ABSTRACTS

M.S. Bichegkuev

Almost periodic on infinity solutions to integro-differential equations with non-invertible operator at derivative

Abstract. In the paper we consider an integro-differential equation with a non-invertible operator at a derivative in the space of uniformly continuous bounded functions. The integral part of the operator is a convolution of an operator-valued compactly supported Borel measure and a bounded continuous vector function. We obtain sufficient conditions (spectral conditions) of almost periodicity at infinity for bounded solutions of this equation.

The above results are based on the proven statement that if the right-hand side of the equation in question belongs to $C_0(\mathbb{J}, X)$ is a space of functions that tend to zero at infinity, then the Beurling spectrum of each weak solution is contained in the singular set of a characteristic equation. In particular, for equations of the form $\mu * x = \psi$, where the function $\psi \in C_0(\mathbb{J}, X)$ and the support $\operatorname{supp}\mu$ scalar measure μ are compact, we establish that every classical solution is almost periodic at infinity. We show that if the singular set of the characteristic function of the considered equation has no limit points on $\mathbb{R}$, then each weak solution is almost periodic at infinity. We study the structure of bounded solutions in terms of slowly varying at infinity functions.

We provide applications of our results to nonlinear integro-differential equations. We establish that when the right hand side of a nonlinear integro-differential equation is decaying at infinity mapping and a singular set of the characteristic function has no finite limit points on $\mathbb{R}$, a bounded solution of this equation is almost periodic at infinity.

The main results of the paper are obtained by means of the methods of abstract harmonic analysis. The spectral theory of Banach modules is essentially employed.

Keywords: almost periodic at infinity function, Banach space of almost periodic functions at infinity, Beurling spectrum, Bohr periodic function.

S.A. Iskhokov, B.A. Rakhmonov

Solvability and smoothness of solution to variational Dirichlet problem in entire space associated with a non-coercive form

Abstract. In the work we study the solvability of the variational Dirichlet problem for one class of higher order degenerate elliptic operators in an entire n-dimensional Euclidean space. The coefficients of the operator have a power-law degeneracy at the infinity. The formulation of the problem is related with integro-differential sesquilinear form, which may not satisfy the coercivity condition. Earlier, the variational Dirichlet problem for degenerate elliptic operators associated with noncoercive forms was studied mostly for a bounded domain by means of a method based on a finite partition of unity of the domain. In contrast to this, we employ a special infinite partition of unity of the entire Euclidean space of finite multiplicity.

The method used is based on techniques from the theory of spaces of differentiable functions of many real variables with a power weight. The boundary conditions in the problem are homogeneous in the sense that a solution to the problem is sought in a functional space in which the set of infinitely differentiable compactly supported functions is dense.

The differential operator depends on the complex parameter λ, and the existence and uniqueness of a solution of the variational Dirichlet problem is proved in the case as λ belongs to a certain angular sector with a vertex at zero that contains the negative part of the real axis. Under additional conditions on the smoothness of the coefficients and the right-hand side of the equation, the differential properties of the solution are studied.

Keywords: variational Dirichlet problem, elliptic operator, power degeneration, noncoercive form, smoothness of a solution.

Kh. Ishkin, R. Marvanov

Equivalence criterion for two asymptotic formulae

Abstract. We study the equivalence conditions of two asymptotic formulae for an arbitrary non-decreasing unbounded sequence $\{\lambda_n\}$. We show that if g is a non-decreasing and unbounded at infinity function, $\{f_n\}$ is a non-decreasing sequence asymptotically inverse to the function g, then for each sequence of real numbers λ_n satisfying an asymptotic estimate $\lambda_n \sim f_n$, $n \to +\infty$, the estimate $N(\lambda) \sim g(\lambda)$, $\lambda \to +\infty$, holds if and only if g is a pseudo-regularly varying function (PRV-C„CŕPSPєC†PëCЦ). We find a necessary and sufficient condition for the non-decreasing sequence $\{f_n\}$ and the function g, under which the second formula implies the first one. Employing this criterion, we find a non-trivial class of perturbations preserving the asymptotics of the spectrum of an arbitrary closed densely defined in a separable Hilbert space operator possessing at least one ray of the best decay of the resolvent. This result is the first generalization of the a known Keldysh theorem to the case of operators not close to self-adjoint or normal, whose spectra can strongly vary under small perturbations. We also obtain sufficient conditions for a potential ensuring that the spectrum of the Strum-Liouville operator on a curve has the same asymptotics as for the potential with finitely many poles in a convex hull of the curve obeying the trivial monodromy condition. These sufficient conditions are close to necessary ones.

Keywords: asymptotic equivalence, functions preserving equivalence, pseudo-regularly varying (PRV) functions, non-self-adjoint operators, Keldysh theorem, spectrum localization, potentials with trivial monodromy.

A.I. Fedotov

On asymptotic convergence of polynomial collocation method for one class of singular integro-differential equations

Abstract. Among the approximate methods for solving the operator equations the most used methods are collocation and Galerkin methods. Each of them has their own advantages and disadvantages. For instance, Galerkin methods are used for the equations in Gilbert spaces. The estimates for the errors of the solutions obtained by these methods have the order of the best approximations of the exact solutions. However, Galerkin methods are not always constructive, as for their implementation

one needs to calculate integrals which is not always possible to do explicitly. Collocation methods are used for the equations in the spaces of continuous functions and thus are always constructive. However, the estimates for the errors obtained by collocation methods are usually worse than those of the best approximation of the exact solutions.

In the present paper, we justify a polynomial collocation method for one class of singular integro-differential equations on an interval. For the justification, the technic of reducing the polynomial collocation method to Galerkin method is used for the first time for such equations. This technic was first used by the author to justify the polynomial collocation method for a wide class of periodic singular integro-differential and pseudo-differential equations. For the equations on a open interval, this approach is used for the first time. Also for the first time we prove that the interpolative Lagrange operator is bounded in the Sobolev spaces H_q^s, $s > 1/2$, with the Chebyshev weight-function of the second kind. Exactly this result gives an opportunity to show that in non-periodic the polynomial collocation method provides the same convergence rate as the Galerkin method.

Keywords: singular integro-differential equations, justification of the approximate methods.

A.V. Chernov

On preservation of global solvability of controlled second kind operator equation

Abstract. For a controlled evolutionary second kind operator equation in a Banach space considered on a finite time segment, we obtain sufficient conditions for the preservation of global solvability under small (with respect to the right-hand side increment with a fixed state) control variations. In addition, we establish an estimate for the global solution increment under a control variation and conditions for uniqueness of the solution corresponding to an arbitrary fixed control. Most essential differences from former results on the preservation of global solvability of controlled distributed systems are as follows. A solution to the abstract equation representing an evolutionary controlled distributed system can be sought in arbitrary space $W[0;T]$ of time functions with values in a Banach space X and not necessarily in the space of continuous functions with values in X or in a Lebesgue space. An estimate for the solution increment under a control variation is also obtained with respect to the norm of the space $W[0;T]$. Moreover, the right hand sides of the partial differential equations associated with a controlled distributed system may include not only the function of state but also its generalized derivatives. As examples, we study the preservation of global solvability for the nonlinear Navier–Stokes system, the Benjamin–Bona–Mahony–Burgers equation, and also for certain strongly nonlinear pseudo-parabolic equations.

Keywords: nonlinear evolutionary operator equation of second kind in a Banach space, preservation of global solvability, nonlinear Navier–Stokes system, Benjamin–Bona–Mahony–Burgers equation, strongly nonlinear pseudo-parabolic equations.

V.V. Shcherbina

Algebraicity of lattice of τ-closed totally ω-saturated formations of finite groups

Abstract. All groups considered in this paper are assumed to be finite. The symbol ω denotes some nonempty set of primes, and τ is a subgroup functor in the sense of A.N. Skiba. Recall that a *formation* is a class of groups that is closed under taking homomorphic images and finite subdirect products. Functions of the form $f : \omega \cup \{\omega'\} \to \{\text{formations of groups}\}$ are called *ω-local satellites* (*formation ω-functions*). Such functions are used to study the structure of the *ω-saturated formations*.

The paper is devoted to studying the properties of the lattice of all closed functorially totally partially saturated formations related to the concept of being algebraic for a lattice of formations. We prove that for each subgroup functor τ, the lattice $l^{\tau}_{\omega_\infty}$ of all τ-closed totally ω-saturated formations is algebraic. This generalizes the results by V.G. Safonov. In particular, we show that the lattice $l^{\tau}_{p_\infty}$ of all τ-closed totally p-saturated formations is algebraic as well as the lattice l^{τ}_{∞} of all τ-closed totally saturated formations. Similar results are obtained for lattices of functorially closed totally partially saturated formations corresponding to certain subgroup functors τ. Thus, we find new classes of algebraic lattices of formations of finite groups.

Keywords: formation of finite groups, totally ω-saturated formation, lattice of formations, τ-closed formation, algebraic lattice.

B.P. Allahverdiev, H. Tuna

Existence of solutions for nonlinear singular q-Sturm-Liouville problems

Abstract. In this paper, we study a nonlinear q- Sturm-Liouville problem on the semi-infinite interval, in which the limit-circle case holds at infinity for the q-Sturm-Liouville expression. This problem is considered in the Hilbert space $L^2_q(0,\infty)$. We study this problem by using a special way of imposing boundary conditions at infinity. In the work, we recall some necessary fundamental concepts of quantum calculus such as q-derivative, the Jackson q-integration, the q-Wronskian, the maximal operator, etc. We construct the Green function associated with the problem and reduce it to a fixed point problem. Applying the classical Banach fixed point theorem, we prove the existence and uniqueness of the solutions for this problem. We obtain an existence theorem without the uniqueness of the solution. In order to get this result, we use the well-known Schauder fixed point theorem.

Keywords: Nonlinear q-Sturm-Liouville problem, singular point, Weyl limit-circle case, completely continuous operator, fixed point. theorems.

A.B. Yakhshimuratov, B.A. Babajanov

Integration of equations of Kaup system kind with self-consistent source in class of periodic functions

Abstract. In this paper, we consider the equations of Kaup system kind with a self-consistent source in the class of periodic functions. We discuss the complete integrability of the considered nonlinear system of equations, which is based on the transformation to the spectral data of an associated quadratic pencil of Sturm-Liouville equations with periodic coefficients. In particular, Dubrovin-type equations are derived for the time-evolution of the spectral data corresponding to the solutions of equations of Kaup system kind with self-consistent source in the class of periodic functions. Moreover, it is shown that spectrum of the quadratic pencil of Sturm-Liouville equations with periodic coefficients associated with considering nonlinear system does not depend on time. In a one-gap case, we write the explicit formulae for solutions of the problem under consideration expressed in terms of the Jacobi elliptic functions. We show that if $p_0(x)$ and $q_0(x)$ are real analytical functions, the lengths of the gaps corresponding to these coefficients decrease exponentially. The gaps corresponding to the coefficients $p(x,t)$ and $q(x,t)$ are same. This implies that the solutions of problem $p(x,t)$ and $q(x,t)$ are real analytical functions in x.

Keywords: equations of Kaup system kind, quadratic pencil of Sturm-Liouville equations, inverse spectral problem, trace formulas, periodical potential.

A. Rathod

Uniqueness theorems for meromorphic functions on annuli

Abstract. In this paper, we discuss the uniqueness problems of meromorphic functions on annuli. We prove a general theorem on the uniqueness of meromorphic functions on annuli. An analogue of a famous Nevanlinna's five-value theorem is proposed. The main result in this paper is an analog of a result on the plane $\mathbb{C}$ obtained by H.S. Gopalkrishna and Subhas S. Bhoosnurmath for an annuli. That is, let $f_1(z)$ and $f_2(z)$ be two transcendental meromorphic functions on the annulus $\mathbb{A} = \left\{z : \frac{1}{R_0} < |z| < R_0\right\}$, where $1 < R_0 \leqslant +\infty$. Let a_j, $j = 1, 2, \ldots, q)$, be q distinct complex numbers in $\overline{\mathbb{C}}$, and k_j, $j = 1, 2, \ldots, q$ be positive integers or ∞ satisfying

$$k_1 \geqslant k_2 \geqslant \ldots \geqslant k_q.$$

If

$$\overline{E}_{k_j)}(a_j, f_1) = \overline{E}_{k_j)}(a_j, f_2), \qquad j = 1, 2, \ldots, q,$$

and

$$\sum_{j=2}^{q} \frac{k_j}{k_j + 1} - \frac{k_1}{k_1 + 1} > 2,$$

then $f_1(z) \equiv f_2(z)$.

Keywords : Nevanlinna theory, meromorphic functions, annuli.