ABSTRACTS

W.E. Admasu, E.I. Galakhov

Conditions for absence of solutions to some higher order elliptic inequalities with singular coefficients in $\mathbb{R}^n$

Abstract. In the present paper we study the theorems of Liouville type theorems for elliptic higher order inequalities with singular coefficients and gradient terms in $\mathbb{R}^n$. Our approach is based on the Pokhozhaev method of nonlinear capacity, which is widely used in studying various nonlinear elliptic inequalities. We obtain apriori estimates for solutions of an elliptic inequality using the method of test functions. An optimal choice of the test function leads us to a nonlinear minimax problem, which generates a nonlinear capacity induced by a corresponding nonlinear problem. The existence of the zero limit of the corresponding apriori estimate ensures the absence of a nontrivial solution to the problem. Our result provide a new view on the behavior of solutions of higher order elliptic inequalities with singular coefficients and gradient terms and this approach can be useful to studying other types of nonlinear elliptic inequalities.

Keywords: Liouville type theorems, apriori estimate, nonlinear capacity, singular coefficients, gradient terms.

Zh.A. Balkizov

Nonlocal problems with shift for matching two second order hyperbolic equations

Abstract. In this work we study two nonlocal problems with a shift for two second order hyperbolic equations being a wave equation in one part of the domain and a degenerate hyperbolic equation of the first kind in the other. As a nonlocal boundary condition, in the considered problems we use a linear combination with variable coefficients of the first derivative and fractional derivative (in the Riemann-Liouville sense) of the unknown function on one of the characteristics and one the line of the type changing. By using the methods of integral equations, the solvability issue of the first problem is equivalently reduced to the solvability of a Volterra equation of the second kind with a weak singularity, while the solvability of the second problem is reduced to the solvability of aFredholm equation of the second kind with a weak singularity. For the first problem we prove a uniform convergence of the resolvent for the kernel of the obtained Volterra equation of the second kind and that it solution belongs to a needed class. For the second problem we find sufficient conditions for the given functions ensuring the existence of the unique solution of the Fredholm equation of the second kind with a weak singularity with a weak singularity in the needed class. In some particular cases the solutions of the problems are written explicitly.

Keywords: wave equation, degenerate hyperbolic equation of the first kind, Volterra integral equation, Fredholm integral equation, Trikomi method, method of integral equations, methods of fractional calculus.

Yu.G. Voronova, A.V. Zhiber

On a class of hyperbolic equations with third-order integrals

Abstract. We consider a Goursat problem on classification of nonlinear second order hyperbolic equations integrable by the Darboux method. In the work we study a class of hyperbolic equations with second order y-integral reduced by an differential substitution to equations with first order y-integral. It should be noted that Laine equations are in the considered class of equations. In work we provide a second order y-integral for the second Laine equation and we find a differential substitution relating this equation with one of the Moutard equations.

We consider a class of nonlinear hyperbolic equations possessing first order y-integrals and third order x-integrals. We obtain three conditions under which the equations in this class possess first order and third order integrals. We find the form of such equations and obtain the formulae for x- and y-integrals. In the paper we also provide differential substitutions relating Laine equations.

Keywords: Laplace invariants, x- and y-integrals, differential substitutions.

R.A. Gaisin

On rate of decreasing of extremal function in Carleman class

Abstract. We study the issues related with Levinson-Sjoberg-Wolf type theorems in the complex analysis and, in particular, we discuss a famous question posed in 70s by E.M. Dyn'kin on an effective bound for majorant of the growth of an analytic function in the vicinity of the set of singular points and another close problem on the rate of decaying of an extremal function in a non-quasianalytic Carleman class in the vicinity of the point, at which all the derivatives of the functions from this class vanish. Exact asymptotic estimates for the best majorant for the growth in the vicinity of the singularities were found by V. Matsaev and M. Sodin in 2002.

Some bound, both from above and below, for an extremal function in the Carleman class were obtained by A.M. Gaisin in 2018 but they turned out to be not very close to exact values of this function. In the present paper we obtain sharp two-sided estimates for the extremal function.

Keywords: non-quasianalytic Carleman class, Levinson-Sjoberg type theorem, extremal function, regular sequence, associated weight.

A.R. Danilin

Asymptotics for solutions to problem on optimally distributed control in convex domain with small parameter at one of higher derivatives

Abstract. We consider a problem on optimally distributed control in a planar strictly convex domain with a smooth boundary and a small parameter at one of the higher derivatives in the elliptic operator. On the boundary of the domain the homogeneous Dirichlet condition is imposed, while the control is additive involved in a inhomogeneity. As a set of admissible controls we use a unit ball in the corresponding space of square integrable functions. The solutions of the studied boundary value problem are treated in the generalized sense as elements of some Hilbert space. As the optimality criterion, we employ the sum of squared norm of the deviation of a state from a prescribed one and the squared norm of the control with some coefficient. Such

structure of the optimality criterion allows, if this is needed, to strengthen the role of the first or the second term in this criterion. In the first case it is more important to achieve a prescribed state, while in this second case it is more important to minimize the resource expenses. We study in details the asymptotics of the problem generated by the differential operator with a small coefficient at one of the higher derivatives, to which a zero order differential operator is added.

Keywords: small parameter, optimal control, boundary value problems for systems of partial differential equations, asymptotic expansions.

R.Sh. Kalmetev, Yu.N. Orlov, V.Zh. Sakbaev

Averaging of random affine transformations of variables in functions

Abstract. We study the averaging of Feynmann-Chernov iterations of random operator-valued strongly continued functions, the values of which are bounded linear operators on separable Hilbert space. In this work we consider averaging for a certain system of such random operator-valued functions. Linear operators, being the values of the considered functions, act in the Hilbert space of square integrable functions on a finite-dimensional Euclidean space and they are defined by random affine transformations of the variables in the functions. At the same time, the compositions of independent identically distributed random affine transformations are a non-commuting analogue of random walk.

For an operator-valued function being an averaging of Feynmann-Chernov iterations, we prove upper bound for its norm and we also establish that the closure of the derivative of this operator-valued function at zero is a generator a strongly continuous semigroup. In the work we obtain sufficient conditions for the convergence of the mathematical expectation of the sequence of Feynmann-Chernov iterations to the semigroup resolving the Cauchy problem for the corresponding Fokker-Planck equations.

Keywords: Feynman-Chernoff iterations, Chernoff theorem, operator-valued random process, Fokker-Planck equation.

E.G. Rodikova

On continuous linear functionals in some spaces of functions analytic in a disk

Abstract. The issue on description of linear continuous functionals on the spaces of analytic functions has been studied since the middle of 20th century. First there was found a structure of linear continuous functionals on the Hardy spaces H^p as $p \geqslant 1$ in work by A. Taylor in 1951. In the spaces H^p, $0 < p < 1$, this problem was solved by P. Duren, B. Romberg, A. Shields. We note that in the proof an estimate for the coefficient multipliers in these spaces was employed. In the present paper, by developing the method proposed in the work by P. Duren et al, we describe linear continuous functionals on planar Privalov classes and classes of Nevanlinna-Dzhrbashjan type. The considered classes generalize the well-known in scientific literature planar Nevanlinna classes. The idea of the proof of the main result is as follows: the issue on finding the general form a continuous linear functional is reduced to finding a form of an arbitrary coefficient multiplier acting from a studied space into the space of bounded analytic functions. The latter problem in a simplified form can be formulated as follows: by what factors we should multiply the Taylor coefficients

of the functions in a studied class in order to make them Taylor coefficients of some bounded analytic function.

M.Sh. Shabozov, Z.Sh. Malakbozov

SHARP JACKSON–STECHKIN TYPE INEQUALITIES IN THE HARDY SPACE H_2 AND WIDTHS OF FUNCTIONAL CLASSES

Abstract. In the work we obtain sharp Jackson–Stechkin type inequalities relating the best joint polynomial approximation of functions analytic in the unit disk and a special generalization of the continuity modulus, which is defined by means of the Steklov function.

While solving a series of problems in the theory on approximation of periodic functions by trigonometric polynomials in the space L_2, a modification of the classical definition of the continuity modulus of mth order generated by the Steklov function was employed by S.B. Vakarchuk, M.Sh. Shabozov and A.A. Shabozova. Here the proposed construction is employed for defining a modification of the continuity modulus of mth order for functions analytic in the unit disk generated by the Steklov function in the Hardy space H_2.

By using this smoothness characteristic we solve a problem on finding a sharp constant in the Jackson–Stechkin type inequalities for joint approximation of the functions and their intermediate derivatives.

For the classes of function, averaged with a weight, the generalized continuity moduli of which are bounded by a given majorant, we find exact values of various n-widths. We also solve the problem on finding sharp upper bounds for best joint approximations of the mentioned classes of functions in the Hardy space H_2.

M.G. Yumagulov, S.V. Akmanova

ON THE STABILITY OF EQUILIBRIUM POINTS OF NONLINEAR CONTINUOUS-DISCRETE DYNAMICAL SYSTEMS

Abstract. In this paper the main attention is paid to discussing the issues on sufficient conditions for Lyapunov stability of nonlinear hybrid (continuous-discrete) systems, that is, the systems, the processes in which have several levels of different descriptions, while the states involve both continuous and discrete components. It is well-known that by switchings between unstable regimes in a continuous dynamical system one can achieve a stability and vice versa, even when all regimes of the continuous system are stable, under the switching there can appear unstable regimes in the system. This is why it is important to make a detailed analysis on the stability issues while passing from continuous to the hybrid system.

In the present paper we propose new tests for Lyapunov stability of stationary regimes of nonlinear hybrid system with a constant discretization step $h > 0$. These tests are based on the methods of studying the stability by the linear approximation and on the formulae from the perturbation theory, which allow us to analyse the equilibria and cycles of the dynamical systems depending on a small parameter. The proposed approaches are based on passage from the original hybrid system to

equivalent in a natural sense dynamical system with a discrete time. We discuss relations between dynamical characteristics of hybrid and discrete systems. While studying the main problem on Lyapunov stability of an equilibrium of the hybrid system, we consider two formulations: the stability for small $h > 0$ and stability for arbitrary fixed $h = h_0 > 0$. Moreover, we discuss some questions on scenarios of bifurcation behavior of the hybrid system under the stability loss of the equilibrium. We adduce an example illustrating the efficiency of the obtained results in the problem on studying the stability of the equilibria of the hybrid systems.

Keywords: continuous-discrete system, hybrid system, equilibrium, periodic solutions, stability, bifurcation.

A. Chouia, A. Azeb Ahmed, F. Yazid

Analysis of a thermo-elasto-viscoplastic contact problem with wear and damage

Abstract. This paper presents a quasistatic problem of a thermo-elaso-visco-plastic body in frictional contact with a moving foundation. The contact is modelled with the normal compliance condition and the associated law of dry friction. The model takes into account wear of the contact surface of the body caused by the friction and which is described by the Archard law. The mechanical damage of the material, caused by excessive stress or strain, is described by the damage function, the evolution of which is determined by a parabolic inclusion. We list the assumptions on the data and derive a variational formulation of the mechanical problem. Existence and uniqueness of the weak solution for the problem is proved using the theory of evolutionary variational inequalities, parabolic variational inequalities, first order evolution equation and Banach fixed point.

Keywords: Thermo-elasto-viscoplastic material, damage, wear, frictional contact, existence and uniqueness, fixed point arguments, weak solution

D.K. Durdiev, J.J. Jumaev, D.D. Atoev

Inverse problem of determining two kernels in the integro-differential equation of heat flow

Abstract. We study the inverse problem on determining the energy-temperature relation $\chi(t)$ and the heat conduction relation $k(t)$ functions in the one-dimensional integro-differential heat equation. The direct problem is an initial-boundary value problem for this equation with the Dirichlet boundary conditions. The integral terms involve the time convolution of unknown kernels and a direct problem solution. As an additional information for solving inverse problem, the solution of the direct problem for $x = x_0$ and $x = x_1$ is given. We first introduce an auxiliary problem equivalent to the original one. Then the auxiliary problem is reduced to an equivalent closed system of Volterra-type integral equations with respect to the unknown functions. Applying the method of contraction mappings to this system in the continuous class of functions, we prove the main result of the article, which a local existence and uniqueness theorem for the inverse problem.

Keywords: Banach principle, resolvent, Volterra equation, operator equation, initial-boundary problem, inverse problem, Green function.

H. Serrai, B. Tellab, Kh. Zennir

On two-order fractional boundary value problem with generalized Riemann-Liouville derivative

Abstract. In this paper we focus our study on the existence, uniqueness and Hyers-Ulam stability for the following problem involving generalized Riemann-Liouville operators:

$$\mathcal{D}_{0^+}^{\rho_1,\Psi}\left(\mathcal{D}_{0^+}^{\rho_2,\Psi} + \nu\right)u(t) = f(t, u(t)).$$

It is well known that the existence of solutions to the fractional boundary value problem is equivalent to the existence of solutions to some integral equation. Then it is sufficient to show that the integral equation has only one fixed point. To prove the uniqueness result, we use Banach fixed point Theorem, while for the existence result, we apply two classical fixed point theorems due to Krasnoselskii and Leray-Scauder. Then we continue by studying the Hyers-Ulam stability of solutions which is a very important aspect and attracted the attention of many authors. We adapt some sufficient conditions to obtain stability results of the Hyers-Ulam type.

Keywords: fractional derivatives, generalized Riemann-Liouville derivative, fixed point theorem, fractional Boundary value problem, Hyers-Ulam stability.