

## ABSTRACTS

**A.V. Atanov**

ORBITS OF DECOMPOSABLE 7-DIMENSIONAL  
LIE ALGEBRAS WITH  $\mathfrak{sl}(2)$  SUBALGEBRA

**Abstract.** The problem on complete classification of holomorphically homogeneous real hypersurfaces in two-dimensional complex spaces was resolved by E. Cartan in 1932. A similar description in the three-dimensional case was recently obtained by A. Loboda. In this work we discuss a part of classification of locally holomorphic homogeneous hypersurfaces in 4-dimensional complex space being orbits in  $\mathbb{C}^4$  by one family of 7-dimensional Lie algebra. As it was shown in works by Beloshapka, Kossovskii, Loboda and other, the ideas by E. Cartan allow one to obtain rather simply the descriptions of the orbits for the algebras having Abelian ideals for rather large dimensions. In particular, the presence of 4-dimensional Abelian ideal in 7-dimensional Lie algebra of holomorphic vector fields in  $\mathbb{C}^4$  often gives rise to the tubularity property for all orbits of such algebra. The Lie algebras in the family we consider are direct sums of the algebra  $\mathfrak{sl}(2)$  and several 4-dimensional Lie algebras and they have at most 3-dimensional Abelian subalgebras. By means of a technique of the simultaneous «flattening» of vector field we obtain a complete description of all Levi non-degenerate holomorphically homogeneous hypersurfaces being the orbits of the considered algebras in  $\mathbb{C}^4$ . Many of the obtained homogeneous hypersurfaces turn out to be tubular manifolds. At the same time, the issue on possible reduction of other hypersurfaces to tubes requires further studying. As an effective tool for such study, as well as for a detailed investigation of issues on holomorphic equivalent of the obtained orbits, the technique of Moser normal forms can serve. By means of this technique, we study the issue on the sphericity for representatives of one of the obtained family of hypersurfaces. However, the application of the method of normal forms for the hypersurfaces in complex spaces of dimension 4 and higher requires a further developing of this technique.

**Keywords:** homogeneous hypersurface, holomorphic transformation, decomposable Lie algebra.

**N.I. Zhukova, K.I. Sheina**

THE STRUCTURE OF FOLIATIONS WITH INTEGRABLE EHRESMANN CONNECTION

**Abstract.** We study foliations of arbitrary codimension  $q$  on  $n$ -dimensional smooth manifolds admitting an integrable Ehresmann connection. The category of such foliations is considered, where isomorphisms preserve both foliations and their Ehresmann connections. We show that this category can be considered as that of bifoliations covered by products. We introduce the notion of a canonical bifoliation and we prove that each foliation  $(M, F)$  with integrable Ehresmann connection is isomorphic to some canonical foliation. We introduce the concept of a structural

group of  $(M, F)$ . A category of triples is constructed and we prove that it is equivalent to the category of foliations with integrable Ehresmann connection. In this way, the classification of foliations with integrable Ehresman connection is reduced to the classification of associated diagonal actions of discrete groups of diffeomorphisms of the product of manifolds. We indicate the classes of foliations with integrable Ehresmann connection are indicated. We consider an application to  $G$ -foliations.

**Keywords:** Cartan foliation, integrable Ehresmann connection for a foliation, global holonomy group, structure group of a foliation, canonical foliation.

### B.I. Islomov, F.M. Juraev

LOCAL BOUNDARY VALUE PROBLEMS FOR A LOADED EQUATION  
OF PARABOLIC-HYPERBOLIC TYPE DEGENERATING INSIDE THE DOMAIN

**Abstract.** In the beginning of 21st century, boundary value problems for non-degenerating equations of hyperbolic, parabolic, hyperbolic-parabolic and elliptic-hyperbolic types were studied. Recently this direction is intensively studied since rather important problems in mathematical physics and biology lead to boundary value problems for non-degenerate loaded partial differential equations. Boundary value problems for second order degenerating equation of a mixed type were not studied before. This is first of all because of the fact that there is no representation for the general solution to this equations. On the other hand, such problems are reduced to poorly studied integral equations with a shift. The present work is devoted to formulating and studying local boundary value problems for loaded equation of parabolic-hyperbolic type degenerating inside the domain.

In the present work we find a new approach for obtaining a representation for the general solution to a degenerating loaded equation of a mixed type. The uniqueness of the formulated problem is proved by the methods of energy integrals. The existence of solutions to the formulated problems is equivalently reduced to a second order integral Fredholm and Volterra equations with a shift. We prove the unique solvability of the obtained integral equations.

**Keywords:** loaded equation of parabolic-hyperbolic type, loaded equation with a degeneration, representation of general solution, method of energy integrals, extremum principle, integral equation with a shift.

### A.V. Loboda, V.K. Kaverina

ON DEGENERACY OF ORBITS OF NILPOTENT LIE ALGEBRAS

**Abstract.** In the paper we discuss 7-dimensional orbits in  $\mathbb{C}^4$  of two families of nilpotent 7-dimensional Lie algebras; this is motivated by the problem on describing holomorphically homogeneous real hypersurfaces. Similar to nilpotent 5-dimensional algebras of holomorphic vector fields in  $\mathbb{C}^3$ , the most part of algebras considered in the paper has no Levi non-degenerate orbits. In particular, we prove the absence of such orbits for a family of decomposable 7-dimensional nilpotent Lie algebra (31 algebra). At the same time, in the family of 12 non-decomposable 7-dimensional nilpotent Lie algebras, each containing at least three Abelian 4-dimensional ideals, four algebras has non-degenerate orbits. The hypersurfaces

of two of these algebras are equivalent to quadrics, while non-specific non-degenerate orbits of other two algebras are holomorphically non-equivalent generalization for the case of 4-dimensional complex space of a known Winkelmann surface in the space  $\mathbb{C}^3$ . All orbits of the algebras in the second family admit tubular realizations.

**Keywords:** homogeneous manifold, holomorphic function, vector field, Lie algebra, Abelian ideal.

## G.U. Urazboev, I.I. Baltaeva

### INTEGRATION OF CAMASSA-HOLM EQUATION WITH A SELF-CONSISTENT SOURCE OF INTEGRAL TYPE

**Abstract.** The work is devoted to studying Camassa-Holm equation with a self-consistent of integral type.

The source of the consistent equation corresponds to the continuous spectrum of a spectral problem related with the Camassa-Holm equation. As it is known, integrable systems admit operator Lax representation  $L_t = [L, A]$ , where  $L$  is a linear operator, while  $A$  is some skew-symmetric operator acting in a Hilbert space. A generalized Lax representation for the considered equation is of the form  $L_t = [L, A] + C$ , where  $C$  is the sum of differential operators with coefficients depending on solutions of spectral problems for the operator  $L$ . The construction of self-consistent source for the considered operator is based on the fact that exactly squares of eigenfunctions of the spectral problems are essential while solving integrable equations by the inverse scattering transform. Moreover, for the considered type of equations the evolution of the eigenfunctions in the generalized Lax representation has a singularity.

The application of the inverse scattering transform is based on the spectral problem related with the classical Camassa-Holm equation. We describe the evolution of scattering data of this spectral problem with a potential being a solution of the Camassa-Holm equation with a self-consistent source. While describing the evolution of the spectral data, we employ essentially Sokhotski-Plemelj formula. The results of the work on the evolution of the scattering data related with the discrete spectrum are based on the methods used in the previous works by the authors. The obtained results, formulated as a main theorem, allow us to apply the inverse scattering transform for solving the Cauchy problem for the considered equation. Our technique can be easily extended to higher analogues of the Camassa-Holm equation.

**Keywords:** Camassa-Holm equation, Yost solution, self-consistent source, evolution of scattering data, inverse scattering transform.

## B.N. Biyarov, Z.A. Zakariyeva, G.K. Abdrasheva

### NON SELF-ADJOINT WELL-DEFINED RESTRICTIONS AND EXTENSIONS WITH REAL SPECTRUM

**Abstract.** In this paper we study the spectral properties of relatively bounded well-defined perturbations of the well-defined restrictions and extensions. The work is devoted to the study of the similarity of a well-defined restriction to some self-adjoint operator in the case when the minimal operator is symmetric. We show that the system of eigenvectors forms a Riesz basis in the case of discrete spectrum. The resulting theorem is applied to the Sturm-Liouville operator and the Laplace operator.

Singular perturbations for differential operators have been studied by many authors for the mathematical substantiation of solvable models of quantum mechanics, atomic physics, and solid state physics. For the Sturm-Liouville operator with a potential from the Sobolev space  $W_2^\alpha[0, 1]$  with  $-1 \leq \alpha \leq 0$ , the Riesz basis property of the system of eigenvectors in the Hilbert space  $L_2(0, 1)$  was proved. In all those cases, the problems were self-adjoint. In this paper, we consider non-self-adjoint singular perturbation problems for the Sturm-Liouville operator with a potential from the Sobolev space  $W_2^\alpha[0, 1]$  with  $-2 \leq \alpha \leq 0$ . We also obtained a similar result for the Laplace operator. A new method has been developed that allows investigating the considered problems. It is shown that the spectrum of a non-self-adjoint singularly perturbed operator is real and the corresponding system of eigenvectors forms a Riesz basis in the considered Hilbert space.

**Keywords:** maximal (minimal) operator, correct restriction, correct extension, real spectrum, non self-adjoint operator, perturbation.

## Ali Naziri-Kordkandi

### COMMUTATIVITY CONDITIONS IN PSEUDO-MICHAEL ALGEBRAS

**Abstract.** We consider the commutativity conditions in unital pseudo-Michael algebras. These kinds of algebras have interesting properties regarding the commutativity criteria. We prove several results, which generalize known results in the case of unital Arens-Michael algebras to the pseudo-convex cases. In this paper, we first derive some specific results for the differentiable and entire functions in pseudo-Michael algebras. Then we show how such results can be applied to obtain commutativity conditions for these algebras. In Section 3, we give simple conditions implying commutativity in the unital pseudo-Michael algebras. These conditions are equivalent to similar cases in unital locally  $m$ -convex algebras, in particular, in Banach algebras. The most outstanding results in this direction are due to Toma, who generalized the commutativity criteria of Banach algebras to locally  $m$ -convex algebras. Our conditions ensure the commutativity of pseudo-Michael algebras. In the proofs of some theorems, we apply the exponential functions and Liouville theorem for bounded holomorphic functions. The use of them allows us to give a very striking short proof. Finally as a consequence, we show that some commutativity results hold for  $k$ -Banach algebras.

**Keywords:** pseudo-michael algebra,  $k$ -seminorm,  $k$ -banach algebra, commutative.