

ABSTRACTS

A.A. Allakhverdyan, A.B. Shabat

PRODUCTS OF EIGENFUNCTIONS AND WRONSKIANS

Abstract. We consider new Wronskian identities found recently in Maikop city. We discuss the relations of these identities with the theory of integrable systems and with the general theory of invertible Darboux transforms for linear differential operators with one independent variable. The object of our study are relations of two Wronskians of orders N and $N' > N$ homogeneous with respect to a dilatation group. The elements of the first Wronskian of the order N are arbitrary functions, which enlarges essentially the means of the theory, while the elements of other Wronskian are formed by the products of these functions of a given order $n \geq 2$.

The dilatation group allows us to pass to projective coordinates in the considered quotient of the Wronskians and in particular, to include symmetric functions and polynomials into the considered theory.

The simplest case is naturally $N = 2$, in which the second Wronskian turns out to be a power of the original Wronskian and hence, the considered quotient becomes independent of the choice of the elements in the second Wronskian. In this case we obtain new equations for the third and other powers of the eigenfunctions of an one-dimensional Schrödinger equation generalizing known formulae for squares related with the Schwarz derivative and KdV hierarchy.

The case $N = 3$ seems to be very interesting from various points of view, but to study it, a further developing of the methods in the projective theory of Wronskians is needed by employing logarithmic derivatives and their higher analogues.

Keywords: factorization, Wronsky matrix, Schwarz derivative, Riccati equation, Darboux transform.

V.V. Volchkov, Vit.V. Volchkov

OVERDETERMINED NEUMANN BOUNDARY VALUE PROBLEM
IN UNBOUNDED DOMAINS

Abstract. The studying of overdetermined boundary value problems for elliptic partial differential equations was initiated by J. Serrin in 1971. In his work, he established a property of radial symmetry for solutions of some overdetermined Poisson problem. Apart of an essential independent interest, the problems of such kind have important applications in the potential theory, integral geometry, hydrodynamics and capillarity theory. Usually, the resolving of these problems is based on Hopf lemma on an angular boundary point and the method of hyperplanes motion introduced by A.A. Alexandrov for studying some geometric problems related with characterizing the spheres. Among others, more modern methods not involving the maximum principle for the considered problems, we mention the duality method, the method of volume derivative as well as an integral method.

In the present paper we consider an overdetermined Neumann problem for the Laplace equation $\Delta f = 0$ in planar unbounded domains. We show that under some

conditions, see Theorem 1 in Section 1, such problem is solvable only for the exterior of a ball. A specific feature of Theorem 1 is that in this theorem, for the first time, we obtain an exact condition for the growth of f at infinity. Moreover, as Theorem 2 in Section 2 shows, other conditions in Theorem 1 are also necessary. Opposite to the earlier works, the proof of Theorem 1 employs some boundary properties of conformal mappings, Smirnov theorem on functions in a class H_p and Fejer-Riesz theorem on non-negative trigonometrical polynomials.

Keywords: overdetermined problems, Neumann problem, harmonic functions, boundary behavior.

F.N. Garif'yanov, E.V. Strezhneva

ON MOMENT PROBLEM FOR ENTIRE FUNCTIONS GENERATED BY DOUBLY PERIODIC GROUP

Abstract. We consider a lacunar problem for Stieltjes moments with an exponential weight. The solution is sought in the class of entire functions of exponential type, the indicator diagram of which is a some square. We construct nontrivial solutions of the corresponding homogeneous problem. This problem is reduced to the study of a linear total equation in the class of functions holomorphic outside four squares. At infinity, they have zero of a multiplicity at least three. Their boundary values satisfy the Hölder condition on any compact set containing no square vertices. At most logarithmic singularities are allowed at these vertices. The solution is sought in the form of an Cauchy type integral over the boundary of these squares with an unknown density. A method for regularizing the total equation is proposed. The condition of equivalence of this regularization is clarified. We find particular case when the obtained Fredholm equation of the second kind is solvable. In order to do this, we employ the principle of contracting mappings in a Banach space.

Keywords: regularization method, boundary value problems for elliptic functions, moments of entire functions of exponential type.

O.E. Mirzaev, A.B. Khasanov

ON FAMILIES OF ISOSPECTRAL STURM-LIOUVILLE BOUNDARY VALUE PROBLEMS

Abstract. The work is devoted to describing all boundary value Sturm-Liouville problems on a finite segment with the same spectrum. Such problems are called isospectral and they were studied in works by E.L. Isaacson, H.P. McKean, B.E. Dahlberg, E. Trubowitz, M. Jodeit, B.M. Levitan, Y.A. Ashrafyan, T.N. Harutyunyan. Nowadays, there are various methods for solving inverse spectral problems: the method of transformation operator, that is, Gelfand-Levitan method, the method of spectral mappings, the method of etalon models and others. V.A. Marchenko showed, that the Sturm-Liouville operator on a finite segment is determined uniquely by its eigenvalues and a sequence of normalizing constants, that is, by its spectral function. I.M. Gelfand and B.M. Levitan found necessary and sufficient conditions on recovering boundary value Sturm-Liouville problems by their spectral functions. This method is based on recovering a potential and boundary conditions by spectral data by means of a Fredholm integral equation of a second kind with parameters. While constructing isospectral boundary value Sturm-Liouville problems with a prescribed spectrum n^2 , $n \geq 0$, we have employed the Gelfand-Levitan method. The main result of the work is an algorithm for recovering a family

of boundary value Sturm-Liouville problems $L = L(q(x), h, H)$, whose spectra satisfy the condition $\sigma(L) = \{n^2, n \geq 0\}$. At that, the found coefficients $q = q(x, \gamma_1, \gamma_2, \dots)$, $h = h(\gamma_1, \gamma_2, \dots)$, $H = H(\gamma_1, \gamma_2, \dots)$ depend on infinitely many parameters γ_j , $j = \overline{1, \infty}$.

Keywords: Sturm-Liouville problem, eigenvalues, normalizing constants, spectral data, inverse spectral problem, integral equation, isospectral operators.

A.E. Salimova, B.N. Khabibullin

GROWTH OF SUBHARMONIC FUNCTIONS ALONG THE LINE AND DISTRIBUTION OF THEIR RIESZ MEASURES

Abstract. Let $u \not\equiv -\infty$ and $M \not\equiv -\infty$ be two subharmonic functions on a complex plane \mathbb{C} with Riesz measures ν_u and μ_M , respectively, such that $u(z) \leq O(|z|)$ and $M(z) \leq O(|z|)$ as $z \rightarrow \infty$, and q is some positive continuous function on a real axis \mathbb{R} , and mes is a linear Lebesgue measure on \mathbb{R} . We assume that the following condition for the growth of function u along the imaginary axis $i\mathbb{R}$ of the form

$$u(iy) \leq \frac{1}{2\pi} \int_0^{2\pi} M(iy + q(y)e^{i\theta}) d\theta + q(y) \quad \text{for all } y \in \mathbb{R} \setminus E,$$

where $E \subset \mathbb{R}$ is some small set, for instance, $\text{mes}(E \cap [-r, r]) \leq q(r)$ as $r \geq 0$. Under such restrictions for the function u it is natural to expect that the Riesz measure ν_u is in some sense majorized by the Riesz measure μ_M of the function M or by integral characteristics of the function M . We provide a rigorous quantitative form of such majorizing. The necessity in estimates of such kind arises naturally in the theory of entire functions in its applications to the completeness issues of exponential systems, analytic continuation, etc. Our results are formulated in terms of special "logarithmic" characteristics of measures ν_u and μ_M arisen earlier in classical works by P. Malliavin, L.A. Rubel and other for sequences of points and also in terms of special "logarithmic" characteristics of the behavior of the function M along the imaginary axis and of the function q along the real axis. The obtained results are new also for distribution of the zeroes of entire functions of exponential type under restrictions for the growth of such function along a line. The latter is demonstrated by a new uniqueness theorem for entire functions of exponential type employing so-called logarithmic block-densities of the distribution of the points on the complex plane.

Keywords: subharmonic function of a finite type, Riesz measure, entire function of exponential type, distribution of zeroes, uniqueness theorem.

A.G. Tayshieva, T.R. Myrzakul, G.N. Nugmanova

ON EQUIVALENCE OF ONE SPIN SYSTEM AND TWO-COMPONENT KAMASS-HOLM EQUATION

Abstract. The work is devoted to studying an equivalence of a two-component Kamass-Holm equation (KHE) and a spin system being a generalization of Heisenberg ferromagnet equation. It is known that the equivalence between two nonlinear integrable equations provides a possibility of an extended search of their various exact solutions. For KHE, a method of inverse scattering problem can be applied via a system of linear partial differential equations with scalar coefficients. Contrary to HKE, the coefficients of linear system corresponding to spin equations are related with symmetric matrix Lax representations. This is why, while establishing an equivalence

between two above equations, additional difficulties arise. In view of this, we propose a matrix Lax representation for HKE in a symmetric space. Employing this result, we establish a gauge equivalence between two-component HKE and a spin system. We describe a relation between their solutions.

Keywords: two-component Kamass-Holm equation, matrix Lax representation, spin system, gauge equivalence.

M.V. Falaleev

FUNDAMENTAL OPERATOR FUNCTIONS OF INTEGRO-DIFFERENTIAL OPERATORS UNDER SPECTRAL OR POLYNOMIAL CONSTRAINTS

Abstract. We study a Cauchy problem for a degenerate higher order integro-differential equation in Banach spaces. The operator core of the integral part of the equation is a linear combination of the operator coefficients of its differential part, which corresponds to the physical meaning of some technological processes. The solution is constructed in the space of generalized functions (distributions) in Banach spaces using the methods of the theory of fundamental operands. The convolutional representation of the original equation leads to a further active use of the convolutional technique and its properties. For the considered equations, the corresponding fundamental operator functions are constructed. By means of this operator, a unique generalized solution to the original Cauchy problem in the class of distributions with a left-bounded support is recovered. The analysis of the resulting generalized solution allows us to study the solvability problem in the classical sense. The fundamental operator function is constructed in terms of the theory of semigroups of operators with kernels. Abstract results are illustrated by examples of initial-boundary value problems of visco-elasticity theory.

Keywords: Banach space, generalized function, distribution, fundamental operator-function, integro-differential operator, spectral boundedness, polynomial boundedness.

P.A. Shaikhullina

SECTORIAL NORMALIZATION OF SIMPLEST GERMS OF SEMIHYPERBOLIC MAPS IN A HALF-NEIGHBORHOOD

Abstract. We consider a problem on analytic classification of semi-hyperbolic maps on the plane for an example of the simplest class of such germs, namely, the class of germs that are formally equivalent to F_λ , which is the 1-time shift along the vector field $x^2 \frac{\partial}{\partial x} + \lambda y \frac{\partial}{\partial y}$, $\lambda \in \mathbb{R}_+$). A key step in the classification is an analytic normalization of the germs on sectorial domains forming a cut neighbourhood of the origin $(\mathbb{C}^2, 0) \setminus \{x = 0\}$. For this class, in the present work, we prove a theorem on a sectorial analytic normalization in the half-neighbourhood invariant with respect to F_λ^{-1} . We also show that a formal normalizing change of the coordinates is asymptotic for the constructed sectorial normalizing change.

Keywords: analytic classification, semi-hyperbolic maps, sectorial normalization.

B. Halim, A.Senouci, M. Sofrani

SOME CHEBYSHEV TYPE INEQUALITIES FOR GENERALIZED RIEMANN-LIOUVILLE OPERATOR

Abstract. In this paper we are interested in the famous inequality introduced by Chebyshev. This inequality has several generalizations and applications in different fields of mathematics and others. In particular it is important for us the applications of fractional calculus for the different integral Chebyshev type inequalities.

We establish and prove some theorems and corollaries relating to fractional integral, by applying more general fractional integral operator than Riemann-Liouville one:

$$K_{u,v}^{\alpha,\beta} = \frac{v(x)}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} \left[\ln \left(\frac{x}{t} \right) \right]^{\beta-1} f(t)u(t)dt, \quad x > 0$$

where $\alpha > 0$, $\beta \geq 1$, u and v locally integrable non-negative weight functions, Γ is the Euler Gamma-function. First, fractional integral Chebyshev type inequalities are obtained for operator $K_{u,v}^{\alpha,\beta}$ with two synchronous or two asynchronous functions and by induction for several functions. Second, we consider an extended Chebyshev functional

$$\begin{aligned} T(f, g, p, q) := & \int_a^b q(x)dx \int_a^b p(x)f(x)g(x)dx + \int_a^b p(x)dx \int_a^b q(x)f(x)g(x)dx \\ & - \left(\int_a^b q(x)f(x)dx \right) \left(\int_a^b p(x)g(x)dx \right) \\ & - \left(\int_a^b p(x)f(x)dx \right) \left(\int_a^b q(x)g(x)dx \right), \end{aligned}$$

where p, q are positive integrable weight functions on $[a, b]$. In this case fractional integral weighted inequalities are established for two fractional integral operators $K_{u_1, v_1}^{\alpha_1, \beta_1}$ and $K_{u_2, v_2}^{\alpha_2, \beta_2}$, with two synchronous or asynchronous functions, where $\alpha_1 \neq \alpha_2$, $\beta_1 \neq \beta_2$ and $u_1 \neq u_2$, $v_1 \neq v_2$. In addition, a fractional integral Hölder type inequality for several functions is established using the operator $K_{u,v}^{\alpha,\beta}$. At the end, another fractional integral Chebyshev type inequality is given for increasing function f and differentiable function g .

Keywords: Chebyshev functional, Integral Inequalities, RL-fractional operator.

È. Muhamadiev, M. Nazarov

REGULARITY OF ALMOST PERIODIC SOLUTIONS OF POISSON EQUATION

Abstract. This paper discusses some regularity of almost periodic solutions of the Poisson equation $-\Delta u = f$ in \mathbb{R}^n , where f is an almost periodic function. It was proved by Sibuya [*Almost periodic solutions of Poisson $B\bar{B}^{TM}$ s equation*. Proc. Amer. Math. Soc., 28:195–198, 1971.] that if u is a bounded continuous function and solves the Poisson equation in the distribution sense, then u is an almost periodic function. In this work, we weaken the assumption of the usual boundedness to boundedness in the sense of distribution, which we refer to as a *bounded generalized function*. The set of bounded generalized functions are wider than the set of usual

bounded functions. Then, assuming that u is a bounded generalized function and solves the Poisson equation in the distribution sense, we prove that this solution is bounded in the usual sense, continuous and almost periodic. Moreover, we show that the first partial derivatives of the solution $\partial u/\partial x_i$, $i = 1, \dots, n$, are also continuous, bounded and almost periodic functions. The technique is based on extending a representation formula using Green function for Poisson equation for solutions in the distribution sense. Some useful properties of distributions are also shown that can be used in studying other elliptic problems.

Keywords: Poisson equation, almost periodic solutions, generalized solutions.

V.M. Savchin, P.T. Trinh

NONPOTENTIALITY OF THE SOBOLEV SYSTEM AND THE CONSTRUCTION OF A SEMIBOUNDED FUNCTIONAL

Abstract. This paper discusses some regularity of almost periodic solutions of the Poisson equation $-\Delta u = f$ in \mathbb{R}^n , where f is an almost periodic function. It was proved by Sibuya [*Almost periodic solutions of Poisson's equation*. Proc. Amer. Math. Soc., 28:195–198, 1971.] that if u is a bounded continuous function and solves the Poisson equation in the distribution sense, then u is an almost periodic function. In this work, we weaken the assumption of the usual boundedness to boundedness in the sense of distribution, which we refer to as a *bounded generalized function*. The set of bounded generalized functions are wider than the set of usual bounded functions. Then, assuming that u is a bounded generalized function and solves the Poisson equation in the distribution sense, we prove that this solution is bounded in the usual sense, continuous and almost periodic. Moreover, we show that the first partial derivatives of the solution $\partial u/\partial x_i$, $i = 1, \dots, n$, are also continuous, bounded and almost periodic functions. The technique is based on extending a representation formula using Green function for Poisson equation for solutions in the distribution sense. Some useful properties of distributions are also shown that can be used in studying other elliptic problems.

Keywords: Poisson equation, almost periodic solutions, generalized solutions.