

## ABSTRACTS

**M.Ju. Balakhnev**

ON THE SYMMETRY CLASSIFICATION  
OF INTEGRABLE EVOLUTION EQUATIONS OF THE 3RD ORDER

**Abstract.** We present new results in the framework of symmetry classification of integrable evolution vector equations of the 3rd order. A technique proposed by G.A. Meshkov and V.V. Sokolov allowed us to find 12 equations satisfying the necessary integrability conditions. We provide a short review of all known nowadays equations of the considered type and also clarify all computational difficulties not allowing us to complete the classification problem in the general form.

By imposing reasonable additional restrictions for the form of equations while classifying them we succeed to complete the calculations. The found equations possess several nontrivial preserved densities and they are likely exactly integrable. As the proof of their integrability, the Lax representation or Backlund autotransform could serve but to find them is a rather complicated problem requiring a sufficient motivation, for instance, an application value of some of these equations.

**Keywords:** integrable vector equations, canonical densities, conservation laws.

**D.A. Baranov, E.V. Nozdrinova, O.V. Pochinka**

SCENARIO OF A STABLE TRANSITION FROM THE TORUS  
ISOTOPIC IDENTITY DIFFEOMORPHISM TO THE SKEW PRODUCT  
OF ROUGH TRANSFORMATIONS OF THE CIRCLE

**Abstract.** In this paper, we consider gradient-like diffeomorphisms of a two-dimensional torus isotopic to the identical one. The isotopicity of diffeomorphisms  $f_0, f_1$  given on  $n$ -manifold  $M^n$  means the existence of some arc  $\{f_t : M^n \rightarrow M^n, t \in [0, 1]\}$  connecting them in the space of diffeomorphisms. If isotopic diffeomorphisms are structurally stable (qualitatively not changing their properties with small perturbations), then it is natural to expect the existence of a stable arc (qualitatively not changing its properties with small movements) connecting them. In this case, one says that the isotopic diffeomorphisms  $f_0, f_1$  are stably isotopic or belong to the same class of stable isotopic connectivity. The simplest structurally stable diffeomorphisms on surfaces are gradient-like transformations having a finite hyperbolic non-wandering set, stable and unstable manifolds of various saddle points of which do not intersect. However, even on a two-dimensional sphere, where all orientation-preserving diffeomorphisms are isotopic, gradient-like diffeomorphisms are generally not stably isotopic. The countable number of pairwise different classes of stable isotopic connectivity is constructed on the base of a rough transformation of the circle  $\varphi_{\frac{k}{m}}$  with exactly two periodic orbits of the period  $m$  and the rotation number  $\frac{k}{m}$ , which can be extended to the diffeomorphism  $F_{\frac{k}{m}} : \mathbb{S}^2 \rightarrow \mathbb{S}^2$ , having two fixed sources at the north and south poles. On the torus  $\mathbb{T}^2$ , the model representative

in the considered class is the skew products of rough transformations of a circle. We show that any isotopic gradient-like diffeomorphism of a torus is connected by a stable arc with some model transformation.

**Keywords:** diffeomorphisms, torus, stable arcs.

## S.V. Grishin

### RANDOM WALKS ON A LINE AND ALGEBRAIC CURVES

**Abstract.** This work is devoted to the studying the generating function of the first hitting time of the positive semi-axis under the homogeneous discrete integer random walk on a line. In the first part of the work the increments are supposed to be independent. Recurrent relations for the probabilities allow us to write the system of equations for the sought generating function. Applying the resultants technique, we succeed to reduce this system to a single equation. Then we can study it by calculating the genus of the corresponding planar algebraic curve via analyzing its singularities. In the work we write the sought equations for some random walks and we show that if the increments take all integer values from  $-2$  to  $2$ , or from  $-1$  to  $3$  with equal probabilities or they take equally probable values  $-1$  and  $4$ , then the curve is rational, while this is not true in the first case.

In the second part of the work we consider a symmetric process, the increments take the values  $-1, 0, 1$ , but then we suppose a non-zero correlation of each next increment with the previous one. For such process the equation for the generating function defines an elliptic curve depending on the square of the correlation coefficient for neighbouring increments if all increments are non-zero and it defines a hyperelliptic curve of genus 2. The degeneration criterion of the latter is the presence of multiple roots of a sixth order polynomial under general symmetrically distributed conditional probabilities.

**Keywords:** random walks, algebraic curves.

## M.B. Zvereva, M.I. Kamenskii

### PROBLEM ON STRINGS SYSTEM VIBRATIONS ON A STAR-SHAPED GRAPH WITH A NONLINEAR CONDITION AT A NODE

**Abstract.** We consider a system of  $n$  strings located in the equilibrium position along a geometrical star-graph. We suppose that the edges of the graph have the same lengths and the graph is oriented to the node. We study the case when the initial velocity of each string is zero. The initial shape of each string is defined by means of given functions on the edges. We assume that at the boundary vertices the strings are fixed. We study the oscillatory process for the case, when the node point of the string system is located inside the motion limiter. At the same time we suppose that the limiter can move in the direction perpendicular to the graph plane. While the limiter does not touch with the node point of the string system, the transmission condition holds (the Kirchoff condition). Once the limiter touches the node, their joint motion begins and an additional restriction for the sign of the sum of derivatives at the node appears. Thus, at the node, a hysteresis type condition is satisfied.

In the work we obtain a representation for the solution and prove its existence. For a particular case we consider a case on periodic oscillations of the node point of the string system. We solve a problem on boundary control of the oscillatory process under the assumption that the oscillation time does not exceed the length of the string.

**Keywords:** wave equation, oscillatory process, hysteresis, geometric graph.

## S.A. Nazarov

### INFLUENCE OF THE WINKLER–STEKLOV CONDITIONS ON NATURAL OSCILLATIONS OF AN ELASTIC WEIGHTY BODY

**Abstract.** We consider the spectral problem for the spacial system of equations of the elasticity theory. Small parts of the body surface are supplied with the Winkler–Steklov conditions which model spring mount while the remaining part of the boundary is traction-free. In several cases (the relative stiffness of springs and their disposition are varied) we construct asymptotics for eigenfrequencies of the body and for corresponding eigenmodes. The limiting problems are ones for the body (spectral or stationary in some case) and problems of the elasticity theory for the half-spaces with the Winkler–Steklov conditions on flat sets (separated or joined into a single spectral theory in some cases). The discreteness of spectrum in of the problem in the half-space with ensured by a polynomial property of the system of equations of the elasticity theory. We study particular case, formulate open questions and discuss pathological situations, in which the spectrum loses usual properties. We construct asymptotic models of the problem, which are two-terms asymptotics for the eigenpairs of the initial problem and which use the technique of self-adjoint extensions of differential operators or Hilbers spaces with separated asymptotics.

**Keywords:** elastic body, Winkler–Steklov conditions of spring mount, singular perturbation, asymptotics of eigenfrequencies.

## S.N. Timergaliev

### SOLVABILITY OF NONLINEAR BOUNDARY VALUE PROBLEMS FOR NON-SLOPING TIMOSHENKO-TYPE ISOTROPIC SHELLS OF ZERO PRINCIPAL CURVATURE

**Abstract.** We study the solvability of boundary value problem for a system of second order partial differential equations under boundary given conditions describing the equilibrium of elastic non-sloping isotropic inhomogeneous shells with free boundary in the framework of the translation Timoshenko model. The base of the study method are the integral representations for generalized motions involving arbitrary functions, including arbitrary holomorphic functions. The arbitrary functions ate determined so that the generalized motions satisfy a linear system of equations and linear boundary conditions extracted from the original boundary value problem. The holomorphic functions are sought as Cauchy type integrals with real densities. The integral representations allow us to reduce the initial boundary value problem to a nonlinear operator equations for generalized motions in the Sobolev space. While studying the solvability of this operator equation, the most essential point is to invert

it with respect to the linear part. As a result, the work is reduced to an equation, the solvability of which is established on the base of the contracting mapping principle.

**Keywords:** non-sloping isotropic inhomogeneous Timoshenko-type shell of zero principal curvature, nonlinear boundary value problem, partial differential equations, generalized solution, holomorphic function, operator equation, existence theorem.

### A.S. Shamaev, V.V. Shumilova

HOMOGENIZATION OF MOTION EQUATIONS FOR A MEDIUM CONSISTING  
OF AN ELASTIC MATERIAL AND AN INCOMPRESSIBLE KELVIN-VOIGT FLUID

**Abstract.** We consider an initial-boundary problem describing the motion of a two-phase medium with a periodic structure. The first phase of the medium is an isotropic elastic material and the second phase is an incompressible viscoelastic Kelvin-Voigt fluid. This problem consists of the partial differential equations of the second and fourth order, the conditions of continuity of displacements and stresses at the phase boundaries, and the homogeneous initial and boundary conditions. Using the Laplace transform method, we derive a homogenized problem as an initial-boundary value problem for the system of fourth order partial integro-differential equations with constant coefficients. The coefficients and convolution kernels of the homogenized equations are found by using solutions of auxiliary periodic problems on the unit cube. In the case of a layered medium, the solutions of the periodic problems are written explicitly, and this allows us to find analytic expressions for the homogenized coefficients and convolution kernels. In particular, we establish that the type and properties of the homogenized convolution kernels depend on the volume fraction of the fluid layers inside the periodicity cell.

**Keywords:** homogenization, equations of motion, two-phase medium, elastic material, Kelvin-Voigt fluid.

### R.R. Ashurov, M.D. Shakarova

INVERSE PROBLEM FOR THE SUBDIFFUSION EQUATION  
WITH FRACTIONAL CAPUTO DERIVATIVE

**Abstract.** We consider an inverse problem on determining the right-hand side of the subdiffusion equation with the fractional Caputo derivative. The right-hand side of the equation has the form  $f(x)g(t)$  and the unknown is the function  $f(x)$ . The condition  $u(x, t_0) = \psi(x)$  is taken as the over-determination condition, where  $t_0$  is some interior point of the considered domain and  $\psi(x)$  is a given function. By the Fourier method we show that under certain conditions on the functions  $g(t)$  and  $\psi(x)$  the solution of the inverse problem exists and is unique. We provide an example showing the violation of the uniqueness of the solution of the inverse problem for some sign-changing functions  $g(t)$ . For such functions  $g(t)$  we find necessary and sufficient conditions on the initial function and on the function from the over-determination condition, which ensure the existence of a solution to the inverse problem.

**Keywords:** subdiffusion equation, forward and inverse problems, the Caputo derivatives, Fourier method.

**M.Y. Mir, W.M. Shah, S.L. Wali**

## INEQUALITIES FOR MEROMORPHIC FUNCTIONS WITH PRESCRIBED POLES

**Abstract.** The extremal problems for functions of complex variables, as well as approaches for obtaining classical inequalities on the base of various methods of the geometric function theory, are known for various norms and for many classes of functions such as rational functions with various constraints and for various domains in the complex plane. It is important to mention that different types of Bernstein-type inequalities appeared in the literature in more generalized forms in which the underlying polynomial was replaced by a more general class of functions. One such generalization is the passage from polynomials to rational functions. In this paper, we prove some inequalities for meromorphic functions with prescribed poles and restricted zeros. These results not only generalize some Bernstein-type inequalities for rational functions, but also improve and generalize some known polynomial inequalities. These inequalities have their own importance in the approximation theory.

**Keywords:** polynomials, Blaschke product, inequalities, rational functions.