ABSTRACTS

D.I. Borisov, M.N. Konyrkulzhaeva

On infinite system of resonances and eigenvalues with exponential asymptotics generated by distant perturbations

Abstract. We consider an one-dimensional Schrödinger operator with four distant potentials separated by large distance. All distances are proportional to a sam large parameter. The initial potentials are of kink shapes, which are glued mutually so that the final potential vanishes at infinity and between the second and third initial potentials and it is equal to one between the first and the second potentials as well as between the third and fourth potentials. The potentials are not supposed to be real and can be complex-valued. We show that under certain, rather natural and easily realizable conditions on the four initial potentials, the considered operator with distant potentials possesses infinitely many resonances and/or eigenvalues of form $l = k_n^2$, $n \in \mathbb{Z}$, which accumulate along a given segment in the essential spectrum. The distance between neighbouring numbers k_n is of order the reciprocal of the distance between the potentials, while the imaginary parts of these quantities are exponentially small. For the numbers k_n we obtain the representations via the limits of some explicitly calculated sequences and the sum of infinite series. We also prove explicit effective estimates for the convergence rates of the sequences and for the remainders of the series.

Keywords: resonance, exponential asymptotics, distant perturbations, non-self-adjoint operator

N.F. Valeev, Y.Sh. Ilyasov

Inverse spectral problem for partial traces of the Sturm-Liouville operator

Abstract. This work is aimed at studying optimization inverse spectral problems with a so-called incomplete spectral data. As incomplete spectral data, Partial traces of the Sturm - Liouville operator serve. We study the following formulation of the inverse spectral problem with incomplete data (optimization problem): find a potential $\hat{V}$ closest to a given function V_0 such that a partial trace of the SturmвЂ“Liouville operator with the potential $\hat{V}$ has a prescribed value. As a main result, we prove the existence and uniqueness theorem for solutions of this optimization inverse spectral problem. A new type of relationship between linear spectral problems and systems of nonlinear differential equations is established. This allows us to find a solution to the inverse optimal spectral problem by solving the boundary value problem for a system of nonlinear differential equations and to obtain a solvability of the system of nonlinear differential equations. To prove the uniqueness of solutions, we use the convexity property of the partial trace of the Sturm-Liouville operator with the potential $\hat{V}$; the trace is treated as a functional of the potential $\hat{V}$. We obtain a new generalization of the Lidskii-Wielandt inequality to arbitrary self-adjoint semi-bounded operators with a discrete spectrum.

Keywords: spectral theory of differential operators, inverse spectral problem, variational problems, inequalities for eigenvalues

G.A. Gaisina

Growth order of sum of Dirichlet series: dependence on coefficients and exponents

Abstract. We study the sharpness of the conditions under which the order of the sum of the Dirichlet series converging in some half-plane can be calculated by means of certain formula depending only on the coefficients and exponents. For unbounded functions analytic in the unit circle, a formula of such kind was obtained by a series of scientist in different years, in partucilar, by Govorov in 1959, by MacLane in 1966 and by Sheremeta in 1968. Later an analogue of this notion was also introduced for a Dirichlet series converging in some half-plane. But a corresponding formula for the growth order of the Dirichlet series was established by many authors under strict restrictions. In all previous formulae there were provided the conditions, which were only sufficient for the validity of this formula. In the present work we find conditions being not only sufficient but also necessary for the possibility to calculate the growth order for each Dirichlet series by means of this formula.

Keywords: Dirichlet series, half-plane of convergence, formula for the growth order

E.S. Zhukovskiy, W. Merchela

On covering mappings in generalized metric spaces in studying implicit differential equations

Abstract. Let on a set $X \neq \emptyset$ a metric $\rho_X : X \times X \to [0, \infty]$ be defined, while on $Y \neq \emptyset$ a distance $d_Y : Y \times Y \to [0, \infty]$, be given, which satisfies only the identity axiom. We define the notion of covering and of Lipschitz property for the mappings $X \to Y$. We formulate conditions ensuring the existence of solutions $x \in X$ to equations of form $F(x, x) = y$, $y \in Y$, with a mapping $F : X \times X \to Y$, being covering in one variable and Lipschitz in the other. These conditions are employed for studying the solvability of a functional equation with a deviation variable and of a Cauchy problem for an implicit differential equation. In order to do this, on the space S of Lebesgue measurable functions $z : [0, 1] \to \mathbb{R}$ we define the distance

$$d(z_1, z_2) = \operatorname{vrai} \sup_{t \in [0,1]} \theta(z_1(t), z_2(t)), \qquad z_1, z_2 \in S,$$

where each continuous function $\theta : \mathbb{R} \times \mathbb{R} \to [0, \infty]$ satisfies $\theta(z_1, z_2) = 0$ if and only if $z_1 = z_2$.

Keywords: covering mapping, metric space, functional equation with a devating variable, ordinary differential equation, existence of solution.

K.P. Isaev, R.S. Yulmukhametov

Geometry of radial Hilbert spaces with unconditional bases of reproducing kernels

Abstract. We study the geometry of abstract radial functional Hilbert spaces stable with respect to the dividing and possessing an unconditional basis of reproducing kernels. We obtain a simple necessary condition ensuring the existence of such bases in terms of the sequence $\|z^n\|$, $n \in \mathbb{N} \cup \{0\}$. We also we obtain a sufficient condition for the norm and the Bergman function of the space to be recovered by a sequence of the norms of monomial. Two main statements we prove are as follows. Let H be a radial functional Hilbert space of entire functions stable with respect to the dividing and let the system of monomials $\{z^n\}$, $n \in \mathbb{N} \cup \{0\}$, be complete in this space.

1. If the space H possesses an unconditional basis of reproducing kernels, then

$$\|z^n\| \asymp e^{u(n)}, \quad n \in \mathbb{N} \cup \{0\},$$

where the sequence $u(n)$ is convex, that is

$$u(n+1) + u(n-1) - 2u(n) \geqslant 0, \quad n \in \mathbb{N}.$$

2. Let $u_{n,k} = u(n) - u(k) - (u(n) - u(n-1))(n-k)$. If $\mathcal{U}$ is the matrix with entries $e^{2u_{n,k}}$, $n, k \in \mathbb{N} \cup \{0\}$, and

$$\|\mathcal{U}\| := \sup_n \left(\sum_k e^{2u_{n,k}} \right)^{\frac{1}{2}} < \infty,$$

then

2.1. the space H as a Banach space is isomorphic to the space of entire functions with the norm

$$\|F\|^2 = \frac{1}{2\pi} \int_0^\infty \int_0^{2\pi} |F(re^{i\varphi})|^2 e^{-2\widetilde{u}(\ln r)} d\varphi d\widetilde{u}'_+(\ln r),$$

where $\widetilde{u}$ is the Young conjugate of the piecewise-linear function $u(t)$;

2.2. the Bergman function of the space H satisfies the condition

$$K(z) \asymp e^{2\widetilde{u}(\ln |z|)}, \quad z \in \mathbb{C}.$$

Keywords: Hilbert spaces, entire functions, unconditional bases, reproducing kernels.

Kh.K. Ishkin, R.I. Marvanov

On localization conditions for spectrum of model operator for Orr–Sommerfeld equation

Abstract. For a model operator $L(\varepsilon)$ related with Orr-Sommerfeld equation, we study the necessity of known Shkalikov conditions sufficient for a localization of the spectrum at a graph of Y-shape. We consider two types of the potentials, for which a unbounded part Γ_∞ of the limiting spectral graph (LSG) is constructed in an explicit form. The first of form is a piece-wise potential with countably many jumps. We show that if the discontinuity points of this potential converges rather fast to one of the intervals $(0, 1)$, then Γ_∞ consists in countably many rays. The second potential is glued from two holomorphic functions. We show that Γ_∞ consists in two curves if the derivative at the gluing point has a jump and Langer conditions are satisfied in the

domain enveloped by the Stokes lines ensuring the possibility of constructing WKB-expansions. If the gluing is infinitely differentiable, WKB-estimates are insufficient to clarify the spectral picture. Because of this we consider an inverse problem: given some spectral data, clarify analytic properties of the potential in the vicinity of the interval $(0, 1)$. In order to understand the nature of spectral data, we first solve a direct problem extended to a complex ε-plane. It turns out that if we assume the holomorphy of the potential in the vicinity of the segment $[0, 1]$, then for small ε in the sector $\mathcal{E}$ of opening $\pi/2$, the part of the spectrum $L(\varepsilon)$ outside some circle satisfies quantizaion conditions of Bohr-Sommerfeld type. In the concluding part of the work we solve the inverse problem. As spectral data, quantization conditions obtained in the direct problem and taken in a slightly weaker form serve. We prove that if the potential is a monotone continuously differentiable function and the mentioned conditions are satisfied, then the potential admits an analytic continuation into some neighbourhood of the interval $(0, 1)$. This proves the necessity of Shkalikov conditions at least in a local sense.

Keywords: Orr-Sommerfeld equation, localization of spectrum, limiting spectral graph

I.Kh. Musin

On Fourier-Laplace transforms of a class of generalized functions

Abstract. We consider a subspace of Schwartz space of fast decaying infinitely differentiable functions on an unbounded closed convex set in a multidimensional real space with a topology defined by a countable family of norms constructed by means of a family $\mathfrak{M}$ of a logarithmically convex sequences of positive numbers. Owing to the mentioned conditions for these sequence, the considered space is a Fréchet-Schwartz space. We study the problem on describing the strong dual space for this one in terms of the Fourier-Laplace transforms of functionals. Particular cases of this problem were considered by by J.W. De Roever while studying problems of mathematical physics, complex analysis in the framework of a developed by him theory of ultradistributions with supports in an unbounded closed convex set; similar studies were also made by by P.V. Fedotova and by the author of the present paper. Our main result, presented in Theorem I, states that the Fourier-Laplace transforms of the functionals establishes an isomorphism between the strong dual space of the considered space and some space of holomorphic functions in a tubular domain of the form $\mathbb{R}^n + iC$, where C is an open convex acute cone in $\mathbb{R}^n$ with the vertex at the origin; the mentioned holomorphic functions possess a prescribed growth majorants at infinity and at the boundary of the tubular domain. The work is close to the researches by V.S. Vladimirov devoted to the theory of the Fourier-Laplace transformatation of tempered distributions and spaces of holomorphic functions in tubular domains. In the proof of Theorem I we apply the scheme proposed by M. Neymark and B.A. Taylor as well as some results by P.V. Yakovleva (Fedotova) and the author devoted to Paley-Wiener type theorems for ultradistributions.

Keywords: Fourier-Laplace transform of functionals, holomorphic functions.

Z.U. Fazullin, N.F. Abuzyarova

On necessary and sufficient condition in theory of regularized traces

Abstract. The present work is devoted to studying the regularized trace formulae for symmetric L_0-compact perturbations of a discrete self-adjoint lower semi-bounded operator L_0 in a separable Hilbert space. By now, the studies of the regularized trace formulae for the perturbations of abstract self-adjoint discrete operators were mostly aimed on finding a sufficient condition, under which the regularized sum with brackets minus first or several leading terms of the perturbation theory vanished. This condition was formulated in terms of spectral characteristics of an unperturbed operator L_0 depending on the belonging of a perturbing operator V to some class. In particular, recently, the traces formulae for model two-dimensional operators in mathematical physics have been intensively studied with a perturbation described by the multiplication operator. Here we study a necessary and sufficient condition for two cases, namely, as the regularized trace with brackets minus the first corrector of the perturbation theory vanishes or is equal to a finite number. We consider a certain summation bracket, which usually arises in the theory of regularized traces of the perturbations of partial differential operators.

Keywords: trace of an operator, resolvent, trace formula, perturbation theory, discrete spectrum.

V.N. Ushakov, A.A. Ershov

On recovering of unknown constant parameter by several test controls

Abstract. We consider a controlled system involving a constant vector parameter, which is unknown to a controlling person. Only a set of possible values of this unknown parameter is supposed to be known. We study the problem on approaching a targeted set at a prescribed time. To resolve the control problem at the beginning of the motion, we recover the unknown parameter by a successive short-time application of several test controlling vectors to the controlled system and observing then the reaction of the system. The choice of test vectors is proposed to make by minimizing the error of recovering of the unknown parameter. In contrast to previous works, we consider a more general case, when one test controlling vector is not enough for the unique recovering of the unknown parameter and moreover, for approximating the velocity of the motion, we employ a central difference derivative instead of the right difference one. As an example, we consider the problem on controlling a pendulum with unknown dissipation coefficient and elasticity coefficient of the spring.

Keywords: controlled system, problem on approaching, unknown constant parameter, test control

B.N. Khabibullin

Liouville-type theorems for functions of finite order

Abstract. A convex subharmonic of pluri-subharmonic function on a real axis, on a finite dimensional real of complex space is called a function of a finite order if it grows not faster than some positive power of the absolute value of the variable as the latter tends to infinity. An entire function on a finite-dimensional complex space is called a function of a finite order if the logarithm of its absolute value is a (pluri-)subharmonic function of a finite order. A measurable set in an m-dimensional space is called a set of a zero density with respect to the Lebesgue density if the Lebesgue

measure of the part of this set in the ball of a radius r is of order $o(r^m)$ as $r \to +\infty$. In this paper we show that convex function of a finite order on the real axis and subharmonic functions of a finite order on a finite-dimensional real space bounded from above outside some set of a zero relative Lebesgue measure are bounded from above everywhere. This implies that subharmonic functions of a finite order on the complex plane, entire and subharmonic functions of a finite order, as well as convex and harmonic functions of a finite order bounded outside some set of a zero relative Lebesgue measure are constant.

Keywords: entire function, subharmonic function, pluri-subharmonic function, convex function, harmonic function of entire order, Liouville theorem.

D. Serikbaev

Inverse problem for fractional order pseudo-parabolic equation with involution

Abstract. In this paper, we consider an inverse problem on recovering the right-hand side of a fractional pseudo-parabolic equation with an involution operator. The major obstacle for considering the inverse problems is related with the well-posedness of the problem. Inverse problems are often ill-posed. For example, the inverse heat equation, deducing a previous distribution of temperature from final data, is not well-posed since the solution is highly sensitive to variations in the final data.

The advantage of this paper is two-fold. On the one hand, we investigate the solvability of the direct problem and prove the solvability to this problem. On the other hand, we study the inverse problem based on this direct problem and prove the solvability results in this problem, too.

First, we investigate the Cauchy problem for the time-fractional pseudo-parabolic equation with the involution operator, and secondly, we consider the inverse problem on recovering the right-hand side from an overdetermined final condition and prove that it is solvable.

To achieve our goals, we use methods corresponding to the different areas of mathematics such as the theory of partial differential equations, mathematical physics, and functional analysis. In particular, we use the $\mathcal{L}$-Fourier analysis method to establish the existence and uniqueness of solutions to this problem on the Sobolev space.

The classical and generalized solutions of the inverse problem are studied.

Keywords: fractional differential equation, inverse problem, involution, pseudo–parabolic equation.