ABSTRACTS

G.G. Amosov, E.L. Baitenov

On rank one perturbations of the semigroup of shifts on the half axis

Abstract. We study a special case of perturbations of the semigroup of translations on the half-axis changing the domain of its generator. We consider a rank one perturbation of generator defined by an exponent. We show that such perturbation of the generator always produces the generator of some C_0-semigroup, the action of which is described explicitly. The criterion of isometricity and contractivity of the perturbed semigroup is obtained. For the contractive case, we show that the considered generator perturbation produces a rank one perturbation of the cogenerator. The studied special case is used to build a model of perturbation for the semigroup of translations defined by an integral equation with respect to some operator-valued measure. In the case when the domain of the generator remains unchanged, this integral equation is reduced to a well-known equation of the perturbation theory, where the integration is made by the usual Lebesgue measure. If the domain is changed, the perturbation satisfies an integral equation with a nontrivial measure that having no density with respect to the Lebesgue measure. We study completely the problem of constructing an operator-valued measure that defines the integral equation relating the perturbed semigroup with the original one. The measure, when it exists, is obtained explicitly and we show that it is defined non-uniquely. We study the possibility of choosing an operator-valued measure with values in the set of self-adjoint and positive operators.

Keywords: the semigroup of translations, rank one perturbations of generator, perturbations changing the domain of generator.

S.N. Askhabov

Nonlinear convolution type integral equations in complex spaces

Abstract. We study various classes of nonlinear convolution type integral equations appearing in the theory of feedback systems, models of population genetics and other. By the method of monotone in the Browder-Minti operators we prove global theorems on existence, uniqueness and estimates for the solutions to the considered equations in complex Lebesgue spaces $L_p(\mathbb{R})$ under rather simple restrictions for the nonlinearities. Subject to the considered class of equations, weassume that either $p \in (1, 2]$ or $p \in [2, \infty)$. The conditions imposed on nonlinearities are necessary and sufficient to ensure that the generated superposition operators act from the space $L_p(\mathbb{R})$, $1 < p < \infty$, into the dual space $L_q(\mathbb{R})$, $q = p/(p-1)$, and are monotone. In the case of the space $L_2(\mathbb{R})$, we combine the method of monotone operator and contracting mappings principle to show that the solutions can be found by the successive approximation method of Picard type and provide estimates for the convergence rate. Our proofs employ essentially the criterion of the Bochner positivity of a linear convolution integral operator in the complex space $L_p(\mathbb{R})$ as $1 < p \leqslant 2$ and the

coercitivity of the operator inverse to the Nemytskii operator. In the framework of the space $L_2(\mathbb{R})$, the obtained results cover, in particular, linear convolution integral operators.

Keywords: nonlinear integral equations, convolution operator, criterion of positivity, monotone operator, coercive operator.

G.G. Braichev

Joint estimates of zeros and Taylor coefficients of entire function

Abstract. In the paper, for an entire function $f(z) = \sum\limits_{n=0}^{\infty} f_n z^n$, we provide asymptotic and uniform bounds of commensurability of the growth of zeroes and the decaying of the Taylor coefficients one with respect to the other. As an initial point for these studies, the following Hadamard statement served: if the coefficients of the series obey the inequality $|f_n| \leqslant \varphi(n)$ with some function $\varphi(x)$, then the absolute values of the zeroes grows faster than $1/\sqrt[n]{\varphi(n)}$. In the present work we improve recently obtained lower bound for joint growth of the zeroes and the coefficients via the maximal term of the Taylor series of the function $f(z)$ or via the counting function of its zeroes. The employing of flattened by Hadamard coefficients of the series give an opportunity to establish corresponding two-sided estimates. By the methods developing classical ideas we find a numerical dependence of such estimates on the sizes of the gaps of the power series representing the entire function. In particular, we find asymptotic identities relating the zeroes and the coefficients of an entire function. The obtained estimates are sharp and strengthen the known results by other authors.

Keywords: Taylor coefficients, flattened by Hadamard zeroes of entire function.

S.A. Budochkina

On connection between variational symmetries and algebraic structures

Abstract. In the work we present a rather general approach for finding connections between the symmetries of B_u-potentials, variational symmetries, and algebraic structures, Li-admissible algebras and Li algebras. In order to do this, in the space of the generators of the symmetries of the functionals we define such bilinear operations as $(\mathcal{S}, \mathcal{T})$-product, $\mathcal{G}$-commutator, commutator. In the first part of the work, to provide a complete description, we recall needed facts on B_u-potential operators, invariant functionals and variational symmetries. In the second part we obtain conditions, under which $(\mathcal{S}, \mathcal{T})$-product, $\mathcal{G}$-commutator, commutator of symmetry generator of B_u-potentials are also its symmetry generator. We prove that under some conditions $(\mathcal{S}, \mathcal{T})$-product turns the linear space of the symmetry generators of B_u-potential into a Li-admissible algebra, while $\mathcal{G}$-commutator and commutator do into a Lie algebra. As a corollary, similar results were obtained for the symmetry generators of potentials, $B_u \equiv I$, where the latter is the identity mapping. Apart of this, we find a connection between the symmetries of functionals with Lie algebras, when they have bipotential gradients. Theoretical results are demonstrated by examples.

Keywords: variational symmetry, transformation generator, Lie-admissible algebra, Lie algebra, $(\mathcal{S}, \mathcal{T})$-product, $\mathcal{G}$-commutator, commutator.

M.R. Langarshoev, S.S. Khorazmshoev

Sharp inequalities of Jackson-Stechkin type and the diameters of classes of functions in L_2

Abstract. Some problems of the approximation theory requires estimating the best approximation of 2π-periodic functions by trigonometric polynomials in the space L_2, and while doing this, instead of the usual modulus of continuity $\omega_m(f,t)$, sometimes it is more convenient to use an equivalent characteristic $\Omega_m(f,t)$ called the generalized modulus of continuity. Similar averaged characteristic of the smoothness of a function was considered by K.V. Runovskiy and E.A. Storozhenko, V.G. Krotov and P. Oswald while studying important issues of constructive function theory in metric space L_p, $0 < p < 1$. In the space L_2, in finding exact constants in the Jackson-type inequality, it was used by S.B. Vakarchuk. We continue studies of problems approximation theory and consider new sharp inequalities of the type Jackson–Stechkin relating the best approximations of differentiable periodic functions by trigonometric polynomials with integrals containing generalized modules of continuity. For classes of functions defined by means of these characteristics, we calculate exact values of some known n-diameters are calculated.

Keywords: best polynomial approximation, generalized modulus of continuity, extremal characteristic, diameters.

K.G. Malyutin, M.V. Kabanko, V.A. Maliutin

Extremal problems in the theory of central Wiman-Valiron index

Abstract. We consider some properties of central index in Wiman-Valiron index. We introduce a notion of a determining sequence of a central index $\nu(r)$ corresponding to a fixed transcendental function f and the notion of a determining sequence for an arbitrary fixed central index $\nu(r)$. Let $\rho_1, \rho_2, \ldots, \rho_s, \ldots$ be the points of the jumps of the function $\nu(r)$ taken counting their multiplicities. This means that at a point ρ_s the jump is equal to m_s, then the quantity ρ_s appears m_s times in this sequence. Such sequence is called determining sequence of the function $\nu(r)$. We introduce the notion of the regularization of the function $\nu(r)$, which is employed for proving main statements. We study two extremal problems in the class of functions with a prescribed central index. We obtain the expression for the maximum of the modulus of the extremal function in terms of its central index. The main obtained results are as follows. Let T_ν be the set of all transcendental functions f with a prescribed central index $\nu(r)$, $M(r,f) = \max\{|f(re^{i\theta})| : 0 \leqslant \theta \leqslant 2\pi\}$, and let $M(r,\nu) = \sup\{M(r,f) : f \in T_\nu\}$. Then for each $r > 0$, in the class of the functions T_ν, the quantity $M(r,\nu)$ is attained at the same function for all $r > 0$. We describe the form of such extremal function. We also prove that for each fixed $r_0 > 0$ and for each prescribed central index $\nu(r)$, in the class T_ν there exists a function $f_0(z)$ such that $M(r_0, f_0) = \inf\{M(r_0, f) : f \in T_\nu\}$.

Keywords: Wiman-Valiron theory, central index, determining sequence, regularization, extremal problem.

N.M. Poluboyarova

Relations between length and instability of tubular extremal surfaces

Abstract. In the paper we study surfaces being extremals of the potential energy functional. In our case, the potential energy is the sum of two functionals, one being a functional of the area type, and the other being a functional of the volume density of forces. Extremal surfaces are stable if the second variation of the functional is sign-definite, otherwise they are instable. In order to obtain the instability, we impose additional conditions on the surface and integrands, then we apply the properties of positive definite symmetric matrices, employ the Kronrod-Federer formula, the Cauchy-Bunyakovsky inequality, and the Weingarten homomorphism estimate. This allows us to estimate the second variation of the functional. Such technique, being a developing of approach proposed by V.A. Klyachin, allow us to obtain conditions ensuring the instability. We establish that the length of the tubular extremal surface can be estimated in terms of the minimal and maximal $(n-1)$-dimensional measure of the cross-sections of the surface by hyperplanes. The obtained statement means that too long tubes with a non-zero mean curvature are unstable. The physical aspects of this phenomenon were considered in a work by V.A. Saranin.

Keywords: variation of a functional, extremal surface, area-type functional, volume density functional, potential energy functional, stability, instability, tubular surface, hyperplane, measure of surface section, length of tubular surface.

E.H. Khalilov, M.N. Bakhshaliyeva

Study of approximate solution to integral equation associated with mixed boundary value problem for Laplace equation

Abstract. We consider an approximate solution of the integral equation arising after reduction of a mixed problem for the Laplace equation. The main advantage of applying the method of integral equations to studying external boundary value problems is that such approach allows one to reduce the problem posed in an unbounded domain to a problem in a domain of a smaller dimension. In the work we study an approximate solution to the integral equation, to which the mixed problem for the Laplace equation is reduced. We seek its solution as a combination of logarithmic single layer potentials and double layer potential, we reduce the problem to an integral equations depending not only on the operators generated by the logarithmic potentials but also on the composition of such operators. We prove that the obtained integral equation has the unique solution in the space of continuous functions. Since the integral equations can be solved in the closed form only in very rare cases, it is of a high importance to develop approximate methods for solving integral equations and give their appropriate theoretical justification. We partition a curve into elementary parts and by certain nodes with construct quadrature formulae for a class of curvilinear potentials and for the composition of the integrals generated by logarithmic potentials and we also estimate the errors of these formulae. Employing these quadrature formulae, the obtained integral equation is replaced by the system of algebraic equations. Then by means of Vainikko's convergence theorem for linear operator equations, we establish the existence and uniqueness of solutions to this system. We prove the convergence of the obtained system of algebraic equations to

the values of the exact solution of the integral equation at the chosen nodes. Moreover, we find the convergence rate of this method. As a result, we find a sequence converging to the solution of the mixed boundary value problem for the Laplace equation and its convergence rate is known.

Keywords: curvilinear integral, integral equation method, collocation method, mixed boundary value problem, Laplace equation.

R.A. Aliev, A.N. Ahmadova

Boundedness of discrete Hilbert transform on discrete Morrey spaces

Abstract. The Hilbert transform plays an important role in the theory and practice of signal processing operations in continuous system theory because of its relevance to such problems as envelope detection and demodulation, as well as because of its use in relating the real and imaginary components, and the magnitude and phase components of spectra. The Hilbert transform is a multiplier operator and is widely used in the theory of Fourier transforms. The Hilbert transform was the motivation for the development of modern harmonic analysis. Its discrete version is also widely used in many areas of science and technology and plays an important role in digital signal processing. The essential motivation behind thinking about discrete transforms is that experimental data are most frequently not taken in a continuous manner but sampled at discrete time values. Since much of the data collected in both the physical sciences and engineering are discrete, the discrete Hilbert transform is a rather useful tool in these areas for the general analysis of this type of data.

The Hilbert transform has been well studied on classical function spaces Lebesgue, Morrey, etc. But its discrete version, which also has numerous applications, has not been fully studied in discrete analogues of these spaces. In this paper we discuss the discrete Hilbert transform on discrete Morrey spaces. In particular, we obtain its boundedness on the discrete Morrey spaces using boundedness of the Hilbert transform on Morrey spaces.

Keywords: discrete Hilbert transform, Morrey spaces, discrete Morrey spaces, boundedness.

Y. Ahmed, W.A. Dudek

Generalized multiplicative derivations in inverse semirings

Abstract. In this note we consider inverse semirings, i.e. semirings S in which for each $a \in S$ there exists a uniquely determined element $a' \in S$ such that $a + a' + a = a$ and $a' + a + a' = a$. If additionally the commutator $[x, y] = xy + y'x$ satisfies both Jordan identities, then such semirings are called Jacobi semirings. The problem of commutativity of such semirings can be solved by specifying easily verifiable conditions which must be satisfied by the commutator or some additive homomorphisms called derivations, or by a pair of nonzero mappings from S to S.

We consider the pair (F, f) of nonzero mappings $S \to S$ such that $F(xy) = F(x)y + xf(y)$ for all $x, y \in S$ and determine several simple conditions under which the pair (F, f) of such mappings (called a generalized multiplicative derivation) forces the commutativity of a semiring S. We show that semiring will be commutative if the

conditions we find are satisfied by the elements of a solid ideal, i.e. a nonempty ideal I with the property that for every $x \in I$ elements $x + x'$ are in the center of I.

For example, a prime Jacobi semiring S with a solid ideal I and a generalized multiplicative derivation (F, f) such that $a(F(xy) + yx) = 0$ for all $x, y \in I$ and some nonzero $a \in S$, is commutative. Moreover, in this case $F(s) = s'$ for all $s \in S$ (Theorem 3.2). A prime Jacobi semiring S with a generalized multiplicative derivation (F, f) is commutative also in the case when S contains a nonzero ideal I (not necessarily solid) such that $a(F(x)F(y) + yx) = 0$ for all $x, y \in I$ and some nonzero $a \in S$ (Theorem 3.3). Also prime Jacobi semirings with a non zero ideal I and a nonzero derivation d such that $[d(x), x] = 0$ for $x \in I$ are commutative.

Keywords: Inverse semirings, multiplicative derivations, annihilators, prime semirings, Jacobi semirings, solid ideals.

B. Bayraktar, M. Emin Özdemir

Generalization of Hadamard-type trapezoid inequalities for fractional integral operators

The role of convexity theory in applied problems, especially in optimization problems, is well known. The integral Hermite-Hadamard inequality has a special place in this theory since it provides an upper bound for the mean value of a function. In solving applied problems from different fields of science and technology, along with the classical integro-differential calculus, fractional calculus plays an important role. A lot of research is devoted to obtaining an upper bound in the Hermite-Hadamard inequality using operators of fractional calculus.

The article formulates and proves the identity with the participation of the fractional integration operator. Based on this identity, new generalized Hadamard-type integral inequalities are obtained for functions for which the second derivatives are convex and take values at intermediate points of the integration interval. These results are obtained using the convexity property of a function and two classical integral inequalities, the Hermite-Hadamard integral inequality and its other form, the power mean inequality. It is shown that the upper limit of the absolute error of inequality decreases in approximately n^2 times, where n is the number of intermediate points. In a particular case, the obtained estimates are consistent with known estimates in the literature. The results obtained in the article can be used in further researches in the integro-differential fractional calculus.

Keywords: convexity, Hermite–Hadamard inequality, Hölder inequality, power–mean inequality, Riemann–Liouville fractional Integrals.

H. Gunawan, D.I. Hakim, A.S. Putri

On geometric properties of Morrey spaces

Abstract. The study of Morrey spaces is motivated by many reasons. Initially, these spaces were introduced in order to understand the regularity of solutions to elliptic partial differential equations. In line with this, many authors study the boundedness of various integral operators on Morrey spaces. In this article, we are interested in their geometric properties, from functional analysis point of view. We show constructively that Morrey spaces are not uniformly non-ℓ_n^1 for any $n \geqslant 2$.

This result is sharper than earlier results, which showed that Morrey spaces are not uniformly non-square and also not uniformly non-octahedral. We also discuss the n-th James constant $C_{\mathrm{J}}^{(n)}(X)$ and the n-th Von Neumann-Jordan constant $C_{\mathrm{NJ}}^{(n)}(X)$ for a Banach space X, and obtain that both constants for any Morrey space $\mathcal{M}_q^p(\mathbb{R}^d)$ with $1 \leqslant p < q < \infty$ are equal to n.

Keywords: Morrey spaces, uniformly non-ℓ_n^1-ness, n-th James constant, n-th Von Neumann-Jordan constant.

A.A. Imomov, A.Kh. Meyliev

On asymptotic structure of continuous-time Markov branching processes allowing immigration without higher-order moments

Abstract. We consider a continuous-time Markov branching process allowing immigration. Our main analytical tool is the slow variation (or more general, a regular variation) conception in the sense of Karamata. The slow variation property arises in many issues, but it usually remains rather hidden. For example, denoting by $p(n)$ the perimeter of an equilateral polygon with n sides inscribed in a circle with a diameter of length d, one can check that the function $\boldsymbol{\pi}(n) := p(n)/d$ converges to π in the sense of Archimedes, but it slowly varies at infinity in the sense of Karamata. In fact, it is known that $p(n) = dn \sin(\pi/n)$ and then it follows $\boldsymbol{\pi}(\lambda x)/\boldsymbol{\pi}(x) \to 1$ as $x \to \infty$ for each $\lambda > 0$. Thus, $\boldsymbol{\pi}(x)$ is so slowly approaching π that it can be suspected that "π is not quite constant".

Application of Karamata functions in the branching processes theory allows one to bypass severe constraints concerning existence of the higher-order moments of the infinitesimal characteristics of the process under study. Zolotarev was one of the first who demonstrated an encouraging prospect of application of the slow variation conception in the theory of Markov branching processes and has obtained principally new results on asymptote of the survival probability of the process without immigration.

In this paper, delving deeply in the nature of the Karamata functions, we study more subtle properties of branching processes allowing immigration. In particular, under quite admissible conditions, we find explicit forms for the generating functions of invariant measures for the process under consideration.

Keywords: Markov branching process, immigration, transition functions, state space classification, generating functions, slowly varying function, invariant measures.