

ABSTRACTS

V.M. Adukov

NORMALIZATION OF WIENER–HOPF FACTORIZATION
FOR 2×2 MATRIX FUNCTIONS AND ITS APPLICATION

Abstract. In work we cover a gap existing in the Wiener-Hopf general factorization theory of matrix functions. It is known that the factors in such factorization are not found uniquely and in the general case, there are no known ways of normalization of factorization ensuring its uniqueness. In the work we introduce the notion of P -normalized factorization. Such normalization ensures the uniqueness of the Wiener-Hopf factorization and gives an opportunity to find the Birkhoff factorization. For the second order matrix function we show that the factorization of each matrix function can be reduced to the P -normalized factorization. We describe all possible types of such factorizations, obtain the conditions ensuring the existence of such normalization and find the form of the factors for such type of the normalization. We study the stability of P -normalization under a small perturbation of the initial matrix function. The results are applied to specify the Shubin theorem on the continuity of the factors and for obtaining the explicit estimates of the absolute errors of the factors for an approximate factorization.

Keywords: Wiener-Hopf factorization, partial indices, continuity of factors, normalization of factorization.

A.O. Bagapsh, K.Yu. Fedorovskiy

ON ENERGY FUNCTIONALS FOR SECOND-ORDER ELLIPTIC SYSTEMS

Abstract. We consider the Dirichlet problem for second-order elliptic systems with constant coefficients. We prove that non-reducible strongly elliptic systems of this type admit no non-negatively definite energy functionals of the form

$$f \mapsto \int_D \Phi(u_x, v_x, u_y, v_y) dx dy,$$

where D is the domain in which the problem is considered, Φ is some quadratic form in \mathbb{R}^4 and $f = u + iv$ is a function of the complex variable. The proof is based on reducing the considered system to a special (canonical) form when the differential operator defining this system is represented as a perturbation of the Laplace operator with respect to two small real parameters, the canonical parameters of the considered system. In particular, the obtained result shows that a direct extension of the classical Lebesgue theorem on the regularity of an arbitrary bounded simply connected domain in the complex plane with respect to the Dirichlet problem for harmonic functions to strongly elliptic second-order equations with constant complex coefficients of a general form is not possible. This clarifies a number of difficulties arising in this problem, which is quite important for the theory of approximations by analytic functions.

Keywords: second-order elliptic system, canonical representation of second-order elliptic system, Dirichlet problem, energy functional.

D.I. BorisovASYMPTOTIC EXPANSION OF DIRICHLET PROBLEM
IN PERFORATED DOMAIN: STRANGE TERM CASE

Abstract. We consider an elliptic operator in a multi-dimensional space periodically perforated by closely spaced small cavities. The coefficients of the differential expression are varying and infinitely differentiable functions bounded uniformly with all their derivatives. For the coefficients at higher derivatives a uniform ellipticity condition is supposed. On the boundaries of the cavities we impose the Dirichlet condition. The sizes of the cavities and the distances between them are characterized by two small parameters. They are chosen so that to ensure the appearance of a strange term under the homogenization, which is an additional potential in the homogenized operator. The main result of the work is the scheme of constructing two-parametric asymptotics for the resolvent of the considered operator and its application for determining the leading terms in the asymptotics. The scheme is based on applying a combination of the multi-scaled method and the method of matching asymptotic expansions. The former is used to take into consideration the distribution of the cavities, while the latter takes into account the geometry of the cavities and the Dirichlet condition on its boundary.

Keywords: perforated domain, elliptic operator, asymptotic expansion, strange term.

J.D. DekhkonovON (k_0) -TRANSLATION-INVARIANT AND (k_0) -PERIODIC GIBBS MEASURES
FOR POTTS MODEL ON CAYLEY TREE

Abstract. As a rule, the solving of problem arising while studying the thermodynamical properties of physical and biological system is made in the framework of the theory of Gibbs measure. The Gibbs measure is a fundamental notion defining the probability of a microscopic state of a given physical system defined by a given Hamiltonian. It is known that to each Gibbs measure one phase of a physical system is associated to, and if this Gibbs measure is not unique then one says that a phase transition is present. In view of this the study of the Gibbs measure is of a special interest. In this paper we study (k_0) -translation-invariant (k_0) -periodic Gibbs measures for the Potts model on the Cayley tree. Such measures are constructed by means of translation-invariant and periodic Gibbs measures. For the ferromagnetic Potts model, in the case $k_0 = 3$ we prove the existence of (k_0) -translation-invariant, that is, (3)-translation-invariant Gibbs measures. For antiferromagnetic Potts model and also in the case $k_0 = 3$ we prove the existence of (k_0) -periodic ((3)-periodic) Gibbs measures on the Cayley tree.

Keywords: Cayley tree, Gibbs measure, Potts model, (k_0) -translation-invariant Gibbs measure, (k_0) -periodic Gibbs measure.

A.S. Krivosheev, O.A. Krivosheeva

CONVERGENCE OF SERIES OF EXPONENTIAL MONOMIALS

Abstract. In the paper we study the convergence of series of exponential monomials, special cases of which are the series of exponentials, Dirichlet series and power series. We provide a description of the space of coefficients of series of exponential monomials converging in a given convex domain in the complex plane is described. Under a single natural restriction on the degrees of monomials, we provide a complete analogue of the Abel theorem for such series, which, in particular, implies results on the continued convergence of series of exponential monomials. We also obtain a complete analogue of the Cauchy-Hadamard theorem, in which we give a formula allowing to recover the convergence domain of these series by their coefficients. The obtained results include, as special cases, all previously known results related with the Abel and Cauchy-Hadamard theorems for exponential series, Dirichlet series and power series.

Keywords: exponential monomial, convex domain, Abel theorem, Cauchy-Hadamard theorem.

A.V. Lutsenko, I.Kh. Musin, R.S. Yulmukhametov

ON A CLASS OF PERIODIC FUNCTIONS IN \mathbb{R}^n

Abstract. By means of some family \mathcal{H} of separately radially convex in \mathbb{R}^n functions we defined a space $G(\mathcal{H})$ of 2π -periodic in each variable infinitely differentiable in \mathbb{R}^n functions with prescribed estimates on all partial derivatives. We describe the space $G(\mathcal{H})$ in terms of the Fourier coefficients. We find conditions on the family \mathcal{H} , under the functions from $G(\mathcal{H})$ can be continued to functions holomorphic in a tubular domain in \mathbb{C}^n . We obtain an inner description of the space of such continuations. The considered problems are directly related with works by P.L. Ul'yanov in the end of 1980s, in which he succeeded to describe completely the classes of 2π -periodic functions of Gevrey type on the real axis not only by the decay rate of the Fourier coefficients but also in terms of the best trigonometric approximations. The obtained results are new both for the case of many variables and the case of a single variable. In particular, the novelty is owing to imposing condition i_4) on the family \mathcal{H} .

Keywords: Fourier series, Fourier coefficients, best possible approximation by trigonometric polynomials, entire functions, convex functions.

A.Yu. Popov, V.B. Sherstyukov

LOWER BOUND FOR MINIMUM MODULUS OF ENTIRE FUNCTION OF GENUS ZERO WITH POSITIVE ROOTS IN TERMS OF DEGREE OF MAXIMUM MODULUS AT FREQUENT SEQUENCE OF POINTS

Abstract. We consider entire function of genus zero, the roots of which are located at a single ray. On the class of all such functions, we obtain close to optimal lower bounds for the minimum of the absolute value on a sequence of the circumferences in terms of a negative power of the maximum of the absolute value on the same circumferences under a restriction on the ratio $a > 1$ of the radii of neighbouring circumferences. We introduce the notion of the optimal exponent $d(a)$ as an extremal exponent of the maximum of the absolute value in this problem. We prove two-sided estimates for the optimal exponent for a «test» value $a = 9/4$ and for $a \in (1, 9/8]$. We find an asymptotics for $d(a)$ as $a \rightarrow 1$. The obtained result differ principally from the

classical $\cos(\pi\rho)$ -theorem containing no restrictions for the frequencies of the radii of the circumferences, on which the minimum of the absolute value of an entire function of order $\rho \in [0, 1]$ is estimated by a power of the maximum of its absolute value.

Keywords: entire function, minimum of absolute value, maximum of absolute value.

M.S. Tikhov

NEGATIVE BINOMIAL REGRESSION IN DOSE-EFFECT RELATIONSHIPS

Abstract. This paper is devoted to problem on estimating the distribution function and its quantiles in the dose-effect relationships with nonparametric negative binomial regression. Most of the mathematical researches on dose-response relationships concerned models with binomial regression, in particular, models with binary data. Here we propose a kernel-based estimates for the distribution function, the kernels of which are weighted by a negative binomial random variable at each covariate. These covariates are quasirandom van der Corput and Halton low-discrepancy sequences. Our estimates are consistent, that is, they converge to their optimal values in probability as the number of observations n grows to infinity. The proposed estimates are compared by their mean-square errors. We show that our estimates have a smaller asymptotic variance in comparison, in particular, with estimates of the Nadaraya-Watson type and other estimates. We present nonparametric estimates for the quantiles obtained by inverting a kernel estimate of the distribution function. We show that the asymptotic normality of these bias-adjusted estimates is preserved under some regularity conditions. We also provide a multidimensional generalization of the obtained results.

Keywords: negative binomial response model, effective dose level, nonparametric estimate.

I.T. Habibullin, A.R. Khakimova

ALGEBRAIC REDUCTIONS OF DISCRETE EQUATIONS OF HIROTA-MIWA TYPE

Abstract. For nonlinear discrete equations in the dimension $1 + 1$ there are easily checked symmetry criterions of integrability which lie in the base of the classification algorithms. The relevant problem of creating effective methods for classifying integrable discrete equations with three or more independent variables remains open, since in the multidimensional the symmetry approach loses its effectiveness due to difficulties related with non-localities.

In our recent works we discovered a specific property of discrete equations in the three-dimensional case which seems to be an effective criterion for the integrability of three-dimensional equations. It turned out that many known integrable chains including equations like two-dimensional Toda chain, equation of Toda type with one continuous and two discrete independent variables, equations of Hirota-Miwa type, where all independent variables are discrete are characterized by the fact that they admit breaks of special form in one of discrete variables which reduce the chain to a system of equations with two independent variables possessing an increased integrability; they possess complete sets of the integrals in each of the characteristics, that is, they are integrable in the Darboux sense. In other words, the characteristic algebras of the obtained finite-field systems have a finite dimension. In this paper, we give examples that confirm the conjecture that the presence of a hierarchy of two-dimensional reductions integrable in the sense of Darboux is inherent in all integrable

discrete equations of the Hirota-Miwa type. Namely we check that the lattice Toda equation and its modified analogue also admit the aforementioned reduction.

Keywords: integrability, lattice Toda equation, characteristic integrals, characteristic algebra.

B.P. Allahverdiev, H. Tuna

SINGULAR HAHN–HAMILTONIAN SYSTEMS

Abstract. In this work, we study a Hahn–Hamiltonian system in the singular case. For this system, the Titchmarsh–Weyl theory is established. In this context, the first part provides a summary of the relevant literature and some necessary fundamental concepts of the Hahn calculus. To pass from the Hahn difference expression to operators, we define the Hilbert space $L^2_{\omega, q, W}((\omega_0, \infty); \mathbb{C}^{2n})$ in the second part of the work. The corresponding maximal operator L_{\max} are introduced. For the Hahn–Hamiltonian system, we proved Green formula. Then we introduce a regular self-adjoint Hahn–Hamiltonian system. In the third part of the work, we study Titchmarsh–Weyl functions $M(\lambda)$ and circles $\mathcal{C}(a, \lambda)$ for this system. These circles proved to be embedded one to another. The number of square-integrable solutions of the Hahn–Hamilton system is studied. In the fourth part of the work, we obtain boundary conditions in the singular case. Finally, we define a self-adjoint operator in the fifth part of the work.

Keywords: Hahn–Hamiltonian system, singular point, Titchmarsh–Weyl theory.

V.I. Oseledets, V.L. Kulikov, E.F. Olekhova

REMARKS ON GARSIA ENTROPY AND MULTIDIMENSIONAL ERDÖS MEASURES

Abstract. We conjecture that the Garsia entropy coincides with the entropy of the invariant multidimensional Erdős measure. This conjecture is true for all Garsia numbers. We also specify the algebraic unit being non-Pisot number, for which this conjecture is true.

We prove a theorem, which generalizes the Garsia theorem on the absolute continuity of the infinite Bernoulli convolution for the Garsia numbers. The proof uses relations between the multidimensional Erdős problem and the one-dimensional Erdős problem.

We discuss a connection between the entropy of the invariant Erdős measure and the conditional Ledrappier–Young entropies. We also formulate three conjectures and obtain some consequences from them. In particular, we conjecture that the Hausdorff dimension of the Erdős measure is equal to the Ledrappier–Young dimension of conditional measure for the multidimensional invariant Erdős measure along the unstable foliation corresponding to the top Lyapunov exponent of multiplicity 1. For 2-numbers, we obtain formulae for the Hausdorff dimension of Erdős measures on the unstable plane.

Keywords: Garsia entropy, Hausdorff dimension of the measure, Erdős measure, Hochman formula, Lyapunov exponent.