

ABSTRACTS

F.G. Avkhadiev

UNIVERSAL INEQUALITIES ON DOMAINS
IN EUCLIDEAN SPACE AND THEIR APPLICATIONS

Abstract. In domains in Euclidean spaces, for test functions, we construct and prove several new Gagliardo-Nirenberg type inequalities with explicit constants. These inequalities are true in any domain, they are nonlinear, integrand functions involve the powers of the absolute values of the gradient and the Laplacian of a test function u , as well as factors of type $f(|u(x)|)$, $f'(|u(x)|)$, where f is a continuously differentiable non-decaying function, $f(0) = 0$. As weight functions, the powers of the distance from a point to the boundary of the domain serve as well as the powers of the varying hyperbolic (conformal) radius.

As applications of universal inequalities of Gagliardo-Nirenberg type we obtain new integral Rellich type inequalities in planar domains with uniformly perfect boundaries. For these Rellich type L_p -inequalities we establish the criterions of the positivity of the constants, obtain two-sided estimates for these constants depending on the Euclidean maximal modulus of the domain and on the parameter $p \geq 2$. In the proof we use several scalar characteristics for domains with uniformly perfect boundaries.

Keywords: Gagliardo-Nirenberg type inequality, distance to the boundary, hyperbolic radius, uniformly perfect set.

G.G. Braichev, O.V. Sherstyukova

ON THE LEAST TYPE OF AN ENTIRE FUNCTION
WITH A GIVEN SUBSEQUENCE OF ZEROS

Abstract. This note is based on the authors' report on International Scientific Conference «Ufa Autumn Mathematical School – 2021». We discuss the following problem. Let we are given a non-integer number $\rho > 0$ and a sequence of complex numbers Λ having a finite upper ρ -density. Then, as it is known by the classical Lindelöf theorem, there exists a (not identically zero) entire function f of a finite type with respect to the order ρ , for which Λ is a sequence of all its zeroes. The question is how much can the type of such function change if, apart of the elements in Λ , it can have other zeroes of an arbitrary multiplicity. We show the possibilities of applying one general theorem proved by B.N. Khabibullin in 2009. In order to do this we use recent results containing the exact formulae for calculating extremal type in classes of entire functions with various restrictions on the distribution of zeroes. The case of entire ρ possesses certain features and in this work we almost not consider it.

Keywords: entire function, sequence of zeroes, subsequence of zeroes, type of entire function, extremal problem.

A.M. Gaisin, T.I. Belous

THE MAXIMUM TERM OF A DIRICHLET SERIES CONVERGING IN THE HALF-PLANE: A THEOREM ON STABILITY

Abstract. We consider a problem on equivalence of logarithms of maximal terms in the Hadamard composition (modified series) $\sum_n a_n b_n e^{\lambda_n z}$ of the Dirichlet series $\sum_n a_n e^{\lambda_n z}$ and $\sum_n b_n e^{\lambda_n z}$ with positive exponents, the convergence domain of which is a half-plane. A similar problem for entire Dirichlet series was first studied by A.M. Gaisin in 2003 and there was obtained a criterion of the stability of the maximal term $\mu(\sigma) = \max_{n \geq 1} \{|a_n| e^{\lambda_n \sigma}\}$. This result turned out to be useful in studying asymptotic properties of the Dirichlet series on arbitrary curves going to infinity, namely, in the proof of the famous Pólya conjecture.

Both in the case of entire Dirichlet series and ones converging only in the half-plane, a key role in such problems is played by Leontiev formulae for the coefficients. The functions of the corresponding biorthogonal system contains a factor, which the derivative of a characteristic function at the points λ_n , $n \geq 1$. This fact naturally leads to the considered here problem on the stability of the maximal term.

We obtain a criterion ensuring the equivalence of logarithm of the maximal term in the Dirichlet series, the convergence domain of which is a half-plane, to the logarithm of the maximal term of the modified series on an asymptotic series.

Keywords: Dirichlet series, convergence half-plane, maximal term, Hadamard composition, asymptotic set.

S.V. Grishin

APPLICATION OF GENERATING FUNCTIONS TO PROBLEMS OF RANDOM WALK

Abstract. We consider a problem on determining the time of first hitting a positive semi-axis under a homogenous discrete integer random walk on a line. More precisely, the object of our study is the graph of the generating function of the mentioned random variable. For the random walk with the maximal positive increment 1, we obtain the equation on the implicit generating function, which implies the rationality of the inverse generating function. In this case, we find the mathematical expectation and dispersion for the time of first hitting a positive semi-axis under a homogenous discrete integer random walk on a line. We describe a general method for deriving systems of equations for the time of first hitting a positive semi-axis under a homogenous discrete integer random walk on a line. For a random walk with increments $-1, 0, 1, 2$ we derive an algebraic equation for the implicit generating function. We prove that a corresponding planar algebraic curve containing the graph of generating function is rational. We formulate and prove several general properties of the generating function the time of first hitting a positive semi-axis under a homogenous discrete integer random walk on a line.

Keywords: generating function, random walk.

I.Sh. Jabbarov, N.E. Allakhyarova

ON INTEGRAL EQUATIONS OF FREDHOLM KIND
IN BOHR SPACE OF ALMOST PERIODIC FUNCTIONS

Abstract. In the present work we consider a question on extending the notion of the Fredholm integral equation or integral equation of second kind, which allows one to consider the issue on existence of solutions in the space of almost periodic functions. Almost periodic functions are defined on entire line. This is why it seems difficult to describe them by some characteristics on finite intervals.

The Fredholm equations are known to be closely related with first order differential equations. In some particular cases there were posed the questions on finding the solutions in various classes of almost periodic functions. In some known cases there are no solutions in the Bohr class for such equations with almost periodic coefficients.

There are known examples of almost periodic functions (in Besicovitch sense), which can not be solutions for a rather wide class of differential equations. It is natural to expect that in the general case the integral equations are also not solvable in Bohr class of almost-periodic functions. This is why a more specific approach is needed for the problem in the space of almost-periodic functions.

Keywords: almost-periodic functions, Bohr classes, Fredholm equation, integral equation, differential equation.

O.A. Ivanova, S.N. Melikhov

HADAMARD TYPE OPERATORS
IN SPACE OF HOLOMORPHIC FUNCTIONS ON A BALL

Abstract. We study Hadamard type operators in the spaces of functions holomorphic in an open ball in \mathbb{C}^N centered at the origin. These linear continuous operators, for which each monomial is an eigenvector. We obtain a representation of Hadamard operators in the form of a multiplicative convolution. The proof of this representation employs essentially Fantappie transformation realizing dual to the spaces of holomorphic functions and the holomorphy property of the characteristic function of a linear continuous operator in them. The applied method allows us to reduce the problem on representation of a Hadamard operator to the problem on holomorphic continuation of a function holomorphic at the point 0 into a given open ball in \mathbb{C}^N with l_1 -norm. We prove that the space of the Hadamard type operator from one mentioned space into another with the topology of the bounded convergence is linearly topologically isomorphic to the strong dual to the space of the germs of all functions holomorphic on a closed polycircle.

Keywords: Hadamard type operator, space of holomorphic functions.

A.A. Klyachin

ON C^1 -CONVERGENCE OF PIECEWISE POLYNOMIAL SOLUTIONS TO A FOURTH ORDER VARIATIONAL EQUATION

Abstract. In the present work we consider a boundary value problem in a polygonal domain for a fourth order variational equation. We assume that this domain is partitioned into finitely many triangles forming its triangulation. We introduce a class of piece-wise polynomial functions of a given degree and for a considered equation we define the notion of a piece-wise polynomial solution on a triangle net. We prove a theorem on existence and uniqueness of such solution. Moreover, we establish that under certain conditions for the triangulation of the domain, the second derivatives of the piece-wise polynomial solutions are estimated by a constant independent of the fineness of the partition. This fact allows us to prove C^1 -convergence of piece-wise polynomial solutions to the equations as the fineness of the net partition tends to zero.

Keywords: biharmonic functions, triangular net, piece-wise polynomial function, error of calculation.

A.B. Kostin, V.B. Sherstyukov

ON TAYLOR COEFFICIENTS OF ANALYTIC FUNCTION RELATED WITH EULER NUMBER

Abstract. We consider a classical construction of second remarkable limit. We pose a question on asymptotically sharp describing the character of such approximation of the number e . In view of this we need the information on behavior of the coefficients in the power expansion for the function $f(x) = e^{-1}(1+x)^{1/x}$ converging in the interval $-1 < x < 1$. We obtain a recurrent rule regulating the forming of the mentioned coefficients. We show that the coefficients form a sign-alternating sequence of rational numbers $(-1)^n a_n$, where $n \in \mathbb{N} \cup \{0\}$ and $a_0 = 1$, the absolute values of which strictly decay. On the base of the Faà di Bruno formula for the derivatives of a composition of functions we propose a combinatorial way of calculating the numbers a_n as $n \in \mathbb{N}$. The original function $f(x)$ is the restriction of the function $f(z)$ on the real ray $x > -1$ having the same Taylor coefficients and being analytic in the complex plane \mathbb{C} with the cut along $(-\infty, -1]$. By the methods of the complex analysis we obtain an integral representation for a_n for each value of the parameter $n \in \mathbb{N}$. We prove that $a_n \rightarrow 1/e$ as $n \rightarrow \infty$ and find the convergence rate of the difference $a_n - 1/e$ to zero. We also discuss the issue on choosing the contour in the integral Cauchy formula for calculating the Taylor coefficients $(-1)^n a_n$ of the function $f(z)$. We find the exact values of arising in calculations special improper integral. The results of the made study allows us to give a series of general two-sided estimates for the deviation $e - (1+x)^{1/x}$ consistent with the asymptotics of $f(x)$ as $x \rightarrow 0$. We discuss the possibilities of applying the obtained statements.

Keywords: Euler number, analytic function, Taylor coefficients, Faà di Bruno formula, integral representation, asymptotic behavior.

Y.M. Mukhsinov

ABOUT ONE DIFFERENTIAL GAME OF NEUTRAL TYPE WITH INTEGRAL RESTRICTIONS IN HILBERT SPACE

Abstract. In the theory of differential games, when the game is defined in a finite-dimensional space, the fundamental works belong to academicians L.S. Pontryagin and N.N. Krasovskii. The works by N.N. Krasovskii and his students are mostly devoted to position games. In works by L.S. Pontryagin and his students the differential game is considered separately from the point of views of the pursuer and the evader and this unavoidably relates the differential game with two different problems. It is topical to study the games in finite-dimensional spaces since many important problems on optimal control under the conditions of a conflict or uncertainty governed by distributed systems, the motion of which is described by integro-differential equations and partial differential equations can be studied as differential games in appropriate Banach spaces.

In the present work, in a Hilbert space, we consider a pursuit problem in the Pontryagin sense for a quasilinear differential game, when the dynamics of the game is described by a functional-differential equation of neutral type in the form of J. Hale with a linear closed operator and on the control of the players integral restrictions are imposed. We prove an auxiliary lemma and four theorems on sufficient conditions ensuring the solvability of the pursuit problem. In the lemma we show that the corresponding inhomogeneous Cauchy problem for the considered game has a solution in the sense of J. Hale. In the theorems we employ a construction similar to the Pontryagin first direct method and the idea by M.S. Nikolskii and D. Zonnevend on dilatation of time $J(t)$ and describe the sets of initial states, from which the termination of the pursuit is possible.

Keywords: pursuit problem, differential game of neutral type, integral restrictions for controls of players, Hilbert space.

R.G. Nasibullin

ONE-DIMENSIONAL L_p -HARDY-TYPE INEQUALITIES FOR SPECIAL WEIGHT FUNCTIONS AND THEIR APPLICATIONS

Abstract. We establish one-dimensional L_p -Hardy inequalities with additional terms and use them for justifying their multidimensional analogues in convex domains with finite volumes. We obtain variational inequalities with power-law weights being generalizations of the corresponding inequalities presented earlier in articles by M. Hoffmann-Ostenhof, T. Hoffmann-Ostenhof, A. Laptev and J. Tidblom. We formulate and prove inequalities valid for arbitrary domains, and then we simplify them substantially for the class of convex domains. The constants in the additional terms in these spatial inequalities depend on the volume or on the diameter of the domain. As a corollary of the obtained results we get estimates for the first eigenvalue of the p -Laplacian subject to the Dirichlet boundary conditions.

Keywords: Hardy inequality, additional term, one-dimensional inequality, distance function, volume of a domain, diameter of a domain, first eigenvalue of the Dirichlet problem.

A. Testici

MAXIMAL CONVERGENCE OF FABER SERIES IN WEIGHTED REARRANGEMENT INVARIANT SMIRNOV CLASSES

Abstract. Let K be a bounded set on the complex plane \mathbb{C} with a connected complement $K^- := \overline{\mathbb{C}} \setminus K$. Let $\mathbb{D} := \{w \in \mathbb{C} : |w| < 1\}$ and $\mathbb{D}^- := \overline{\mathbb{C}} \setminus \overline{\mathbb{D}}$. By φ we denote the conformal mapping of K^- onto $\{w \in \mathbb{C} : |w| > 1\}$ normalized by the conditions $\varphi(\infty) = \infty$ and $\lim_{z \rightarrow \infty} \varphi(z)/z > 0$. Let $\Gamma_R := \{z \in K^- : |\varphi(z)| = R > 1\}$ and $G_R := \text{Int} \Gamma_R$. Let also $\Phi_k(z)$, $k = 0, 1, 2, \dots$ be the Faber polynomials for K constructed via conformal mapping φ . As it is well known, if f is an analytic function in G_R , then the representation $f(z) = \sum_{k=0}^{\infty} a_k(f) \Phi_k(z)$, $z \in G_R$ holds. The partial sums of Faber series play an important role in constructing approximations in complex plane and investigating properties of Faber series is one of the essential issue. In this work the maximal convergence of the partial sums of the partial sums of the Faber series of f in weighted rearrangement invariant Smirnov class $E_X(G_R, \omega)$ of analytic functions in G_R is studied. Here the weight ω satisfies the Muckenhoupt condition on Γ_R . The estimates are given in the uniform norm on K . The right sides of obtained inequalities involve the powers of the parameter R and $E_n(f, G)_{X, \omega}$ called the best approximation number of f in $E_X(G_R, \omega)$, defined as $E_n(f, G)_{X, \omega} := \inf \left\{ \|f - P_n\|_{X(\Gamma, \omega)} : P_n \in \Pi_n \right\}$. Here Π_n is the class of algebraic polynomials of degree not exceeding n . These results given in this paper is a kind of generalisation of similar results obtained in the classical Smirnov classes.

Keywords: Maximal convergence, Banach function space, Faber series, weighted rearrangement invariant space.

A. Sukhov

ON BOUNDARY PROPERTIES OF ASYMPTOTICALLY HOLOMORPHIC FUNCTIONS

Abstract. It is well known that for a generic almost complex structure on an almost complex manifold (M, J) all holomorphic (even locally) functions are constants. For this reason the analysis on almost complex manifolds concerns the classes of functions which satisfy the Cauchy-Riemann equations only approximately. The choice of such a condition depends on a considered problem. For example, in the study of zero sets of functions the quasiconformal type conditions are very natural. This was confirmed by the famous work of S. Donaldson. In order to study the boundary properties of classes of functions (on a manifold with boundary) other type of conditions are suitable. In the present paper we prove a Fatou type theorem for bounded functions with $\bar{\partial}_J$ differential of a controlled growth on smoothly bounded domains in an almost complex manifold. The obtained result is new even in the case of \mathbb{C}^n with the standard complex structure. Furthermore, in the case of \mathbb{C}^n we obtain results with optimal regularity assumptions. This generalizes several known results.

Keywords: almost complex manifold, $\bar{\partial}$ -operator, admissible region, Fatou theorem.