ABSTRACTS

N.F. Abuzyarova

On condition of representing a subspace in Schwartz space invariant with respect to differentiation as direct sum of its residual and exponential components

Abstract. In the work we consider the Schwartz space $\mathcal{E}$ of infinitely differentiable functions on the real line and its closed subspaces invariant with respect to the differentiation operator. It is known that each such space possesses, possibly trivial, exponential and residual components, which are defined by a multiple sequence of points $(-\mathrm{i}\Lambda)$ in the complex plane (spectrum W) and by a relatively closed in $\mathbb{R}$ segment I_W (residual interval of the subspace W).

Recent studies showed that under certain restrictions for the behavior of Λ and I_W, the corresponding invariant subspace W is uniquely recovered by these characteristics, that is, it admits a spectral synthesis in a weak sense. In the case when the spectrum $(-\mathrm{i}\Lambda)$ is a finite sequence, the exponential component of the subspace W is finite-dimensional and the subspace W is the algebraic sum of the residual subspace and a finite-dimensional span of the set of exponential monomials contained in W. In the case of an infinite discrete spectrum we obtained the conditions, under which the algebraic sum of the residual and exponential subspaces in W is closed, and hence, it is a direct topological sum coinciding with W. These conditions were general but not convenient enough for straightforward checking. Here we obtain transparent easily checked conditions for the infinite sequence Λ, under which the invariant subspace W with the spectrum $(-\mathrm{i}\Lambda)$ and the residual interval I_W is a direct algebraic and topological sum of its exponential and residual components, that is, each element in W is uniquely represented as a sum of two functions, one of which is the limit of a sequence of exponential monomials in $\mathcal{E}$, while the other vanishes identically on I_W.

Keywords: invariant subspace, spectral synthesis, entire function, Schwartz space.

G.A. Gaisina

Representation of analytic functions by exponential series in half-plane with given growth majorant

Abstract. In the paper we study representations of analytic in the half-plane $\Pi_0 = \{z = x+iy\colon\ x > 0\}$ functions by the exponential series taking into consideration a given growth.

In the theory of exponential series one of fundamental results is the following the most general result by A.F. Leontiev: for each bounded convex domain D there exists a sequence $\{\lambda_n\}$ of complex numbers depending only on the given domain such that each function F analytic in D can be expanded into an exponential series $F(z) = \sum_{n=1}^{\infty} a_n e^{\lambda_n z}$, the convergence of which is uniform on compact sets D. Later a similar results on expansions into exponential series, but taking into consideration the growth, was also obtained by A.F. Leontiev for the space of analytic functions of finite order in a convex polygon. He also showed that the series of absolute values $\sum_{n=1}^{\infty} \left|a_n e^{\lambda_n z}\right|$ admits the same upper bound as the initial function F. In 1982, this fact was extended to the half-plane Π_0^+ by A.M. Gaisin.

In the present paper we study a similar case, when as a comparing function, some decreasing convex majorant serves and this majorant is unbounded in the vicinity of zero. In order to do this, we employ the methods of estimating based on the Legendre transform.

We prove a statement which generalizes the corresponding result by A.M. Gaisin on expanding analytic in half-plane functions into exponential series taking into consideration the growth order.

Keywords: analytic functions, exponential series, growth majorant, bilogarithmic Levinson condition.

F.N. Garif'yanov, E.V. Strezhneva

Sum-difference equation for analytic functions generated by a triangle and its applications

Abstract. Let D be a triangle and Γ by the half of its boundary ∂D. We consider an element-wise linear sum-difference equation in the class of functions holomorphic outside Γ and vanishing at infinity. The solution is sought in the form of a Cauchy-type integral over Γ with an unknown density. The boundary values satisfy the Hölder condition on each compact subset in Γ containing no nodes. At most logarithmic singularities are admitted at the nodes. In order to regularize the equation to ∂D, we introduce a piecewise linear Carleman shift. It maps each side into itself changing the orientation. In this case, the midpoints of the sides are fixed points. We regularize the equation and find its solvability condition for. We consider a particular case when the number of solvability conditions can be counted exactly. We provide applications to interpolation problems for entire functions of exponential type. Previously, similar problems were investigated for quadrilateral, pentagon, and hexagon.

Keywords: sum-difference equation, Carleman problem, equivalent regularization, interpolation problems for entire functions of exponential type.

K.Yu. Zamana

Averaging of random orthogonal transformations of independent variable of functions

Abstract. We consider and study the notion of a random operator, random operator-valued function and a random semigroup defined on a Hilbert space as well as their averaging. We obtain conditions under which the averaging a random strongly continuous function is also strongly continuous. In particular, we show that each random strongly continuous contracting operator-valued function possesses a strongly continuous contracting averaging.

We consider two particular random semigroups: a matrix semigroup of random orthogonal transformations of Euclidean space and a semigroup of operators defined on the Hilbert space of functions square integrable on the sphere in the Euclidean space and these operators describe random orthogonal transformations of the independent of the variables of these functions. The former semigroup is called a

semigroup of random rotations; it can be interpreted as a random walk on the sphere. We prove the existence of the averaging for both random semigroups.

We study an operator-valued function obtained by replacing the time variable t by $\sqrt{t}$ in averaging of the semigroup of random rotations. By means of Chernov theorem, under some conditions, we prove the convergence of the sequence of Feynman – Chernov iterations of this function to a strongly continuous semigroup describing the diffusion on the sphere in the Euclidean space. In order to do this, we first find and study the derivative of this operator-valued function at zero being at the same time the generator of the limiting semigroup. We obtain a simple divergent form of this generator. By means of this form we obtain conditions ensuring that this generator is a second order elliptic operator; under these conditions we prove that it is essentially self-adjoint.

Keywords: random linear operator, random operator-valued function, averaging, Feynman – Chernov iterations.

M.A. Komarov

Convergence rate of one class of differentiating sums

Abstract. We consider a differentiation formula for functions analytic in the circle $|z| < 1$: $azf'(z) = nf(0) - \sum_{k=1}^{n} f(\lambda_k z) + R_n(z)$. Here $a \neq 0$ is a real constant, $n = 1, 2, \ldots$, while complex parameters $\lambda_k = \lambda_{n,k}(a)$, $k = 1, \ldots, n$, are defined as the unique solution of a discrete system of momenta for Newtonian power sums $\lambda_1^m + \cdots + \lambda_n^m = -ma$, $m = 1, \ldots, n$. Under such choice of the parameters, the function $R_n(z) = R_n(a, f; z)$, which is the residual term in the formula, is of order $O(z^{n+1})$ as $z \to 0$. In this work we show that for each fixed $a > 0$ and each $n \geqslant 3\alpha$ ($\alpha := \max\{a; 1\}$) the domain of the applicability of the formula contains the circle $|z| < \exp(-3\sqrt{v} - 2v)$, $v := \alpha/(n+1)$, the radius of which tends to one as $n \to \infty$. We establish an exponential convergence rate of differentiating sums to $nf(0) - azf'(z)$ in the same circle. This result completes and extends essentially previous results by V.I. Danchenko (2008) and P.V. Chunaev (2020), which, respectively for the cases $a = -1$ and $-n \leqslant a < 0$ established the convergence of the differentiating formula but only in the domains contained in fixed compact subsets of the unit circle. The proof of the main results of the paper is based essentially on an approach for constructing a solutions for the mentioned system of momenta; this approach differs from that by Danchenko and Chunaev.

Keywords: differentiation of analytic functions, differentiating sums, h-sums, convergence rate.

A.B. Kostin, V.B. Sherstyukov

Integral representations of quantities associated with Gamma function

Abstract. We study a series of issues related with integral representations of Gamma functions and its quotients. The base of our study is two classical results in the theory of functions. One of them is a well-known first Binet formula, the other is a less known Malmsten formula. These special formulae express the values of the Gamma function in an open right half-plane via corresponding improper integrals. In this work we show that both results can extended to the imaginary axis except for the point $z = 0$. Under such extension we apply various methods of real and complex analysis. In particular, we obtain integral representations for the argument of the complex quantity being the value of the Gamma function in a pure imaginary point. On the base of the mentioned Malmsten formula at the points $z \neq 0$ in the closed right half-plane, we provide a detailed derivation of the integral representation for a special quotient expressed via the Gamma function: $D(z) \equiv \Gamma(z + 1/2)/\Gamma(z + 1)$. This fact on the positive semi-axis was mentioned without the proof in a small note by Dušan Slavić in 1975. In the same work he provided two-sided estimates for the quantity $D(x)$ as $x > 0$ and at the natural points these estimates coincides with the normalized central binomial coefficient. These estimates mean that $D(x)$ is enveloped on the positive semi-axis by its asymptotic series.

In the present paper we briefly discuss the issue on the presence of this property on the asymptotic series $D(z)$ in a closed angle $|\arg z| \leqslant \pi/4$ with a punctured vertex. By the new formula representing $D(z)$ on the imaginary axis, we obtain explicit expressions for the quantity $|D(iy)|^2$ and for the set Arg $D(iy)$ as $y > 0$. We indicate a way of proving the second Binet formula employing the technique of simple fractions.

Keywords: Gamma function, central binomial coefficient, asymptotic expansion, integral representation, Binet, Gauss, Malmsten formulae, enveloping series in the complex plane.

N.A. Rautian

Exponential stability of semigroups generated by Volterra integro-differential equations

Abstract. We study abstract Volterra integro-differential equations, which are operator models of problems in the viscoelasticity theory. This class includes Gurtin-Pipkin integro-differential equations describing the heat transfer in medias with memory. In particular, as the kernels of integral operators, the sums of decaying exponentials can serve or the sums of Rabotnov functions with positive coefficients having wide applications in the viscoelasticity theory and the theory of heat transfer.

The presented results are based on the approach related with studying one-parametric semi-groups for linear evolution equations. We provide a method for reducing the initial problem for a model integro-differential equation with operator coefficients in the Hilbert space to the Cauchy problem for a first order differential equation. We provide results on existing a strongly continuous contracting semigroup generated by a Volterra integro-differential equation with operator coefficients in a Hilbert space. We establish an exponential decay of the semigroup under known assumptions for the kernels of the integral operators. On the base of the obtained results we establish a well solvability of initial problem for the Volterra integro-differential equation with appropriate estimates for the solution.

The proposed approach can be also employed for studying other integro-differential equations involving integral terms of Volterra convolution type.

Keywords: Volterra integro-differential equations, linear equations in Hilbert space, operator semigroups.

E.G. Rodikova

On coefficient multipliers for planar Privalov classes

Abstract. The problem of describing Taylor coefficients of functions analytic in a circle was first resolved for the Nevanlinna class by an outstanding Soviet mathematician S.N. MNergelyan in beginning of 20th century. Later, the studies devoted to obtaining similar estimates in various classes of analytic functions were made by known Russian and foreign specialists in the complex analysis: G. Hardy, J. Littlewood, A.A. Friedman, N. Yanagihara, M. Stoll, S.V. Shvedenko and others.

In the paper we introduce a planar Privalov class $\tilde{\Pi}_q$, $(q > 0)$, being a generalization of a known planar Nevanlinna class. In the first part of the paper we obtain a sharp estimate for the growth of an arbitrary function in the planar Privalov class, we describe the coefficients of the Taylor expansion for this function. In the second part of the work, on the base of the obtained estimates we completely describe the coefficient multipliers from planar Privalov classes into Hardy classes. In a simplified form this problem can be formulated as follows: by what factors the Taylor coefficients of a function in a given class $\tilde{\Pi}_q$, $q > 0$, should be multiplied in order to get Taylor coefficients of a function in a Hardy class.

Keywords: planar Privalov class, Taylor coefficients, multiplier, growth, analytic functions.

A.I. Fedotov

Justification of Galerkin and collocations methods for one class of singular integro-differential equations on interval

Abstract. We justify the Galerkin and collocations methods for one class of singular integro-differential equations defined on the pair of the weighted Sobolev spaces. The exact solution of the considered equation is approximated by the linear combinations of the Chebyshev polynomials of the first kind. According to the Galerkin method, we equate the Fourier coefficients with respect to the Chebyshev polynomials of the second kind in the right-hand side and the left-hand side of the equation. According to collocations method, we equate the values of the right-hand side and the left-hand side of the equation at the nodes being the roots of the Chebyshev polynomials the second kind.

The choice of the first kind Chebyshev polynomials as coordinate functions is due to the possibility to calculate explicitly the singular integrals with Cauchy kernel of the products of these polynomials and corresponding weight functions. This allows us to construct simple well converging methods for the wide class of singular integro-differential equations on the interval $(-1, 1)$.

The Galerkin method is justified by the Gabdulkhaev – Kantorovich technique. The convergence of collocations method is proved by the Arnold – Wendland technique as a consequence of convergence of the Galerkin method. Thus, the covergence of both methods is proved and effective estimates for the errors are obtained.

Keywords: singular integro-differential equations, justification of approximate methods.

R.S. Yulmukhametov

Dual spaces to weighted spaces of locally integrable functions

Abstract. In this work we consider integral weighted L_2 spaces on convex domains in $\mathbb{R}^n$ and we study the problem on describing the dual space in terms of the Laplace-Fourier transform.

Let D be a bounded convex domain in $\mathbb{R}^n$ and φ be a convex function on this domain. By $L_2(D,\varphi)$ we denote the space of locally integrable functions D with a finite norm

$$\|f\|^2 := \int\limits_D |f(t)|^2 e^{-2\varphi(t)} dt.$$

Under some restrictions for the weight φ we prove that an entire function F is represented as the Fourier – Laplace transform of a function in $L_2(D,\varphi)$, that is,

$$F(\lambda) = \int\limits_D e^{t\lambda - 2\varphi(t)} \overline{f(t)} dt, \qquad f \in L_2(D,\varphi),$$

for some function $f \in L_2(D,\varphi)$ if and only if

$$\|F\|^2 := \int \frac{|F(z)|^2}{K(z)} \det G(\widetilde{\varphi}, x) dy dx < \infty,$$

where $G(\widetilde{\varphi}, x)$ is the Hesse matrix of the function $\widetilde{\varphi}$,

$$K(\lambda) := \|\delta_\lambda\|^2, \qquad \lambda \in \mathbb{C}^n.$$

As an example we show that for the case, when D is the unit circle and $\varphi(t) = (1-|t|)^\alpha$, the space of Fourier – Laplace transforms is isomorphic to the space of entire functions $F(z)$, $z = x+iy \in \mathbb{C}^2$, for which

$$\|F\|^2 := \int |F(x+iy)|^2 e^{-2|x| - 2(a\beta)^{\frac{1}{\beta+1}}(a+1)|x|^{\frac{\beta}{\beta+1}}} (1+|x|)^{\frac{\alpha-3}{2}} dx dy < \infty,$$

where $\alpha = \frac{\beta}{\beta+1}$.

Keywords: weighted spaces, Fourier – Laplace transform, entire functions.

O.Sh. Sharipov, A.F. Norjigitov

Law of large numbers for weakly dependent random variables with values in $D\,[0,1]$

Abstract. Limit theorems in Banach spaces are important, in particular, because of applications in functional data analysis. This paper is devoted to the law of large numbers for the random variables with values in $D\,[0,1]$ space. This space is not separable, if we consider it with supremum norm and it is difficult to prove limit theorems in this space. The law of large numbers is well-studied for the sequences of independent $D\,[0,1]$-valued random variables. It is known that in the case of independent and identically distributed random variables with values in $D\,[0,1]$ the existence of the first moment of the norm of random functions is a necessary and sufficient condition for the strong law of large numbers. Law of large numbers for the sequences of independent and not necessarily identically distributed random variables with values in $D\,[0,1]$ were proved as well.

Our main goal is to prove the law of large numbers for the weakly dependent random variables with values in $D[0,1]$ space. Namely, we consider the sequences of mixing random variables with values in $D[0,1]$. Mixing conditions for $D[0,1]$-valued random variables can be introduced in several ways. One can assume that random functions themselves satisfy mixing conditions. We consider a slightly different condition. In fact, we assume that the finite dimensional projections of the $D[0,1]$-valued random variables satisfy mixing condition. This is a weaker condition than assuming that random functions themselves satisfy mixing condition. In the paper, we prove the law of large numbers for ρ_m-mixing sequences of $D[0,1]$-valued random variables.

Keywords: law of large numbers, mixing sequence, $D[0,1]$ space.