ABSTRACTS

F.A. Abdushukurov

Poisson limit theorems in schemes of distributions of distinguishable particles

Abstract. We consider a random variable $\mu_r(n, K, N)$ being the number of cells containing r particles among first K cells in an equiprobable scheme of distribution of at most n distinguishable particles over N different cells. We find conditions ensuring the convergence of these random variables to a random Poisson variable. We describe a limit distribution. These conditions are of the simplest form, when the number of particles r belongs to a bounded set or as K is equivalent to $\sqrt{N}$. Then random variables $\mu_r(n, K, N)$ behave as the sums of independent identically distributed indicators, namely, as binomial random variables, and our conditions coincide with the conditions of a classical Poisson limit theorem. We obtain analogues of these theorems for an equiprobable scheme of distribution of n distinguishable particles of N different cells. The proofs of these theorems are based on the Poisson limit theorem for the sums of commuting indicators and on an analogue of the local limit Gnedenko theorem.

Keywords: scheme of distribution of distinguishable particles of different cells, Poisson random variable, Gaussian random variable, limit theorem, local limit theorem.

N.F. Abuzyarova

Synthesizable sequence and primary submodules in the Schwratz module

Abstract. We consider a module of entire functions of exponential type and polynomial growth on the real axis, that is, the Schwarz module with a non-metrizable locally convex topology. In relation with the problem of spectral synthesis for the differentiation operator in the space $C^\infty(a; b)$, we study primary submodules in this module. In particular, we find out what functions, apart of products of the polynomials on the generating function, are contained in a primary submodule. The main results of the work is as follows: despite the topology in the Schwarz module is non-metrizable, the primary submodule coincides with a sequential closure of the set of products of its generating function by polynomials. As a corollary of the main result we prove a weight criterion of a weak localizability of the primary submodule. Another corollary concerns a notion of "synthesizable sequence" introduced recently by A. Baranov and Yu. Belov. It follows from a criterion of a synthesizable sequence obtained by these authors that a synthesizable sequence is necessary a zero set of a weakly localizable primary submodule. In the work we give a positive answer to a natural question on the validity of the inverse statement. Namely, we prove that the weak set of a weakly localizable primary submodule is a synthesizable sequence.

Keywords: entire functions, Fourier-Laplace transform, Schwarz space, local description of submodules, spectral synthesis.

B.E. Kanguzhin

Recover of two-point boundary conditions by a finite set of eigenvalues of boundary value problems for higher order differential equations

Abstract. The recovering of boundary conditions for higher order differential equations by some set of spectra is difficult because of two facts. First, opposite to second order differential equations, there are not triangle transformation operators for higher order differential equations. Second, non-separating boundary conditions give additional analytic problems while recovering them by the set of spectra. We note that in the present work we provide a new way of normalizing boundary conditions, which is adapted for further recovering by some set of spectra of boundary value problems. In other words, before posing the issue by which data the boundary conditions can be recovered, one should first reduce them to a canonical form. Then, basing on an assumed canonical form, a system of boundary value problems is to be chosen and by the their spectra boundary conditions are to recovered.

We propose an algorithm of recovering two-point boundary conditions in a boundary value problem for higher order differential equations. As an additional information, a finite set of eigenvalues of special boundary value problems serve. According the terminology by V.A. Sadovnichii, such problems are called etalon problems.

Keywords: boundary conditions, boundary value problems, etalon problems.

A.S. Krivosheev, O.A. Krivosheeva

Invariant subspaces in a half-plane

Abstract. We study subspaces of functions analytic in a half-plane and invariant with respect to the differentiation operators. A particular case of an invariant subspace is a space of solutions a linear homogeneous differential equation with constant coefficients. It is known that each solution of such equations is a linear combination of primitive solutions, which are exponential monomials with exponents being possibly multiple zeroes of characteristic polynomials. The presence of such representation is called Euler fundamental principle. Other particular cases of invariant subspaces are spaces of solutions of linear homogeneous differential, difference and differential-difference equations with constant coefficients of both finite and infinite orders as well as of more general convolution equations and the systems of them. In the work we study the issue on fundamental principle for arbitrary invariant subspaces for arbitrary invariant subspaces of analytic functions in a half-plane. In other words, we study representation of all functions in an invariant subspace by the series of exponential monomials. These exponential monomials are eigenfunctions and adjoint functions for the differentiation operator in an invariant subspace. In the work we obtain a decomposition of an arbitrary invariant subspace of analytic functions into a sum of two invariant subspaces. We prove that the invariant subspace in an unbounded domain can be represented as a sum of two invariant subspaces. Their spectra correspond to a bounded and unbounded parts of a convex domain. On the base of this result we obtain a simple geometric criterion of the fundamental principle for an invariant subspace of analytic functions in a half-plane. It is formulated just in terms of the Krisvosheev condensation index for the sequence of exponents of the mentioned exponential monomials.

Keywords: invariant subspace, fundamental principle, exponential monomial, entire function, series of exponentials.

A.M. Kytmanov, S.G. Myslivets

On a family of complex curves sufficient for existence of holomorphic continuation of continuous functions on boundary of domain

Abstract. The problem on holomorphic continuation of functions defined on the boundary of a domain into this domain is topical in the multi-dimensional complex analysis. It has a long history beginning from works by Poincaré and Hartogs. In the present work we consider continuous functions defined on a boundary of a bounded domain D in $\mathbb{C}^n$, $n > 1$, and possessing a generalized Morera property along the family of complex curves intersecting the germ of a real analytic manifold of codimension 2 lying away of the boundary of the domain. The Morera property is the vanishing of the integral of this function over the intersection of the boundary of the domain with the complex curve. We show that such function possesses a holomorphic continuation into the domain D. For functions of one complex variable, the Morera property obviously does not imply the existence of holomorphic continuation. This is why such problem can be considered only in the multi-dimensional case ($n > 1$).

Keywords: holomorphic continuation, Morera boundary condition, Bochner-Martinelli kernel.

E.Yu. Mashkov

On approach for studying stochastic Leontieff type equations with impulse actions

Abstract. We study a system of Itô stochastic differential equations having a degenerating constant linear operator in the left hand side. The right hand side of the system contains a constant linear operator and a deterministic term depending on the time only as well as impulse actions. We assume that the diffusion coefficient of this system is described by a square matrix depending on time only. These systems of equations arise in many applications. The system we study can be reduced to a canonical form by applying a transformation of a regular matrix pencil to a generalized real Schur form. The study of the obtained canonical equations requires considering the derivatives of rather higher orders for free terms including the Wiener process. Because of this, in order to differentiate the Wiener process, we apply the Nelson mean derivatives for random processes and this allows us to avoid using the theory of generalized functions. As a result we obtain analytic formulae for solutions of equations in terms of mean derivatives for random processes.

Keywords: mean derivative, current velocity, Wiener process, stochastic equations of Leontieff type

E. Mukhamadiev, A.B. Nazimov, A.N. Naimov

On solvability class of nonlinear equations with small parameter in Banach space

Abstract. We study the solvability of one class of nonlinear equations with a small parameter in a Banach space. The main difficulty is that the principal linear part of this equation is non-invertible. To study the solvability of the considered class of equations we apply a new method combining the Pontryagin method from the theory of autonomous systems on the plane and the methods of calculating the rotations of vector fields. At that we employ a scheme for matrix representations of split operators known in the bifurcation theory for solutions of nonlinear equations. In contrast to the Pontryagin method, we do not assume the differentiability for a nonlinear mapping and apply methods for calculating the rotations of vector fields. On the base of the proposed method we formulate and prove a theorem on solvability conditions for the considered class of nonlinear equations. As application, we study two periodic problems for nonlinear differential equations with a small parameter, namely, a periodic problem for the system of ordinary differential equations in a resonance case and a periodic problem for a nonlinear elliptic equations with a non-invertible linear part.

Keywords: nonlinear equation with small parameter, Pontryagin method, rotation of vector field, periodic problem.

G.G. Petrosyan

On antiperiodic boundary value problem for a semilinear differential inclusion of fractional order with a deviating argument in a Banach space

Abstract. We consider a boundary value problem for a semi-linear differential inclusion of fractional Caputo derivative and a deviating coefficient in a Banach space. We assume that the linear part of the inclusion generates a bounded C_0-semigroup. A nonlinear part of the inclusion is a multi-valued mapping depending on the time and the history of the function before the current time. The boundary condition is functional and anti-periodic in the sense that one function is equals to another with an opposite sign. To resolve the considered problem, we employ the theory of fractional mathematical analysis, the properties of Mittag-Leffler as well as the theory of topological power for multi-valued condensing maps. The idea is as follws: the original problem is reduced to the issue on existence of fixed points of a corresponding resolving multi-valued integral operator in the space of continuous functions. To prove the existence of fixed points of the resolving multi-operator we employ a generalized theorem of Sadovsky type on a fixed point. This is why we show that the resolving integral multi-operator is condensing with respect to a vector measure of non-compactness in the space of continuous functions and maps a closed ball in this space into itself.

Keywords: Caputo fractional derivative, semi-linear differential inclusion, boundary value problem, fixed point, condensing multi-mapping, measure of non-compactness.

I.M.Khamdamov

Properties of convex hull generated by inhomogeneous Poisson point process

Abstract. The paper is devoted to the limit distribution study of the exterior of a convex hull generated by independent observations of two-dimensional random points having Poisson distributions above the parabola. Following P. Groeneboom, we note that near the boundary of support, the Binomial point process is almost indistinguishable from the Poisson point process. Therefore, the approximation of a Binomial point process to a Poisson process is not considered here; it is believed that it is sufficient to study the functionals of the convex hull generated by the Poisson point process. Using the modified P. Groeneboom technique, the so-called strong mixing and martingale properties of the vertex Markovian jump stationary process, the asymptotic expressions are obtained for the expectation and variance of the external part of the area of the convex hull inside the parabola. This is a continuation of results by H. Carnal, where an asymptotic expression was found only for mean values of basic functionals of a convex hull. The asymptotic expression for the variance of the area of a convex hull was later obtained by J. Pardon as no regularity conditions were imposed on the boundary of the support of a uniform distribution. The asymptotic expressions obtained here are used in the proofs of the central limit theorem for the area of the convex hull. Similar results were established in the studies by A.J. Cabo and P. Groeneboom for the case as the initial distribution in a convex polygon is uniform.

Keywords: convex hull, random points, Poisson point process.

Kwok-Pun Ho

Exponential Rosenthal and Marcinkiewicz-Zygmund inequalities

Abstract. We extend the Rosenthal inequalities and the Marcinkiewicz-Zygmund inequalities to some exponential Orlicz spaces.The Rosenthal inequalities and the Marcinkiewicz-Zygmund inequalities are fundamental estimates on the moment of random variables on Lebesgue spaces. The proofs of the Rosenthal inequalities and the Marcinkiewicz-Zygmund inequalities on the exponential Orlicz spaces rely on two results from theory of function spaces and probability theory. The first one is an extrapolation property of the exponential Orlicz spaces. This property guarantees that the norms of some exponential Orlicz spaces can be obtained by taking the supremum over the weighted norms of Lebesgue spaces. The second one is the sharp estimates for the constants involved in the Rosenthal inequalities and the Marcinkiewicz-Zygmund inequalities on Lebesgue spaces. Our results are applications of the extrapolation property of the exponential Orlicz spaces and the sharp estimates for the constants involved in the Rosenthal inequalities and the Marcinkiewicz-Zygmund inequalities on Lebesgue spaces. In addition, the sharp estimates for the constants involved in the Rosenthal inequalities and the Marcinkiewicz-Zygmund inequalities on Lebesgue spaces provide not only some sharpened inequalities in probability, but also yield some substantial contributions on extending those probability inequalities to the exponential Orlicz spaces.

Keywords: Rosenthal inequality, Marcinkiewicz-Zygmund inequalities, martingale, exponential spaces, Orlicz spaces.

K.R. Prasad, M. Rashmita, N. Sreedhar

Solvability of Higher order three-point iterative systems

Abstract. In this paper, we consider an iterative system of nonlinear n^{th} order differential equations:

$$y_i^{(n)}(t) + \lambda_i p_i(t) f_i(y_{i+1}(t)) = 0, \qquad 1 \leqslant i \leqslant m, \qquad y_{m+1}(t) = y_1(t), \qquad t \in [0,1],$$

with three-point non-homogeneous boundary conditions

$$y_i(0) = y_i'(0) = \cdots = y_i^{(n-2)}(0) = 0,$$
$$\alpha_i y_i^{(n-2)}(1) - \beta_i y_i^{(n-2)}(\eta) = \mu_i, \qquad 1 \leqslant i \leqslant m,$$

where $n \geqslant 3$, $\eta \in (0,1)$, $\mu_i \in (0,\infty)$ is a parameter, $f_i : \mathbb{R}^+ \to \mathbb{R}^+$ is continuous, $p_i : [0,1] \to \mathbb{R}^+$ is continuous and p_i does not vanish identically on any closed subinterval of $[0,1]$ for $1 \leqslant i \leqslant m$. We express the solution of the boundary value problem as a solution of an equivalent integral equation involving kernels and obtain bounds for these kernels. By an application of Guo–Krasnosel'skii fixed point theorem on a cone in a Banach space, we determine intervals of the eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_m$ for which the boundary value problem possesses a positive solution. As applications, we provide examples demonstrating our results.

Keywords: boundary value problem, iterative system, kernel, three-point, eigenvalues, cone, positive solution.