variable and substituting $\eta = c_2y_1+(1-c_2)y_2$, we obtain the inequality

$$
\begin{aligned}
&\big\|f(c_1x_1+(1-c_1)x_2,c_2y_1+(1-c_2)y_2) \\
&\qquad - c_2f(c_1x_1+(1-c_1)x_2,y_1)-(1-c_2)f(c_1x_1+(1-c_1)x_2,y_2)\big\| \\
&\quad \leqslant \frac{3}{2}|c_2(1-c_2)|m_2(y_2-y_1)^2.
\end{aligned} \tag{4.3}
$$

Using inequalities (4.1), (4.2) and (4.3), we estimate the difference

$$
\begin{aligned}
&\big\|f(c_1x_1+(1-c_1)x_2,c_2y_1+(1-c_2)y_2) \\
&\qquad - c_1c_2f(x_1,y_1)-(1-c_1)c_2f(x_2,y_1)-c_1(1-c_2)f(x_1,y_2)-(1-c_1)(1-c_2)f(x_2,y_2)\big\| \\
&\quad \leqslant \big\|f(c_1x_1+(1-c_1)x_2,c_2y_1+(1-c_2)y_2)-c_2f(c_1x_1+(1-c_1)x_2,y_1) \\
&\qquad - (1-c_2)f(c_1x_1+(1-c_1)x_2,y_2)\big\| \\
&\qquad + \big\|c_2f(c_1x_1+(1-c_1)x_2,y_1)-c_1c_2f(x_1,y_1)-(1-c_1)c_2f(x_2,y_1)\big\|
\end{aligned}
$$

$$+\big\|(1-c_2)f(c_1x_1+(1-c_1)x_2,y_2)-c_1(1-c_2)f(x_1,y_2)-(1-c_1)(1-c_2)f(x_2,y_2)\big\|$$
$$\leqslant \frac{3}{2}\big|c_2(1-c_2)\big|m_2(y_2-y_1)^2$$
$$+\frac{3}{2}\big|c_1(1-c_1)c_2\big|m_2(x_2-x_1)^2+\frac{3}{2}\big|c_1(1-c_1)(1-c_2)\big|m_2(x_2-x_1)^2.$$

This completes the proof. □

Corollary 4.1. *If, under the assumption of Lemma 4.1, we additionally restrict* $0\leqslant c_1\leqslant 1$, $0\leqslant c_2\leqslant 1$, *then*

$$\big\|f(c_1x_1+(1-c_1)x_2,c_2y_1+(1-c_2)y_2)-c_1c_2f(x_1,y_1)-(1-c_1)c_2f(x_2,y_1)$$
$$-c_1(1-c_2)f(x_1,y_2)-(1-c_1)(1-c_2)f(x_2,y_2)\big\|\leqslant \frac{3}{8}m_2\|P_2-P_1\|^2.$$

Theorem 4.1. *Let system (2.1) satisfy Conditions* A, B, C, D, E, *and let it be controlled under the information conditions listed in Section 2. And let, while solving problem 1 at Step 5) of Algorithm 3.1, the existence of an admissible resolving control is established, and then, using Algorithm 3.2, program control* $\hat{u}(t)$ *generating motion* $\hat{x}(t)$ *is constructed. Then*

$$\rho(\hat{x}(\vartheta),M)\leqslant \Delta_x+\delta_x e^{L(\vartheta-t_0)}+\frac{\omega^{(3)}\Big(\frac{3}{8}M_2\Delta_\alpha^2\Big)+\omega^{(4)}(\delta_\alpha)}{L}\big(e^{L(\vartheta-t_0)}-1\big).$$

Proof. In accordance with Step 3) of Algorithm 3.1 there exists a number $j\in\{1,\dots,N_\alpha\}$ such that

$$\alpha^*=c_1c_2\alpha^{(j,-,-,)}+(1-c_1)c_2\alpha^{(j,+,-,)}+c_1(1-c_2)\alpha^{(j,-,+,)}+(1-c_1)(1-c_2)\alpha^{(j,+,+,)},$$

where $0\leqslant c_1\leqslant 1$, $0\leqslant c_2\leqslant 1$.

By $\hat{x}(t)$ we denote the motion of system (2.1) corresponding to the control

$$\hat{u}(t)=c_1c_2u^{(j,-,-,)}(t)+(1-c_1)c_2u^{(j,+,-,)}(t)+c_1(1-c_2)u^{(j,-,+,)}(t)+(1-c_1)(1-c_2)u^{(j,+,+,)}(t),$$

exact value of the parameter α and initial state $x(t_0)$. We note that under our notations $\hat{x}(t_0)=x(t_0)$ is the exact initial state of system.

By construction

$$\rho(\overline{x}(\vartheta),M)\leqslant \Delta_x. \tag{4.4}$$

Under our notation estimate (2.3) is

$$\|\hat{x}(t_0)-\overline{x}(t_0)\|=\|x(t_0)-x^*(t_0)\|\leqslant \delta_x. \tag{4.5}$$

By Condition E there exists some ideal compensator vector $\overline{w}^{(1,j)}\in B^p(\mathbf{0},\rho(\Delta))$ such that the state of system $\overline{x}(t_0)$ under the action of a constant control $\overline{u}^{(1,j)}+\overline{w}^{(1,j)}$ on the segment $[t_0,t_1)$ and under the parameter α^* is moved to the point $\overline{x}(t_1)$ over some trajectory $\overline{x}(t)$. For further purposes, by $\overline{x}(t)$ we denote entire trajectory of system (2.1) passing through the points $\overline{x}(t_0)$, $\overline{x}(t_1)$, …, $\overline{x}(t_N)$ under the action of piece-wise constant control $\overline{u}(t)=\overline{u}^{(k,j)}+\overline{w}^{(k,j)}$, $t\in[t_{k-1},t_k)$, $k=\overline{1,N}$.

However, according to Algorithm 3.2, instead of the ideal compensator vector $\overline{w}^{(1,j)}$ we use the compensator vector

$$\hat{w}^{(1,j)}=c_1c_2w^{(1,j,-,-,)}+(1-c_1)c_2w^{(1,j,+,-,)}+c_1(1-c_2)w^{(1,j,-,+,)}+(1-c_1)(1-c_2)w^{(1,j,+,+,)}.$$

Since it is a convex combination $w^{(1,j,-,-)}$, $w^{(1,j,-,+)}$, $w^{(1,j,+,-)}$ and $w^{(1,j,+,+)}$, it also belongs to $B^p(\mathbf{0},\rho(\Delta))$. By Corollary 4.1, the estimate holds:

$$\|\hat{w}^{(1,j)}-\overline{w}^{(1,j)}\|\leqslant \frac{3}{8}M_2\Delta_\alpha^2.$$

Since, according to formulas (3.1) and (3.2),

$$\hat{u}(t) = \overline{u}^{(1,j)} + \hat{w}^{(1,j)}, \qquad t \in [t_0, t_1),$$

then

$$\|\hat{u}(t) - \overline{u}(t)\| = \|\hat{w}^{(1,j)} - \overline{w}^{(1,j)}\| \leqslant \frac{3}{8} M_2 \Delta_\alpha^2, \qquad t \in [t_0, t_1).$$

Similarly, for each $k = \overline{1, N}$ by Condition E there exists some ideal compensator vector $\overline{w}^{(k,j)} \in B^p(\mathbf{0}, \rho(\Delta))$ such that the state of system $\overline{x}(t_{k-1})$ under the action of a constant control $\overline{u}(t) = \overline{u}^{(k,j)} + \overline{w}^{(k,j)}$ on the segment $[t_{k-1}, t_k)$ and under $\alpha = \alpha^*$ maps the point $\overline{x}(t_k)$ along the trajectory $\overline{x}(t)$. However, by Algorithm 3.2 on the segment $[t_{k-1}, t_k)$ we use the control

$$\begin{aligned}\hat{u}(t) = \overline{v}^{(k,j)} + \hat{w}^{(k,j)} =& \overline{v}^{(k,j)} + c_1 c_2 w^{(1,j,-,-,)} + (1 - c_1) c_2 w^{(1,j,+,-,)} \\ &+ c_1 (1 - c_2) w^{(1,j,-,+,)} + (1 - c_1)(1 - c_2) w^{(1,j,+,+,)},\end{aligned}$$

for which, by Corollary 4.1, the estimate holds:

$$\|\hat{u}(t) - \overline{u}(t)\| = \|\hat{w}^{(k,j)} - \overline{w}^{(k,j)}\| \leqslant \frac{3}{8} M_2 \Delta^2, \qquad t \in [t_{k-1}, t_k), \qquad k = \overline{1, N}. \tag{4.6}$$

In other words, estimate (4.6) holds on entire segment $[t_0, \vartheta]$. We also recall that the value α^* of the parameter α is known with the accuracy δ_α, see (2.2).

Thus, taking into consideration (4.5), (4.6) and (2.2) for $t \in [t_0, \vartheta]$ we obtain the following integral estimate for the mismatch of movements:

$$\begin{aligned}\|\hat{x}(t) - \overline{x}(t)\| \leqslant& \left\| \hat{x}(t_0) + \int_{t_0}^{t} f(\tau, \hat{x}(\tau), \hat{u}(\tau), \alpha)\, d\tau - \overline{x}(t_0) - \int_{t_0}^{t} f(\tau, \overline{x}(\tau), \overline{u}_j(\tau), \alpha^*)\, d\tau \right\| \\ \leqslant& \|\hat{x}(t_0) - \overline{x}(t_0)\| + \int_{t_0}^{t} \big(\|f(\tau, \hat{x}(\tau), \hat{u}(\tau), \alpha) - f(\tau, \overline{x}(\tau), \hat{u}(\tau), \alpha) \\ &+ f(\tau, \overline{x}(\tau), \hat{u}(\tau), \alpha) - f(\tau, \overline{x}(\tau), \overline{u}(\tau), \alpha) \\ &+ f(\tau, \overline{x}(\tau), \overline{u}(\tau), \alpha) - f(\tau, \overline{x}(\tau), \overline{u}(\tau), \alpha^*)\| \big)\, d\tau \\ \leqslant& \delta_x + \int_{t_0}^{t} L \|\hat{x}(\tau) - \overline{x}(\tau)\|\, d\tau + \int_{t_0}^{t} \omega^{(3)}\big(\hat{u}(\tau) - \overline{u}(\tau)\big)\, d\tau + \int_{t_0}^{t} \omega^{(4)}(\alpha - \alpha^*)\, d\tau \\ \leqslant& \delta_x + L \int_{t_0}^{t} \|\hat{x}(\tau) - \overline{x}(\tau)\|\, d\tau + (t - t_0) \cdot \omega^{(3)}\Big(\frac{3}{8} M_2 \Delta_\alpha^2\Big) + (t - t_0) \cdot \omega^{(4)}(\delta_\alpha).\end{aligned}$$

This by a strong Grönwall lemma [16, Ch. 1, Sect. 2] implies that

$$\|\hat{x}(\vartheta) - \overline{x}(\vartheta)\| \leqslant \delta_x e^{L(\vartheta - t_0)} + \frac{\omega^{(3)}\Big(\frac{3}{8} M_2 \Delta_\alpha^2\Big) + \omega^{(4)}(\delta_\alpha)}{L} \big(e^{L(\vartheta - t_0)} - 1\big). \tag{4.7}$$

Relations (4.4) and (4.7) imply the statement of the theorem. □

Remark 4.1. *For modified Algorithm 3.1, in accordance with Remark 3.1 and using Lemma 4.1 we can obtain a weaker estimate*

$$\begin{aligned}&\rho(\hat{x}(\vartheta), M) \leqslant \Delta_x + \delta_x e^{L(\vartheta - t_0)} \\ &+ \frac{e^{L(\vartheta - t_0)} - 1}{L} \Big(\omega^{(3)}\Big(\frac{3}{2} \big(|c_2(1 - c_2)| + |c_1(1 - c_1) c_2| + |c_1(1 - c_1)(1 - c_2)| \big) M_2 \Delta_\alpha^2 \Big) + \omega^{(4)}(\delta_\alpha) \Big).\end{aligned}$$

5. Example

As an example we consider a nonlinear generalization of navigation Zermelo problem. On the segment $[t_0, \vartheta] = [0, 2]$ we consider a controlled problem

$$\begin{cases} \dot{x}(t) = s(x(t)) + u(t), \quad t \in (0,2), \\ x(0) = x^{(0)} = (0,0), \end{cases} \tag{5.1}$$

where t is the time, $x(t) = (x_1(t), x_2(t))$ is the phase variable, $s(x) = (s_1(x), s_2(x)) = \big(\sin(x_2), \cos(x_1)\big)$ is a vector function, $x^{(0)}$ is the initial state of system, $u(t)$ is a Lebesgue measurable vector function with values in the circle $P = \{u = (u_1, u_2) \,:\, u_1^2 + u_2^2 \leqslant 1\}$, which is a vector of controlling actions.

The problem is to quickly present a program control $u(t)$, $t \in [t_0, \vartheta]$, which would translate the motion $x(t)$ of controlled system (5.1) from the initial point $x^{(0)}$ to a small neighborhood of the point $x^{(f)}$, the coordinates of which are given to us at the initial moment $t_0 = 0$. However, we know in advance that $x^{(f)}$ belongs to the circle $B^2\Big((1,2), \dfrac{1}{2}\Big)$.

In this section, we demonstrate the operation of Algorithms 3.1 and 3.2 for solving our problem and model the accuracy of hitting the system motion (5.1) the target set at the final time $\vartheta = 2$. For simplicity, all measurements are considered to be exact, that is, $\delta_\alpha = 0$ and $\delta_x = 0$.

Thus, to move the uncertainty in the target set into an unknown parameter, we introduce a constant parameter $\alpha = (\alpha_1, \alpha_2) = \Big(\dfrac{1}{2}(x_1^{(f)} - 1), \dfrac{1}{2}(x_2^{(f)} - 2)\Big)$ and make the change of the phase variable $\xi = x - t\alpha$. After such change of variables system (5.1) becomes

$$\begin{cases} \dot{\xi}(t) = -\alpha + s\big(\xi(t) + t\alpha\big) + u(t), \qquad t \in (0,2), \\ \xi(0) = x^{(0)} = (0,0). \end{cases} \tag{5.2}$$

Here the target set is the point $\xi^{(f)} = (1,2)$, while the unknown vector parameter α takes an arbitrary value in $\mathscr{L} = B^2\Big((0,0), \dfrac{1}{4}\Big)$.

For such transformed problem we execute Algorithm 3.1.

1) We choose $N = 2$, then $\Gamma = \{t_0 = 0, t_1 = 1, t_0 = 2\}$.

2) We denote $\Delta_\alpha = \dfrac{\sqrt{2}}{8}$. We define with a margin $\rho(\Delta) = \dfrac{3}{4}$, then $\check{P}(\rho(\Delta)) = B^2\Big((0,0), \dfrac{1}{4}\Big)$.

3) We choose $\widetilde{\mathscr{L}} = \{\alpha^{(1)} = (0,0)\}$. Respectively, $\alpha^{(1,\pm,\pm)} = \Big(\pm\dfrac{\sqrt{2}}{8}, \pm\dfrac{\sqrt{2}}{8}\Big)$.

4) We choose $\Delta_x = \dfrac{\sqrt{2}}{50}$ and construct the approximations of the target sets $\widetilde{X}_1(\alpha^{(1)})$ and $\widetilde{X}_2(\alpha^{(1)})$, Figure 1.

5) Since $\rho(\xi^{(f)}, \widetilde{X}_2(\alpha^{(1)})) < \Delta_x$, we continue the execution of Algorithm 3.1.

6) We choose $\overline{\xi}^{(2,1)} = (1.0015, 2)$, a nearest to $\xi^{(f)}$ in $\widetilde{X}_2(\alpha^{(1)})$. We observe that $\overline{\xi}^{(0)} = (0,0)$, $\overline{u}^{(1,1)} = (0,0)$, $\overline{\xi}^{(1,1)} = (0,1)$, $\overline{u}^{(2,1)} = (0.16, 0)$.

7) For each $k = 1,2$ and $j = 1$, in accordance with Remark 2.4, we solve four problems

$$\begin{cases} \dot{\xi}^{(k,j,\pm,\pm)}(t) = f(t, \xi^{(k,j,\pm,\pm)}(t), \overline{u}^{(k,j)} + w^{(k,j,\pm,\pm)}, \alpha^{(j,\pm,\pm)}), \quad t \in (t_{k-1}, t_k), \\ \xi^{(k,j,\pm,\pm)}(t_{k-1}) = \overline{\xi}^{(k-1,j)}, \qquad x^{(k,j,\pm,\pm)}(t_k) = \overline{\xi}^{(k,j)}. \end{cases}$$

Their solutions are

$$w^{(1,1,-,+)} = (-0.70185, 0.19243), \qquad w^{(1,1,+,+)} = (-0.3472, 0.17898),$$
$$w^{(1,1,-,-)} = (-0.56727, -0.16393), \qquad w^{(1,1,+,-)} = (-0.21275, -0.17465),$$

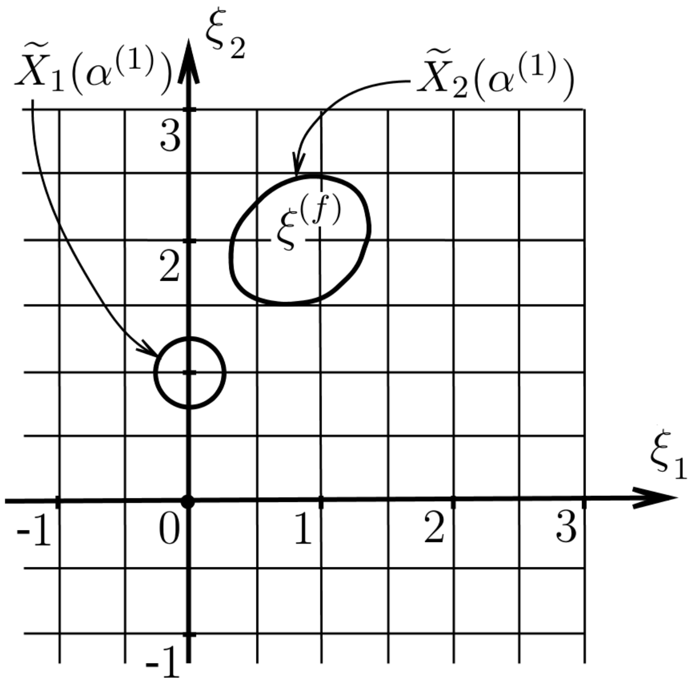

FIGURE 1. Approximations of target sets $\widetilde{X}_1(\alpha^{(1)})$ and $\widetilde{X}_2(\alpha^{(1)})$.

$$w^{(2,1,-,+)} = (-0.09795, 0.23763), \qquad w^{(2,1,+,+)} = (0.26776, 0.5129),$$
$$w^{(2,1,-,-)} = (-0.09828, -0.12595), \qquad w^{(2,1,+,-)} = (0.24169, 0.13066).$$

8) By formula (3.1) we find four node controls

$$u^{(1,-,-)}(t) = \begin{cases} (-0.56727, -0.16393), & t \in [0,1), \\ (0.06172, -0.12595), & t \in [1,2], \end{cases}$$
$$u^{(1,-,+)}(t) = \begin{cases} (-0.70185, 0.19243), & t \in [0,1), \\ (0.06205, 0.23763), & t \in [1,2], \end{cases}$$
$$u^{(1,+,-)}(t) = \begin{cases} (-0.21275, -0.17465), & t \in [0,1), \\ (0.40169, 0.13066), & t \in [1,2], \end{cases}$$
$$u^{(1,+,+)}(t) = \begin{cases} (-0.3472, 0.17898), & t \in [0,1), \\ (0.42776, 0.5129), & t \in [1,2]. \end{cases}$$

Thus, the execution of Algorithm 3.1 is over.

Then suppose that at the initial time moment $t_0 = 0$ we are reported the coordinates of the target $x^{(f)} = \Big(1, \dfrac{5}{2}\Big)$. These coordinates are associated with the parameter

$$\alpha^* = (\alpha_1^*, \alpha_2^*) = \Big(\frac{1}{2}(x_1^{(f)} - 1), \frac{1}{2}(x_2^{(f)} - 2)\Big) = \Big(0, \frac{1}{4}\Big).$$

To solve the approach problem we execute Algorithm 3.2.

1) By a given α^* we choose a unique $\alpha^{(1)} = (0,0)$.

2) In accordance with Remark 3.1 we represent as a non-convex linear combination

$$\alpha^* = \frac{1+\sqrt{2}}{2}\Big(\frac{1}{2}\alpha^{(1,-,+)} + \frac{1}{2}\alpha^{(1,+,+)}\Big) + \frac{1-\sqrt{2}}{2}\Big(\frac{1}{2}\alpha^{(1,-,-)} + \frac{1}{2}\alpha^{(1,+,-)}\Big).$$

3) As a sought resolving control we use the function

$$\hat{u}(t) = \frac{1+\sqrt{2}}{4}u^{(1,-,+)}(t) + \frac{1+\sqrt{2}}{4}u^{(1,+,+)}(t) + \frac{1-\sqrt{2}}{4}u^{(1,-,-)}(t) + \frac{1-\sqrt{2}}{4}u^{(1,+,-)}(t)$$
$$= \begin{cases} (-0.55238, 0.25923), & t \in [0,1), \\ (0.24764, 0.45250), & t \in [1,2]. \end{cases}$$

The found program control moves system (2.1) to the point $x_*^{(f)} = (1.122, 2.492)$.

The error obtained in the example is very small relative to $\delta = 1$ and $\delta_\alpha = \sqrt{2}/8$, especially in view of the fact that we used a non-convex linear combination for interpolation. Its value indicates the possible fulfillment a part of Condition E on the smoothness of the dependence of the ideal vector-compensator on the parameter α.

6. Conclusion

For simplicity of presentation, in this article we consider the case of a two-dimensional parameter, however, there are no fundamental differences from the same problem with a constant vector parameter of an arbitrary dimension, except for certain technical difficulties in formulating and proving an analogue of Lemma 4.1, although the estimate from Corollary 4.1 is valid for any dimension. Compared to [11], a significant simplification of the algorithm is made: in the new algorithm a "reverse" time is not introduced and an additional "node" resolving controls are not built for points inside the cells of partition of the set of possible values of the parameter.

We note that an inconveniently large norm of the compensator vectors $w^{(1,1,-,+)}$ and $w^{(1,1,+,+)}$ in the analyzed example is related to the large the distance from the point $\overline{\xi}^{(1,1)}$ to the real motion $\xi(1)$ of system (5.2), which took too much control resources to overcome. The solution to this problem can be in replacing the Euler method by the Runge-Kutta method [17] when constructing the points $\overline{x}^{(k,j)}$, as well as to reduce Δ. However, a more significant problem is unverifiable Condition E in the current formulation. In this regard, further research can be aimed on finding sufficient conditions that replace Condition E, and examples of controlled systems for which Condition E is surely satisfied. It is also necessary to explore numerical methods for calculating functions $w = w(\widetilde{\alpha})$ in Remark 2.4.

BIBLIOGRAPHY

1. E.B. Lee, L. Markus. *Foundations of optimal control theory.* Wiley, New York (1967).
2. N.N. Krasovskij. *Game problems on the encounter of motions.* Nauka, Moscow (1970). (in Russian).
3. N.N. Krasovskij, A.I. Subbotin. *Game-theoretical control problems.* Nauka, Moscow (1974). [Springer-Verlag, New York (1988).]
4. V.M. Veliov. *Parametric and functional uncertainties in dynamic systems local and global relationship* // in Proc. 3rd Int. IMACS-GAMM Symposium on Computer Arithmetic and Enclosure Methods, North–Holland, Amsterdam, 427–436 (1992).
5. A.B. Kurzhansky. *Control and observation under uncertainty.* Nauka, Moscow (1977). (in Russian).
6. A.A. Ershov, V.N. Ushakov. *An approach problem for a control system with an unknown parameter* // Matem. Sborn. **208**:9, 56–99 (2017). [Sb. Math. **208**:9, 1312–1352 (2017).]
7. V.N. Ushakov, A.A. Ershov, A.V. Ushakov. *An approach problem with an unknown parameter and inaccurately measured motion of the system* // IFAC-PapersOnLine **51**:32, 234–238 (2018).
8. M.S. Nikol'skii. *A control problem with a partially known initial condition* // Prikl. Matem. Inform. **51**, 16–23 (2016). [Comput. Math. Model. **28**:1, 12–17 (2017).]
9. S.S. Lemak. *Formation of positional strategies for a differential game in Krasovskii's method of extremal aiming* // Vestn. Mosk. Univ. Ser. 1. Matem. Mekh. 6, 61–65 (2015). [Moscow Univ. Mech. Bull. **70**:6, 157–160 (2015).]

10. V.N. Ushakov, A.R. Matviychuk, G.V. Parshikov. *A method for constructing a resolving control in an approach problem based on attraction to the solvability set* // Trudy IMM UrO RAN. **19**:2, 275–284 (2013). [Proc. Steklov Inst. Math. **284**. Suppl. 1, 135–144 (2014).]
11. A.A. Ershov. *Linear parameter interpolation of a program control in the approach problem* // Probl. Matem. Anal. **113**, 17–27 (2022). [J. Math. Sci. **260**:6, 725–737 (2022).]
12. A. Bressan, B. Piccoli. *Introduction to the mathematical theory of control.* Amer. Inst. Math. Sci., New York (2007).
13. S.G. Mikhlin. *Mathematical physics, an advanced course.* Nauka, Moscow (1968). [North-Holland Publ. Co., Amsterdam (1970).]
14. N.S. Bakhvalov, N.P. Zhidkov, G.M. Kobel'kov. *Numerical methods.* Nauka, Moscow (1987). (in Russian).
15. V.N. Ushakov, A.A. Ershov. On the solution of control problems with fixed terminal time // Vestn. Udmurt. Univ. Mat. Mekh. Komp'yut. Nauki. **26**:4, 543–564 (2016). (in Russian).
16. P.I. Lizorkin. *Course of differential and integral equations with additional chapters of analysis.* Nauka, Moscow (1981). (in Russian).
17. A.O. Novikova. *Construction of reachibility sets for two-dimensional nonlinear controlled system by pixel method* // Prikl. Matem. Inform. Trudy VMK MGU. **50**, 62–82 (2015). (in Russian).

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