# ABSTRACTS

## G.A. Akishev

### On estimates for orders of best $M$–term approximations of multivariate functions in anisotropic Lorentz–Karamata spaces

**Abstract.** In the paper we consider a well-known class of weakly varying functions and by these functions we define an anisotropic Lorentz–Karamata space of $2\pi$–periodic functions of many variables. Particular cases of these spaces are anisotropic Lorentz-Zygmund and Lorentz spaces. In the anisotropic Lorentz–Karamata space we define an analogue of Nikolskii-Besov space. The main aim of the paper is to find the sharp orders of best $M$–term trigonometric approximation of functions from Nikolskii-Besov space by the norm of another anisotropic Lorentz-Karamata space. In the paper we establish order sharp two-sided estimates of best $M$–term trigonometric approximations for the functions from the Nikolskii-Besov space in the anisotropic Lorentz-Karamata space in various metrics. In order to prove an upper bound for $M$–term approximations, we employ an idea of the greedy algorithms proposed by V.N. Temlyakov modified for the anisotropic Lorentz-Karamata space.

**Keywords:** Lorentz-Karamata space, Nikolskii-Besov space, $M$–term approximation.

## S.Kh. Gekkieva, M.A. Kerefov, F.M. Nakhusheva

### Local and nonlocal boundary value problems for generalized Aller–Lykov equation

**Abstract.** In mathematical modelling solid media with memory there arise equations describing a new type of wave motion, which is between the usual diffusion and classical waves. Here we mean differential equations with fractional derivatives both in time and spatial variables, which a base for most part of mathematical models in mechanics of liquids, viscoelasticity as well as in processes in media with fractal structure and memory.

In the present work we present a qualitatively new water transfer equation being a generalization of Aller-Lykov equation. This generalization provides an opportunity to reflect specific features of the studied objects in the nature of the equation such, namely, the structure and physical properties of the going processes, by means of introducing a fractal velocity of moisture varying.

The work is devoted to studying local and nonlocal boundary value problems for inhomogeneous Aller-Lykov water transfer equation with variable coefficients and Riemann-Liouville fractional time derivative. For a generalized equation of Aller-Lykov type we consider initial boundary value problems with Dirichlet and Robin boundary conditions as well as nonlocal problems involving nonlocality in time in the boundary conditions. By the method of energy inequalities, assuming the existence of regular solutions, we obtain apriori estimates in terms of Riemann-Liouville fractional derivative, which imply the uniqueness of the solutions to the considered problems as well as their stability in the right hand side and initial data.

**Keywords:** Aller-Lykov water transfer equation, Riemann-Liouville fractional derivative, Fourier method, apriori estimate.

## K.K. Kudaybergenov, B.O. Nurjanov

### Partial orders on $*$-regular rings

**Abstract.** In this work we consider some new partial orders on $*$-regular rings. Let $\mathcal{A}$ be a $*$-regular ring, $P(\mathcal{A})$ be the lattice of all projectors in $\mathcal{A}$ and $\mu$ be a sharp normal normalized measure on $P(\mathcal{A})$. Suppose that $(\mathcal{A}, \rho)$ is a complete metric $*$-ring with respect to the rank metric $\rho$ on $\mathcal{A}$ defined as $\rho(x,y) = \mu(l(x-y)) = \mu(r(x-y))$, $x, y \in \mathcal{A}$, where $l(a)$, $r(a)$ is respectively the left and right support of an element $a$. On $\mathcal{A}$ we define the following three partial orders: $a \prec_s b \Longleftrightarrow b = a + c$, $a \perp c$; $a \prec_l b \Longleftrightarrow l(a)b = a$; $a \prec_r b \Longleftrightarrow br(a) = a$, $a \perp c$ means algebraic orthogonality, that is, $ac = ca = a^*c = ac^* = 0$. We prove that the order topologies associated with these partial orders are stronger than the topology generated by the metric $\rho$. We consider the restrictions of these partial orders on the subsets of projectors, unitary operators and partial isometries of $*$-regular algebra $\mathcal{A}$. In particular, we show that these three orders coincide with the usual order $\leqslant$ on the lattice of the projectors of $*$-regular algebra. We also show that the ring isomorphisms of $*$-regular rings preserve partial orders $\prec_l$ and $\prec_r$.

**Keywords:** partial order, $*$-regular ring, von Neumann algebra, order topology.

## M.M. Rakhmatullaev, B.M. Isakov

### Ground states of the model Ising-Potts on a Cayle tree

**Abstract.** It is known that for low temperatures, a ground state is associated with a limiting Gibbs measure. This is why, the studying of the sets of ground states for a given physical system is a topical issue.

We consider a model of mixed type on the Cayley tree, which is referred to as Ising-Potts model, that is, the Ising and Potts models are related with the parameter $\alpha$, where $\alpha \in [0,1]$. In the paper we study the ground state for the Ising-Potts model with three states on the Cayley tree. It is known that there exists a one-to-one correspondence between the set of the vertices $V$ of the Cayley tree of order $k$ and a group $G_k$ being a free product of $k+1$ cyclic groups of second order. We define periodic and weakly periodic ground states corresponding to normal divisors of the group $G_k$. For the Ising-Potts model we describe the set of periodic and weakly periodic ground states corresponding to normal divisors of index 2 of the group $G_k$. We prove that for some values of the parameters there exist no such periodic (non translation-invariant) ground states. We also prove that for a normal subgroup consisting of even layers there exist periodic (non translation-invariant) ground states and we also prove the existence of weakly-periodic (non periodic) ground states.

**Keywords:** Cayley tree, Ising-Potts model, periodic and weakly periodic ground states.

**J. Merker**

## Inexistence of non-product Hessian rank 1 affinely homogeneous hypersurfaces $H^n \subset \mathbb{R}^{n+1}$ in dimension $n \geqslant 5$

**Abstract.** Equivalences under the affine group $\mathrm{Aff}(\mathbb{R}^3)$ of constant Hessian rank 1 surfaces $S^2 \subset \mathbb{R}^3$, sometimes called *parabolic*, were, among other objects, studied by Doubrov, Komrakov, Rabinovich, Eastwood, Ezhov, Olver, Chen, Merker, Arnaldsson, Valiquette. In particular, homogeneous models and algebras of differential invariants in various branches were fully understood.

*Then what is about higher dimensions?* We consider hypersurfaces $H^n \subset \mathbb{R}^{n+1}$ graphed as $\big\{u = F(x_1, \ldots, x_n)\big\}$ whose Hessian matrix $\big(F_{x_i x_j}\big)$, a relative affine invariant, is similarly of constant rank 1. *Are there homogeneous models?*

Complete explorations were done by the author on a computer in dimensions $n = 2, 3, 4, 5, 6, 7$. The first, expected outcome, was a complete classification of homogeneous models in dimensions $n = 2, 3, 4$ (forthcoming article, case $n = 2$ already known). The second, unexpected outcome, was that in dimensions $n = 5, 6, 7$, there are no affinely homogenous models except those that are affinely equivalent to a product of $\mathbb{R}^m$ with a homogeneous model in dimensions $2, 3, 4$.

The present article establishes such a non-existence result in every dimension $n \geqslant 5$, based on the production of a normal form for $\big\{u = F(x_1, \ldots, x_n)\big\}$, under $\mathrm{Aff}(\mathbb{R}^{n+1})$ up to order $\leqslant n + 5$, valid in any dimension $n \geqslant 2$.

**Keywords:** Affine homogeneity, Normal forms, tangential vector fields.

**Zh. Myrzakulova, G. Nugmanova, N. Serikbayev, K. Yesmakhanova, R. Myrzakulov**

## Integrable Tolkynay equations and related Yajima-Oikawa type equations

**Abstract.** We consider some nonlinear models describing resonance interactions of long waves and short-waves (shortly, the LS waves models). Such LS models were derived and proposed due to various motivations, which mainly come from the different branches of modern physics, especially, from the fluid and plasma physics. In this paper, we study some of integrable LS models, namely, the Yajima-Oikawa equation, the Newell equation, the Ma equation, the Geng-Li equation and their different modifications and extensions. In particular, the gauge equivalent counterparts of these integrable LS models (equations), namely, different integrable spin systems are constructed. In fact, these gauge equivalent counterparts of these LS equations are integrable generalized Heisenberg ferromagnet type models (equations) (HFE) with self-consistent potentials (HFESCP). The associated Lax representations of these HFESCP are presented. Using these Lax representations of these HFESCP, they can be studied by the inverse scattering method. For instance, the equivalence established using the Lax representation also makes it possible to find a connection between the solutions of the corresponding integrable equations.

**Keywords:** Integrable equations, Heisenberg ferromagnet equation, Yajima-Oikawa equation, gauge equivalent, Lax representation.