ABSTRACTS

F.G. Avkhadiev

Hardy type inequalities involving gradient of distance function

Abstract. We prove several new Hardy type inequalities in Euclidean domains; these inequalities involve the gradient of the distance function from a point to the boundary of the domain. For test functions we consider improved inequalities in form proposed by Balinsky and Evans for convex domains. Namely, in Hardy type inequalities, instead of the gradient of the test function, one takes the scalar product of the gradients of the test function and of the distance from a point to the boundary of a given domain.

In the present paper, integral Hardy type inequalities are studied in non-convex n-dimensional domains having a finite inradius. We prove three new Hardy type L_p-inequalities in an improved form with explicit estimates for the constants depending on the dimension of the Euclidean space $n \geqslant 2$, the inradius of the domain and two parameters $p \geqslant 1$, $s \geqslant n$.

Our proofs are based on three key ingredients. The first of them is related with an approximation and a special partition of the domain, in particular, we employ the approximation of the domain by subsets formed by finitely many cubes with sides parallel to the coordinate planes.

The second ingredient is the representation of the domain as a countable union of subdomains with piece-wise smooth boundaries and applying a new theorem by the author on convergence of the gradients of the distance functions for these subdomains. Moreover, we prove three new Hardy type inequalities on a finite interval, which are employed in justifying the inequalities in multi-dimensional domains.

Keywords: Hardy type inequality, inradius, gradient of distance function.

M.A. Artemov, Yu.N. Babkina

Dirichlet boundary value problem for equations describing flows of a nonlinear viscoelastic fluid in a bounded domain

Abstract. We consider a boundary value problem for a mathematical model describing a stationary isotermic flow of a nonlinear viscoelastic liquid with a varying viscosity depending on the velocity of the fluid in a bounded three- or two-dimensional domain with a sufficiently smooth boundary. We assume that the viscosity function is continuous and bounded. The considered model is a system of strongly nonlinear third order partial differential equations. The boundary of the flow region is subject to the homogeneous Dirichlet boundary condition, which corresponds to the standard condition of adhesion of a liquid on the solid walls of a vessel. This boundary value problem is considered in a weak (generalized) sense. A weak solution is a pair of functions "velocity-pressure" satisfying the equations of motion in the distribution sense. Using the regularization method via introducing terms with additional viscosity into the equations, we construct a family of auxiliary approximating problems. We provide an interpretation of the problems of this family

in the form of operator equation with a continuous nonlinear operator α satisfying the monotonicity condition. On the base of a solvability theorem for equations with α-operators, we prove the existence of at least one solution for each positive value of the additional viscosity. We obtain estimates for the norms of solutions independent of the additional viscosity parameter. The solution to the original boundary value problem is obtained as the limit of the sequence of solutions to approximating problems as the additional viscosity tends to zero. The passage to the limit is carried out on the base of well-known results on the compactness of the embedding of Sobolev spaces and Lebesgue theorem on dominated convergence. In addition, we establish an energy-type estimate for the vector velocity function.

Keywords: Dirichlet boundary value problem, existence theorem, weak solution, α-operator, regularization method, additional viscosity, nonlinear viscoelastic fluid, polymer solution.

R.A. Bashmakov, K.P. Isaev, A.A. Makhota

Exponential series in normed spaces of analytic functions

Abstract. There is a classical well-known theorem by A.F. Leontiev on representing functions analytic in a convex domain D and continuous up to the boundary by series of form $\sum_{k=1}^{\infty} f_k e^{\lambda_k z}$ converging in the topology of the space $H(D)$, that is, uniformly on compact subsets in D.

In the paper we prove the possibility of representing the functions in

$$A_0(D) = \left\{ f \in H(D) \bigcap C(\overline{D}) : \ \|f\| := \sup_{z \in \overline{D}} |f(z)| \right\}$$

by the exponential series converging in a stronger topology, namely, there exists an integer number $s > 0$ such that

1) for each bounded convex domain D there exists a system of exponentials $e^{\lambda_k z}$, $k \in \mathbb{N}$, such that each function $f \in H(D) \bigcap C^{(s)}(\overline{D})$ is represented as a series over this system converging in the norm of the space $A_0(D)$;

2) for each bounded convex domain D there exists a system of exponentials $e^{\lambda_k z}$, $k \in \mathbb{N}$ such that each function $f \in A_0(D)$ is represented as a series over this system converging in the norm

$$\|f\| = \sup_{z \in D} |f(z)|(d(z))^s,$$

where $d(z)$ is the distance from a point z to the boundary of the domain D. The number s is related with the existence of entire functions with a maximal possible asymptotic estimate.

In particular cases, when D is a polygon or a domina with a smooth boundary possessing a smooth curvature separated from zero, we can assume that $s = 4$.

Keywords: analytic function, entire function, Fourier–Laplace transform, interpolation, exponential series.

N.V. Zaitseva

Hyperbolic differential-difference equations with nonlocal potentials

Abstract. We consider a three-parametric set of solutions for a two-dimensional hyperbolic differential-difference equation in a half-plane containing the sum of a differential operator and shift operators with respect to a spatial variable ranging on the entire real axis (or a differential-difference equation with nonlocal potentials). All shifts in potentials with respect to the spatial variable are arbitrary real numbers no commensurability are assumed. This is the most general case.

At present, elliptic and parabolic functional-differential equations, and in particular, differential-difference equations, are studied well enough. The aim of this work is to investigate hyperbolic differential-difference equations with shift operators in the space variable, which, as far as we know, have not been studied previously. The nature of the physical problems leading to such equations is fundamentally different from the problems for the classical equations of mathematical physics. To construct solutions, we employ a classical operation scheme is used, according to which the direct and then the inverse Fourier transforms are formally applied to the equation. However, if in the classical case the application of the Fourier transform leads to the study of polynomials with respect to the dual variable, in our case, due to the fact that in the Fourier images a shift operator is a multiplier, the symbol of the differential-difference operator is no longer a polynomial, but a combination of a power function and trigonometric functions with incommensurable arguments. This gives rise to computational difficulties and completely different effects in the solution. Generally speaking, this scheme leads to solutions in the sense of generalized functions. However, in this case it is possible to prove that the obtained solutions are classical.

We prove a theorem that if the real part of the symbol of the differential-difference operator in the spatial variable involved in the equation is positive, then the constructed solutions are classical. Classes of equations for which this condition is satisfied are given. We obtain the relations for the coefficients and shifts in the equation ensuting the required positivity of the real part of the symbol of the differential-difference operator in the equation.

Keywords: hyperbolic equation, differential-difference equation, incommensurable shifts, classical solution.

B.I. Islomov, O.Kh. Abdullaev

On non-local problems for third order equation with Caputo operator and non-linear loaded part

Abstract. This paper is devoted to proving the unique solvability of nonlocal problems with an integral adjoint condition for one class of third-order equations with a parabolic-hyperbolic operator including the Caputo fractional derivative and a nonlinear term containing the trace of the solution $u(x,0)$. Since the considered equation is of third order, in which a first order differential operator with coefficients a, b and c acts on a parabolic-hyperbolic second order operator, the coefficients a, b and c influence essentially a well-defined formulation of boundary value problems. This is why, before providing complete formulation of the studied problems, we present the boundary conditions in their formulation for various cases of the behavior of the coefficients a, b and c.

In the first part of the paper we formulate a nonlocal Problem I with an integral adjoint condition in the case $0 < b/a \leqslant 1$. This problem is equivalently reduced to a

nonlinear integral Volterra type equation and we prove its unique solvability by the successive approximations method.

The second part of the work is devoted to well-defined formulation and studying other nonlocal problems, whose formulations are related with other possible cases of a and b. We provide a detailed study of Problem II. Then as remarks we described the way of studying other formulated problems.

Keywords: parabolic-hyperbolic operator, Capute fractional derivative, nonlinear loaded term, integral adjoint condition, nonlinear integral equation.

A.S. Krivosheev, O.A. Krivosheeva, A.I. Rafikov

Invariant subspaces in the half-plane

Abstract. In this work we consider sequence of specified order $\rho(r)$. We find necessary and sufficient conditions guaranteeing that a sequence $\Lambda^2 \supseteq \Lambda^1$ consists a regularly distributed set Λ with a prescribed angular density containing Λ^1. These results cover a most part of knonw results on constructions of regularly distributed sets.

We consider various applications of the results. On the base of then, we prove theorems on splitting of entire functions of specified order $\rho(r)$. Moreover, we find an asymptotic representation of an entire function with a measurable sequence of zeroes. This generalizes a classical representation by B.Ya. Levin with a regularly distributed zero set for the case of a function with a measurable zero set. This representation is based on the obtained representation for a function, the zero set of which has a zero density. Its implication is the strengthening of a known result by M.L. Cartwright on the type of a function with a zero set having a zero density. Another corollary is the way for constructing entire functions of exponential type with a prescribed indicator and the minimal possible zero density.

Keywords: sequence, specified order, angular density, splitting of functions, entire function, indicator.

A.V. Lutsenko, I.Kh. Musin

On space of holomorphic functions with boundary smoothness and its dual

Abstract. We consider a Fréchet-Schwartz space $A_{\mathcal{H}}(\Omega)$ of functions holomorphic in a bounded convex domain Ω in a multidimensional complex space and smooth up to the boundary with the topology defined by means of a countable family of norms. These norms are constructed via some family $\mathcal{H}$ of convex separately radial weight functions in $\mathbb{R}^n$. We study the problem on describing a strong dual space for this space in terms of the Laplace transforms of functionals. An interest to such problem is motivated by the researches by B.A. Derjavets devoted to classical problems of theory of linear differential operators with constant coefficients and the researches by A.V. Abanin, S.V. Petrov and K.P. Isaev of modern problems of the theory of absolutely representing systems in various spaces of holomorphic functions with given boundary smoothness in convex domains in complex space; these problems were solved by Paley-Wiener-Schwartz type theorems. The main result of our paper is Theorem 1. It states that the Laplace transformation establishes an isomorphism between the strong dual of our functional space and some space of entire functions of exponential type in $\mathbb{C}^n$, which is an inductive limit of weighted Banach spaces of entire functions. This result generalizes the corresponding result of the second author in 2020. To

prove this theorem, we apply the scheme proposed by M. Neymark and B.A. Taylor. On a base of Theorem 1 and Theorem 7.6.11 from monograph by L. Hörmander (L. Hörmander. *An Introduction to Complex Analysis in Several Variables* // North Holland; 3rd edition, 1990) a problem of solvability of systems of partial differential equations in $A^m_{\mathcal{H}}(\Omega)$ is considered. An analogue of theorem 7.6.13 from monograph by L. Hörmander is obtained. In this case, as in the statement of Theorem 1, we employ essentially the properties of the Young-Fenchel transform of functions in the family $\mathcal{H}$.

Keywords: Laplace transform, entire functions.

V.P. Maksimov

Continuous-discrete dynamic models

Abstract. We consider dynamic models with an aftereffect in the form of functional differential equations with continuous and discrete time. We formulate a general control problem with respect to a given system of target functionals and a brief summary of known results on solvability of this problem under polyhedral point control constraints. In concluding section we present results on estimating the set of attainability under integral restrictions for the control. The proposed version of the synthesis of continuous and discrete systems is based on the systematic use of the theory abstract functional differential equation and has certain advantages in the study of systems and processes with aftereffect. Continuous-discrete functional-differential models allow us to take into consideration the aftereffects when modeling, including cases of complete memory, and effects arising when impulse perturbations (shocks) are taken into consideration and they are leading to jump changes in the phase state by components with continuous time.

Keywords: functional-differential systems, control problems, hybrid systems, set of attainability.

A.B. Muravnik

Elliptic differential-difference equations with differently directed translations in half-spaces

Abstract. We study the Dirichlet problem in the half-space for for elliptic differential-difference equations with operators being the compositions of differential operators and shift operators acting on spatial variables, which are independent variables that ranging in the entire real axis. These equations generalize essentially the classical elliptic partial differential equations and they arise in various applications of mathematical physics, which are characterized by nonlocal and (or) inhomogeneous properties of the process or medium. In theoretical terms, an interest in such equations is due to the fact that that they relate the values of the unknown function to each other (and its derivatives) not at one point, but at different points, which makes many classical methods are not applicable.

We establish the solvability for considered in the sense of generalized functions and a classical solvability for the equation. We also find an integral representation of the solution by a Poisson type formula and we prove that the constructed solution is classical outside boundary hyperplane and uniformly tends to zero as the only independent variable, changing on the positive axis, orthogonal to the boundary data hyperplane, tends to infinity. Earlier, there were studied only the cases when

the shift operator acts only in one spatial variable. In this work, the shift operators act on each spatial variable.

To obtain the Poisson kernel, we use classic operation scheme by Gelfand-Shilov: we apply Fourier transform to the problem under over all spatial variables and use the fact that the shift operators, as well as differential operators, are Fourier multipliers. Then we study the obtained Cauchy problem for the ordinary differential equation depending on dual variables as on parameters.

Keywords: elliptic problems, differential-difference equations, multi-directed shifts

A.E. Salimova, B.N. Khabibullin

Growth of entire functions of exponential type and characteristics of points distributions along straight line in complex plane

Abstract. According a classical Weierstrass-Hadamard-Lindelöf theorem, for each distribution of points with a finite upper density in the complex plane, there exists a non-zero entire function of exponential type vanishing on the these points with the multiplicity taken into account. In the beginning of 1960s, in a joint work by P. Malliavin and L.A. Rubel, the following problem was completely solved. Given two distributions of points with finite upper densities on the positive half-line, find under which relations between these distributions for each non-zero entire function of exponential type vanishing on one of the distributions, there exists a non-zero entire function of exponential type vanishing on the other distribution and having the absolute value not exceeding that of the first function. A complete solution of this problem going back to works by F. Carlson, T. Carleman, M. Cartwright, L. Schwartz, J.-P. Kahane and many others, was given in terms of so-called logarithmic characteristics of point distributions, which are expressed via reciprocals to points in these distributions. In this paper we extend these results on complex distributions of the points separated from the imaginary axis by a pair of vertical angles of an arbitrary small opening; here we develop logarithmic characteristics for complex point distributions. We consider three types of possible restrictions on the growth along the imaginary axis, very strict ones, as by P. Malliavin and L.A. Rubel, and less restrictive as in previous works by the second co-author. The main results are of a completed form and are formulated as criterions.

Keywords: entire function of exponential type, distribution of zeroes, growth of entire function, logarithmic characteristics and measures, Lindelöf condition.

A.A. Uspenskii, P.D. Lebedev

On singularity structure of minimax solution to Dirichlet problem for Eikonal type equation with discontinuous curvature of boundary of target set

Abstract. The origin of nonsmooth singularities in the minimax (generalized) solution of the Dirichlet problem for the eikonal equation is due to the existence of pseudo-vertices, the singular points of the boundary of the boundary set. Finding the pseudo-vertices is the first step in the procedure for constructing a singular set for solving a boundary value problem. To find these points, one has to construct local solutions to an equation of the golden ratio type, which establishes a connection between the eikonal operator and the geometry of the boundary set. The problem of identifying local solutions to the equation is related to the problem of finding fixed points of the mappings formed by local reparametrization of the boundary of the boundary set. In this work we obtain necessary conditions for the existence of pseudo-vertices when the smoothness of the curvature of a parametrically given boundary of the boundary set is broken. The conditions are written in various equivalent forms. In particular, we obtain a representation in the form of a convex combination of one-sided derivatives of the curvature. We provide the formulae for the coefficients of a convex combination determined by markers, which scalar characteristics of the pseudo-vertices. We find an algebraic equation, the roots of which are the markers. We adduce an example of the numerical-analytical construction of a minimax solution to the Dirichlet problem and this example demonstrates the effectiveness of the developed methods for solving nonsmooth boundary value problems.

Keywords: first order partial differential equation, minimax solution, speed, wavefront, diffeomorphism, eikonal, optimal result function, singular set, symmetry, pseudo-vertex.

A.V. Chernov

On differentiation of a functional in the problem of parametric coefficient optimization in semilinear global electric circuit equation

Abstract. For the problem on parametric optimization with respect to an integral criterion of the coefficient and the right-hand side of the semilinear global electric circuit equation, we obtain formulae for the first partial derivatives of the cost functional with respect to control parameters. The problem on reconstructing unknown parameters of the equation by the observed data of local sensors can be represented in such form. In the paper we generalize a similar result obtained earlier by the author for the case of linear global electric circuit equation. But it is commonly believed by experts that the right hand side treated as the volumetric density of external currents of the equation depends on the gradient, with respect to the collection of space variables, of the unknown electric potential function. Because of this, it is necessary to study the case of a semilinear equation. We use the conditions of preserving global solvability of the semilinear global electric circuit equation and the estimates for the increment of the solutions, which we have obtained formerly. The mathematical novelty of presented research is due to the fact that, unlike the earlier studied linear case, now the right hand side depends nonlinearly on the state, which, in its turn, depends on the controlled parameters. Such more complicated nonlinear dependence of the state on the control parameters requirs, in particular,

the development of a special technique to estimate the additional terms arising in the increment of solutions of the controlled equation.

Keywords: controlled coefficient and right hand side, parametric optimization, semilinear differential global electric circuit equation.

M.G. Yumagulov, L.S. Ibragimova, A.S. Belova

Perturbation theory methods in problem of parametric resonance for linear periodic Hamiltonian systems

Abstract. We consider the problem on parametric resonance for linear periodic Hamiltonian systems depending on a small parameter. We propose new formulae based on the methods of the perturbation theory for linear operators in the problem on approximate construction of multipliers for linear non-autonomous periodic Hamiltonian systems. We focus on obtaining the formulae for the first correctors of perturbations of multiple definite and indefinite multipliers. The proposed formulae lead to new Lyapunov stability criteria for linear periodic Hamiltonian systems in critical cases. We consider applications to the problem on parametric resonance in main resonances. The obtained results are formulated in terms of the original equations and lead us to effective formulae and algorithms. The effectiveness of the proposed formulae is demonstrated by solving the problem of plotting the boundaries of the stability regions of triangular libration points of a planar bounded elliptic three-body problem.

Keywords: Hamiltonian system, stability, multiplier, small parameter, parametric resonance, perturbation theory, three-body problem, libration point.

G. Khudaiberganov

The boundary Morera theorem for domain $\tau^+(n-1)$

Abstract. In this work, we continue developing an analysis in the future tube and proceed to studying the Lie ball. The Lie ball can be realized as a future tube. These realizations are the subject of our research. These methods turn out to be convenient for computing the Bergman, Cauchy-Szegö and Poisson kernels in this domain.

In the theory of functions, Morera theorems have been proved by many mathematicians. In the complex plane, the functions with one-dimensional holomorphic extension property are trivial but Morera boundary theorems are not available. Therefore, the results of the work are essential in the multidimensional case. In this paper, we prove the boundary Morera theorem for the domain $\tau^+(n-1)$. An analog of Morera theorem is given, in which the integration is made along the boundaries of analytic disks. For this purpose, we use the automorphisms $\tau^+(n-1)$ and the invariant Poisson kernel in the domain $\tau^+(n-1)$. Moreover, an analogue of Stout theorem on functions with the one-dimensional holomorphic continuation property is obtained for the domain $\tau^+(n-1)$. In addition, generalizations of Tumanov theorem is obtained for a smooth function from the given class of CR manifolds.

Keywords: Classic domain, Lie ball, realization, future tube, Shilov boundary, Poisson kernel, holomorphic continuation, Morera theorem, analytic disk, Hardy spaces.