ABSTRACTS

N.N. Aitkuzhina, A.M. Gaisin, R.A. Gaisin

Behavior of entire Dirichlet series of class $\underline{D}(\Phi)$ on curves of bounded K-slope

Abstract. We study an asymptotic behavior of the sum of an entire Dirichlet series $F(s) = \sum\limits_n a_n e^{\lambda_n s}$, $0 < \lambda_n \uparrow \infty$, on curves of a bounded K-slope naturally going to infinity. For entire transcendental functions of finite order having the form

$$f(z) = \sum_n a_n z^{p_n}, \quad p_n \in \mathbb{N},$$

Pólya showed that if the density of the sequence $\{p_n\}$ is zero, then for each curve γ going to infinity there exists an unbounded sequence $\{\xi_n\} \subset \gamma$ such that, as $\xi_n \to \infty$, the relation holds:

$$\ln M_f(|\xi_n|) \sim \ln |f(\xi_n)|\,;$$

here $M_f(r)$ is the maximum of the absolute value of the function f. Later these results were completely extended by I.D. Latypov to entire Dirichlet series of finite order and finite lower order according in the Ritt sense. A further generalization was obtained in works by N.N. Yusupova–Aitkuzhina to more general classes $D(\Phi)$ and $\underline{D}(\Phi)$ defined by the convex majorant Φ. In this paper we obtain necessary and sufficient conditions for the exponents λ_n ensuring that the logarithm of the absolute value of the sum of any Dirichlet series from the class $\underline{D}(\Phi)$ on the curve γ of a bounded K–slope is equivalent to the logarithm of the maximum term as $\sigma = \operatorname{Re} s \to +\infty$ over some asymptotic set, the upper density of which is one. We note that for entire Dirichlet series of arbitrarily fast growth the corresponding result for the case of $\gamma = \mathbb{R}_+$ was obtained by A.M. Gaisin in 1998.

Keywords: Dirichlet series, maximal term, curve of a bounded slope, asymptotic set.

M.Kh. Beshtokov

Numerical solution of initial-boundary value problems for a multi-dimensional pseudoparabolic equation

Abstract. We consider initial boundary value problems for a multi-dimensional pseudoparabolic equation with Dirichlet boundary conditions of a special form. For an approximate solution of the considered problems, the multi-dimensional pseudoparabolic equation is reduced to an integro-differential equation with a small parameter. It is shown that as the small parameter tends to zero, the solution of the corresponding modified problem converges to the solution of the original problem. For each of the problems we construct a locally one-dimensional difference scheme following A.A. Samarsky. The main idea is to reduce the transition from a layer to a layer to the sequential solving a number of one-dimensional problems in each of the coordinate directions. Using the maximum principle, we obtain apriori estimates,

which imply the uniqueness, stability, and convergence of the solution of a locally one-dimensional difference scheme in the uniform metric. We construct an algorithm for numerical solving of the modified problem with conditions of a special form.

Keywords: pseudoparabolic equation, moisture transfer equation, integro-differential equation, initial boundary value problem, difference schemes, apriori estimates, stability and convergence.

A.A. Ershov

Bilinear interpolation of program control in approach problem

Abstract. We consider a controlled system involving a constant two-dimensional vector parameter, the approximate value of which is reported to the controlling person only at the moment of the start of movement. Apriori only the set of possible values of these unknown parameter is given. For this controlled system we pose the problem on approaching the target set at a given time. At the same time, we suppose that the controlling person has no the ability to carry out cumbersome calculations in real time associated with the construction of such resolving structures as reachability sets and integral funnels. Therefore, to solve this problem, it is proposed to calculate in advance several «node» resolving controls for parameter values, which are nodes of a grid covering a set of possible parameter values. If at the moment of the beginning of the movement, the parameter value turns out not coincide with any of the grid nodes, it is proposed to calculate the software control by using linear interpolation formulas. However, this procedure can be effective only if a linear combination of controls corresponding to the same «guide» is used in the terminology of the N.N. Krasovsky extreme aiming method. For the possibility of effective use of linear interpolation, it is proposed to build four «node» resolving controls for each grid node and, in addition, to use the method of dividing the control into the main and compensating ones. Due to the application of the latter method , the computed solvability set turns out to be somewhat less than the actual one, but the accuracy of translating the state of the system to the target set increases. A nonlinear generalization of the Zermelo navigation problem is considered as an example.

Keywords: controlled system, approaching problem, unknown constant parameter, bilinear approximation.

L.A. Kalyakin

Perturbation of a simple wave: from simulation to asymptotics

Abstract. We consider a problem on perturbation of a simple (travelling) wave at the example of a nonlinear partial differential equation that models domain wall dynamics in the weak ferromagnets. The main attention is focused on the case when, for fixed constants coefficients, there are many exact solutions in the form of a simple wave. These solutions are determined by an ordinary differential equation with boundary conditions at infinity. The equation depends on the wave speed as a parameter. Suitable solutions correspond to the phase trajectory that connects the equilibria. The main problem is that the wave velocity is not uniquely determined by the coefficients of the initial equations. For an equation with slowly varying coefficients, the asymptotics of the solution is constructed with respect to a small parameter. In the considered case, the well-known asymptotic construction turns out

to be ambiguous due to the uncertainty of the perturbed wave velocity. For unique identification of the speed, we propose an additional restriction on the structure of the asymptotic solution. This restriction is a stability of the leading edge of the wave is formulated on the base of numerical simulation of the original equation.

Keywords: simple wave, perturbation, small parameter, asymptotics.

A.S. Krivosheev, O.A. Krivosheeva

Necessary condition of the fundamental principle for invariant subspaces on unbounded convex domain

Abstract. In this paper we study the spaces $H(D)$ of analytic functions in convex domains of the complex plane as well as subspaces $W(\Lambda, D)$ of such spaces. A subspace $W(\Lambda, D)$ is the closure in the space $H(D)$ of the linear span of the system $\mathcal{E}(\Lambda) = \{z^n \exp(\lambda_k z)\}_{k=1,n=0}^{\infty,n_k-1}$, where Λ is the sequence of different complex numbers λ_k and their multiplicities n_k. This subspace is invariant with respect to the differentiation operator. The main problem in the theory of invariant subspaces is to represent all its functions by using the eigenfunctions and associated functions of the differentiation operator, $z^n e^{\lambda_k z}$. In this paper we study the problem of the fundamental principle for an invariant subspace $W(\Lambda, D)$, that is, the problem of representing all its elements by using a series constructed by the system $\mathcal{E}(\Lambda)$. We obtain simple geometric conditions, which are necessary for the existence of a fundamental principle. These conditions are formulated in terms of the length of the arc of the convex domain and the maximum density of the exponent sequence.

Keywords: exponential monomial, convex domain, fundamental principle, length of arc.

M.N. Kuznetsova

On nonlinear hyperbolic systems related by Bäcklund transforms

Abstract. In this work we describe pairs of nonlinear hyperbolic system of equations $u_{xy} = f(u, u_x, u_y)$, where $u^i_{xy} = f^i$, $i = 1, 2, \ldots n$, the linearizations of which are related by the first order Laplace transform. On the base of this Laplace transform we construct Bäcklund transforms relating the solutions of nonlinear systems.

The classical Bäcklund transform is defined for a second-order nonlinear differential equation whose solution is a function of two independent variables. The Bäcklund transform for a pair of nonlinear equations is a system of relations involving functions and their first derivatives and it provides a transform of a solution of one equation into the solution of another and vice versa. The Bäcklund transforms preserve integrability. The Bäcklund problem is to list the possible Bäcklund transforms and the equations admitting such transforms.

The Laplace cascade integration method is one of the classical methods for integrating linear partial differential equations. The Laplace transform is a special case of the Bäcklund transform for linear equations. The method used in this paper was previously applied to nonlinear hyperbolic equations. In this paper, this method is employed to describe systems associated with Backlund transforms.

Keywords: nonlinear hyperbolic system, Laplace transform, Bäcklund transform, linearization

A.V. Lutsenko, I.Kh. Musin, R.S. Yulmukhametov

On Gelfand-Shilov spaces

Abstract. In this work we follow the scheme of constructing of Gelfand-Shilov spaces S_α and S^β by means of some family of separately radial weight functions in $\mathbb{R}^n$ and define two spaces of rapidly decreasing infinitely differentiable functions in $\mathbb{R}^n$. One of them, namely, the space $\mathcal{S}_\mathcal{M}$ is an inductive limit of countable-normed spaces

$$\mathcal{S}_{\mathcal{M}_\nu} = \{f \in C^\infty(\mathbb{R}^n) : \|f\|_{m,\nu} = \sup_{\substack{x\in\mathbb{R}^n,\beta\in\mathbb{Z}^n_+,\\ \alpha\in\mathbb{Z}^n_+:|\alpha|\leqslant m}} \frac{|x^\beta(D^\alpha f)(x)|}{\mathcal{M}_\nu(\beta)} < \infty,\ m \in \mathbb{Z}_+\}.$$

Similarly, starting with the normed spaces

$$\mathcal{S}^{\mathcal{M}_\nu}_m = \{f \in C^\infty(\mathbb{R}^n) : \rho_{m,\nu}(f) = \sup_{x\in\mathbb{R}^n,\alpha\in\mathbb{Z}^n_+} \frac{(1+\|x\|)^m|(D^\alpha f)(x)|}{\mathcal{M}_\nu(\alpha)} < \infty\}$$

there is introduced the space $\mathcal{S}^\mathcal{M}$. It is shown that under certain natural conditions on weight functions the Fourier transform establishes an isomorphism between spaces $\mathcal{S}_\mathcal{M}$ and $\mathcal{S}^\mathcal{M}$.

Keywords: Gelfand-Shilov spaces, Fourier transform, convex functions.

A.B. Muravnik

Elliptic differential-difference equations in half-spaces: case of summable functions

Abstract. We study the Dirichlet problem in the half-space for elliptic equations involving, apart of differential operators, the shift operators acting in tangential (spatial-like) variables, that is, in independent variables varying in entire real line. The boundary function in the problem is supposed to be summable, which in the classical case corresponds to the situation, in which only solutions with finite energy are possible.

We consider two principally different cases: the case, in which the studied equation involves superpositions of differential operators and the shift operators and the case, when it involves their sums, that is, it is an equation with nonlocal potentials.

For both types of problems we construct an integral representation of the solution to this problem in the sense of generalized functions and we prove that its infinitely smoothness in an open half-space (i.e., outside the boundary hyperplane) and tends uniformly to zero together with all its derivatives as a time-like variable tends to infinity; this time-like variable is a single independent variable varying on the positive half-axis. The rate of this decay is power-law; the degree is equal to the sum of the dimension of the space-like independent variable and the order of the derivative of the solution.

The most general current results are presented: shifts of independent variables are allowed in arbitrary (tangential) directions, and if there are several shifts, no conditions of commensurability are imposed on their values.

Thus, just as in the classical case, problems with summable boundary functions fundamentally differ from the previously studied problems with essentially bounded boundary functions: the latter, as previously established, admit solutions having no limit when a time-like variable tends to infinity, and the presence or absence of such a limit is determined by the Repnikov-Eidelman stabilization condition.

Keywords: elliptic differential-difference equations, problems in half-space, summable boundary functions.

A.D. Baranov, A.A. Lishanskii

Point spectrum and hypercyclicity problem for a class of truncated Toeplitz operators

Abstract. Truncated Toeplitz operators are restrictions of usual Toeplitz operators onto model subspaces $K_\theta = H^2 \ominus \theta H^2$ of the Hardy space H^2, where θ is an inner function. In this note we study the structure of eigenvectors for a class of truncated Toeplitz operators and discuss an open problem whether a truncated Toeplitz operator on a model space can be hypercyclic, that is, whether there exists a vector with a dense orbit. For the classical Toeplitz operators on H^2 with antianalytic symbols a hypercyclicity criterion was given by G. Godefroy and J. Shapiro, while for Toeplitz operators with polynomial or rational antianalytic part some partial answers were obtained by the authors jointly with E. Abakumov and S. Charpentier.

We find point spectrum and eigenfunctions for a class of truncated Toeplitz operators with polynomial analytic and antianalytic parts. It is shown that the eigenvectors are linear combinations of reproducing kernels at some points such that the values of the inner function θ at these points have a polynomial dependence. Next we show that, for a class of model spaces, truncated Toeplitz operators with symbols of the form $\Phi(z) = a\bar{z}+b+cz$, where $|a| \neq |c|$, have complete sets of eigenvectors and, in particular, are not hypercyclic. Our main tool here is the factorization of functions in an associated Hardy space in an annulus. We also formulate several open problems.

Keywords: Hypercyclic operator, Toeplitz operator, model space, truncated Toeplitz operator.

S.A. Budochkina, T.H. Luu, V.A. Shokarev

On indirect representability of fourth order ordinary differential equation in form of Hamilton-Ostrogradsky equations

Abstract. In the paper we solve the problem on the representability of a fourth order ordinary differential equation in the form of Hamilton-Ostrogradsky equations. Local bilinear forms play an essential role in the investigation of the potentiality property of the considered equation. It is well known that the problem of representing differential equations in the form of Hamilton-Ostrogradsky equations is closely related to the existence of a solution to the inverse problem of the calculus of variations, that is, for a given equation one needs to construct a functional-variational principle. To solve this problem, we first obtain necessary and sufficient conditions for the given equation to admit an indirect variational formulation relative to a local bilinear form and then construct the corresponding Hamilton-Ostrogradsky action. Note that the found conditions are analogous to the Helmholtz potentiality conditions for the given ordinary differential equation. We also define the structure of the considered equation with the potential operator and use the Ostrogradsky scheme to reduce the given equation to the form of Hamilton-Ostrogradsky equations.

It should be noted that applications and extensions of the work are the possibility to establish connections between invariance of the functional and first integrals of the given equation and to extend the proposed scheme to partial differential equations and systems of such equations.

Keywords: Local bilinear form, potential operator, Hamilton-Ostrogradsky action, Hamilton-Ostrogradsky equations.

A. El Ouissari, K. El Moutaouakil

Genetic algorithm applied to fractional optimal control of a diabetic patient

Abstract. Diabetes is a dangerous disease that increases in incidence every year. The aim of this paper is to present and analyze the model of diabetes and its complications with the fractional derivative of Caputo, namely, we propose a mathematical model with a fractional derivative of the type 2 diabetes. The positivity and boundedness of the solutions is demonstrated by the Laplace transform method. We study the existence and uniqueness of the solution of the system. We use the genetic algorithm (GA) to solve the fractional differential equation model and to characterize the optimal control and this is an efficient and simple metaheuristic method to implement. Simulations of the total number of diabetics with the different values of a parameter α show that the combined control strategy leads to a significant decrease. The simulation results also show that the number of uncomplicated diabetics in the fractional model, for the different fractional values of α, decreases more rapidly than the integer derivative model.

Keywords: Diabetic population dynamic system, Optimal control, Fractional derivative, Genetic algorithm, Artificial intelligent.

A.O. Smirnov, A.A. Caplieva

Vector form of Kundu-Eckhaus equation and its simplest solutions

Abstract. Nowadays, new vector integrable models of nonlinear optics are actively investigated. This is motivated by a need to transmit more information per unit of time by using polarized waves. In our work we study one of such models and we construct an hierarchy of integrable vector nonlinear differential equations depending on the functional parameter r by using a monodromy matrix. The first equation of this hierarchy for $r = \alpha(\mathbf{p}^t\mathbf{q})$ is a vector analogue of the Kundu-Eckhaus equation. As $\alpha = 0$, the equations of this hierarchy turn into equations of the Manakov system hierarchy. Other values of the functional parameter r correspond to other integrable nonlinear equations. New elliptic solutions to the vector analogue of the Kundu-Eckhaus and Manakov system are presented. We also give an example of a two-gap solution of these equations in the form of a solitary wave. We show that there exist linear transformations of solutions to the vector integrable nonlinear equations into other solutions to the same equations. This statement is true for many vector integrable nonlinear equations. In particular, this is true for multicomponent derivative nonlinear Schrödinger

equations and for the Kulish-Sklyanin equation. Therefore, the corresponding Baker-Akhiezer function can be constructed from a spectral curve only up to a linear transformation. In conclusion, we show that the spectral curves of the finite-gap solutions of the Manakov system and the Kundu-Eckhaus vector equation are trigonal curves whose genus is twice the number of phases of the finite-gap solution, that is, in the finite-gap solutions of the Manakov system and the vector analogue of the Kundu-Eckhaus equation, only half of the phases contain the variables t, $z_1, \ldots, z_n$. The second half of the phases depends on the parameters of the solutions.

Keywords: Monodromy matrix, spectral curve, derivative nonlinear Schrödinger equation, vector integrable nonlinear equation.