# ABSTRACTS

## V.A. Glazatov, V.Zh. Sakbaev

### Measures on Hilbert space that are invariant with respect to Hamiltonian flows

**Abstract.** We study Hamiltonian flows in a real separable Hilbert space endowed with a symplectic structure. We study measures on the Hilbert space which are invariant with respect to the flows of completely integrable Hamiltonian systems are investigated and which allow one to describe Hamiltonian flows in phase space by means of unitary groups in the space of functions square integrable with respect to an invariant measure. The introduced measures invariant with respect to the flows of completely integrable Hamiltonian systems are applied for studying model linear Hamiltonian systems admitting singularities as unbounded increasing of a kinetic energy in a finite time. Owing to such approach, the solutions of the Hamilton equations having singularities can be described by means of the phase flow in the extended phase space and by the corresponding Koopman representation of the unitary group.

**Keywords:** shift invariant measure, Weyl theorem, Hamiltonian flow, Koopman presentation.

## R.N. Gumerov, R.L. Khazhin

### On divisible quantum dynamical mappings

**Abstract.** In the paper we study quantum dynamical mappings called also quantum processes. The set of values of such mapping is a one-parameter family of completely positive trace-preserving linear operators defined on a finite-dimensional Hilbert space. In quantum information theory such operators are referred to as quantum channels. An important concept for quantum dynamical mappings is their divisibility. There are different types of this concept. The present paper deals with so-called completely positive divisible quantum processes. For two such processes, which are bijective and satisfy a commutativity condition, we construct a compound quantum process. It is shown that this compound quantum process is also completely positive divisible. Endowing a set of quantum channels with the norm topology, we consider continuous quantum processes and continuous completely positive evolutions. The latter are defined as two-parameter families of quantum channels satisfying additional properties. We prove that a continuous bijective completely positive divisible quantum process generates a continuous completely positive evolution. In order to illustrate the considered concepts and the results on them, we provide examples of quantum dynamical mappings with values in the set of qubit channels. In particular, a completely positive divisible compound quantum process is constructed for two bijective commuting quantum processes. Geometric and physical interpretations of this compound quantum process are given.

**Keywords:** Banach algebra, bijective process, completely positive divisible process, compound process, continuous completely positive evolution, positive divisible process, operator norm, quantum channel, quantum dynamical mapping, quantum process, topological group, trace norm.

## K.N. Zhuikov, A.Yu. Savin

### Eta-invariant for parameter-dependent families with periodic coefficients

**Abstract.** On a closed smooth manifold, we consider operator families being linear combinations of parameter-dependent pseudodifferential operators with periodic coefficients. Such families arise in studying nonlocal elliptic problems on manifolds with isolated singularities and/or with cylindrical ends. The aim of the work is to construct the $\eta$-invariant for invertible families and to study its properties. We follow Melrose approach who treated the $\eta$-invariant as a generalization of the number of rotations being equal to the trace of the logarithmic derivative of the family. At the same time, the Melrose $\eta$-invariant is equal to the regularized integral of the regularized trace of the logarithmic derivative of the family. In our situation, for the trace regularization, we employ the operator of difference differentiating instead of the usual differentation used by Melrose. The main technical result is the fact the operator of difference differentiation is an isomorphism between the spaces of the functions with a conormal asymptotics at infinity and this allows us to determine the regularized trace. Since the obtained regularized trace can increase at infinity, we also introduce a regularization for the integral. Our integral regularization involves an averaging operation. Then we establish the main properties of the $\eta$-invariant. Namely, the $\eta$-invariant in the sense of this work satisfies the logarithmic property and is a generalization of Melrose $\eta$-invariant, that is, it coincides with it for usual pseudodifferential operators with a parameter. Finally, we provide a formula for the variation of the $\eta$-invariant under a variation of the family.

**Keywords:** elliptic operator, parameter-dependent operator, $\eta$-invariant, difference differentiation.

## V.L. Leontiev

### Fourier method connected with orthogonal splines in parabolic initial boundary-value problem for region with curvilinear boundary

**Abstract.** The Fourier method allows one to find the solutions of boundary value problems and initial boundary value problems for partial differential equations admitting the separation of variables. The application of the method for problems of many types faces significant difficulties. One of the direction on extending the domain of applicability of the Fourier method is to overcome the mathematical problems related with this method, for instance, ones related with a nature of boundary conditions. Another direction concerns the usage of special functions for the domains of classical forms defined by coordinate lines and surfaces of orthogonal curvilinear coordinates. But in the general case of domains with curved boundaries such approach is not effective. The directions of developing the Fourier method for solving problems in domains with curved boundary are related also, first, with developing and applying

variation grid and projection grid method and second, with a modification of the Fourier method itself. The present paper belongs t to the second direction and is aimed on an extension of the domain of applicability of the Fourier method, which is determined by constructing a sequence of finite generalized Fourier series related with orthogonal splines and giving analytic solutions to a parabolic initial boundary value problem in the domain with a curved boundary. For such problem, we propose and study an algorithm of the Fourier method related with the application of orthogonal splines. A sequence of finite generalized Fourier series generated by this algorithm converges to the exact solution given by an infinite Fourier series at each time moment. While increasing the number of the nodes in the grid in the considered domain with a curved boundary, the structure of the finite Fourier series approaches the structure of an infinite Fourier series being an exact solution of initial boundary value problem. The method provides approximated analytic solutions with an arbitrary small error in the form of orthogonal series, which are generalized Fourier series, and this gives new opportunities of the classical Fourier method.

**Keywords:** parabolic initial boundary value problem, curved boundary, separation of variables, generalized Fourier series, orthogonal splines.

## E.V. Lipacheva

### Trivial extensions of semigroups and semigroup $C^*$-algebras

**Abstract.** The object of the study in the paper is reduced semigroup $C^*$-algebras for left cancellative semigroups. Such algebra is a very natural object because it is generated by isometric shift operators belonging to the image of the left regular representation of a left cancellative semigroup. These operators act in the Hilbert space consisting of all square summable complex-valued functions defined on a semigroup. We study the question on functoriality of involutive homomorphisms of semigroup $C^*$-algebras, that is, the question on the existence of the canonical embedding of semigroup $C^*$-algebras which is induced by an embedding of corresponding semigroups. In order to do this, we investigate the reduced semigroup $C^*$-algebras associated with semigroups which constitute the normal extensions of semigroups by groups. At the same time, in the paper we consider the case of one of the simplest classes of extensions, namely, the class of so-called trivial extensions. It is shown that if a semigroup $L$ is a trivial extension of the semigroup $S$ by means of a group $G$, then there exists the embedding of the reduced semigroup $C^*$-algebra $C_r^*(S)$ into the $C^*$-algebra $C_r^*(L)$ which is induced by an embedding of a semigroup $S$ into a semigroup $L$.

In the work we also introduce and study the structure of a Banach $C_r^*(S)$-module on the underlying space of the reduced semigroup $C^*$-algebra $C_r^*(L)$. To do this, we use a topological grading for the $C^*$-algebra $C_r^*(L)$ over the group $G$. In the case when a semigroup $L$ is a trivial extension of a semigroup $S$ by means of a finite group, we prove the existence of the structure of a free Banach module over the reduced semigroup $C^*$-algebra $C_r^*(S)$ on the underlying Banach space of the semigroup $C^*$-algebra $C_r^*(L)$.

We give examples of extensions of semigroups and reduced semigroup $C^*$-algebras for a more complete characterization of the issues under consideration and for revealing connections with previous results.

**Keywords:** cancellative semigroup, normal extension of a semigroup, trivial extension of a semigroup, reduced semigroup $C^*$-algebra, embedding a semigroup $C^*$-algebra, Banach module, free module.

## S.V. Khabirov, T.F. Mukminov

### Simple waves of conic motions

**Abstract.** Continuous media models of a gas dynamical type admit 11-dimensional Lie algebra of Galileo group extended by an uniform dilatation of all independent variables. The object of the study is the constructing of submodels of the chain of embedded subalgebras with dimensions from 1 till 4 describing conical motions of the gas. For the chose chain with find consistent invariant in the cylindrical coordinate system. On their base of obtain the representations for an invariant solution for each submodel in the chain. By substituting them into the system of gas dynamics equations we obtain embedded invariant submodels of ranks from 0 to 3. We prove that the solutions of submodels constructed by a subalgebra of a higher dimension are solutions to submodels constructed by subalgebras of smaller dimensions.

In the chosen chain, we consider a 4-dimensional subalgebra generating irregular partially invariant solutions of rank 1 defect 1 in the cylindrical coordinates. In the gas dynamics, such solutions are called simple waves. We study the compatibility of the corresponding submodel by means of the system of alternative assumptions obtained from the submodel equations. We obtain solutions depending on arbitrary functions as well as partial solutions which can be invariant with respect to the subalgebras embedded into the considered subalgebra but are not necessarily from the considered chain.

**Keywords:** gas dynamics, chain of embedded subalgebra, consistent invariants, invariant submodels, partially invariant solutions.

## S.G. Haliullin

### Ultraproducts of quantum mechanical systems

**Abstract.** The study of ultraproducts for various spaces is motivated by an interest in methods of non-standard mathematical analysis, which operates on infinitesimal (or infinitely large) sequences as if they were numbers. On the one hand, a space obtained as a set-theoretic ultraproduct of a sequence of spaces becomes very «rich». On the other hand, it loses some attractive properties of factors. In particular, it has no a natural Hausdorff topology generated by its factors, and the natural $\sigma$-algebra of its measurable subsets is not countably generated.

If a space «is embedded» into its ultrapower with the preservation of required properties, then the usage of the ultraproduct technique gives some advantages in proving many «standard» assertions.

In order to preserve various properties of factors, we need to change the construction of an ultraproduct. For example, by changing the construction of an ultraproduct, it becomes possible to preserve the Hausdorff topology, the structure of a normed space, the structure of operator algebras, von Neumann algebras, and so on.

In this paper we discuss the stochastic properties of the so-called quantum mechanical systems in a rather abstract form. Such systems (structures) arise in probability theory, in the theory of operator algebras and in the theory of topological vector spaces. The ultraproducts for sequences of such structures are also defined, and certain properties of these ultraproducts are investigated.

The notion of an observable on an event structure is an analogue of a random variable defined on a probability space. An observable is naturally given in the ultraproduct of quantum mechanical systems which is defined in the present paper. We study its probabilistic characteristics. Moreover, ultraproducts of quantum logics are also considered within the framework of ultraproducts for quantum mechanical systems.

**Keywords:** event structures, ultraproduct, quantum logics.

## Yu.Kh. Eshkabilov, D.J. Kulturaev

### On discrete spectrum of one two-particle lattice Hamiltonian

**Abstract.** Linear self-adjoint operators in the Friedrichs models arise in various fields, for instance, in the perturbation theory of spectra of self-adjoint operators, in the quantum field theory, in theory of two- and three-particle discrete Schrödinger operators, in hydrodynamics, etc. An operator $H$ in the Friedrichs model is a sum of two operators in the Hilbert space $L_2(\Omega)$, that is, $H = H_0 + \varepsilon K$, $\varepsilon > 0$, where $H_0$ is the operator by multiplication by a function and $K$ is a compact integral operator. For the operators in the Friedrichs models we need to solve the following problems:

1) Under which conditions the discrete spectrum is an empty set?

2) Under which conditions the discrete spectrum is a non-empty set?

3) Find conditions ensuring that an operator in the Friedrichs model is a finite set;

4) Find sufficient conditions guaranteeing that an operator in the Friedrichs model is an infinite set.

It is known that if a kernel of an integral operator in the model is degenerate, then the discrete spectum of the correponding operator in the Friedrichs model is a finite set. Therefore, a necessary condition for the operator in the Friedrichs model to possess an infinite discrete spectrum is the non-degeneracy of the integral operator in the model. In the paper we consider linear bounded self-adjoint operator in the Friedrichs model, for which the integral operator has a non-degenerate kernel. In this work, we study the first and the fourth questions. We obtain one sufficient condition guaranteeing that the operators in Friedrichs model possesses an infinite discrete spectrum. We study the spectrum of one two-particle discrete Schrödinger operator $Q(\varepsilon)$ on the lattice $\mathbb{Z}^\nu \times \mathbb{Z}^\nu$, in which the Fourier transform of the operator $Q(\varepsilon)$ is represented as $H = H_0 + \varepsilon K$, $\varepsilon > 0$. It is shown that the structure of the Schrödinger operator $Q(\varepsilon)$ highly depends on the dimension $\nu$ of the lattice. It is proven that in the case $\nu = 1, 2$, for all $\varepsilon > 0$ the discrete spectrum of the Schrödinger operator $Q(\varepsilon)$ is infinite, while in the case $\nu \geqslant 3$, for sufficiently small $\varepsilon > 0$, the discrete spectrum of the Schrödinger operator $Q(\varepsilon)$ is an empty set.

**Keywords:** Friedrichs model, two-partical Hamiltonian, self-adjoint operator, spectrum, essential spectrum, discrete spectrum, non-degenerate kernel.

**A.J. Mkrtchyan**

## Continuability of multiple power series into sectorial domain by means of interpolation of coefficients

**Abstract.** We consider the problem on continuability a multiple power series centered at the origin of $\mathbb{C}^n$ into a sectorial domain. The condition of the mentioned continuability is given in terms of an entire function interpolating the coefficients of power series. We estimate the indicator function of the interpolating function with the help of which the sectorial set is determined. More precisely, the growth of the interpolating function on the imaginary subspace describes the sectorial set on which the series sum is extended. In the study we use methods of multivariate complex analysis, in particular, integral representations (Cauchy, Mellin, and Lindelof representations), multidimensional residues and properties of power series.

**Keywords:** multiple power series, analytic continuation, indicator of an entire function, multidimensional residues.