ABSTRACTS

N.P. Volchkova, Vit.V. Volchkov

A one-radius theorem on a sphere with pricked point

Abstract. We considers local properties of mean periodicity on the two-dimensional sphere $\mathbb{S}^2$. According to the classical properties of periodic functions, each function continuous on the unit circle $\mathbb{S}^1$ possessing zero integrals over any interval of a fixed length $2r$ on $\mathbb{S}^1$ is identically zero if and only if the number r/π is irrational. In addition, there is no non-zero continuous function on $\mathbb{R}$ possessing zero integrals over all segments of fixed length and their boundaries. The aim of this papere is to study similar phenomena on a sphere in $\mathbb{R}^3$ with a pricked point. We study smooth functions on $\mathbb{S}^2 \setminus (0,0,-1)$ with zero integrals over all admissible "spherical caps" and circles of a fixed radius. For such functions, we establish a one-radius theorem, which implies the injectivity of the corresponding integral transform. We also improveme the well-known Ungar theorem on spherical means, which gives necessary and sufficient conditions for the "spherical cap" to belong to the class of Pompeiu sets on $\mathbb{S}^2$. The proof of the main results is based on the description of solutions $f \in C^\infty(\mathbb{S}^2 \setminus (0,0,-1))$ of the convolution equation $(f * \sigma_r)(\xi) = 0$, $\xi \in B_{\pi-r}$, where $B_{\pi-r}$ is the open geodesic ball of radius $\pi - r$ centered at the point $(0,0,1)$ on $\mathbb{S}^2$ and σ_r is the delta-function supported on ∂B_r. The key tool used for describing f is the Fourier series in spherical harmonics on $\mathbb{S}^1$. We show that the Fourier coefficients $f_k(\theta)$ of the function f are representable by series in Legendre functions related to the zeros of the function $P_\nu(\cos r)$. Our main results are consequence of the above representation of the function f and the corresponding properties of the Legendre functions. The results obtained in the work can be used in solving problems associated with ball and spherical means.

Key words: spherical means, Pompeiu transform, Legendre functions, convolution equations.

R.K. Gazizov, A.A. Kasatkin, S.Yu. Lukashchuk

Group classification and symmetry reduction of three-dimensional nonlinear anomalous diffusion equation

Abstract. The work is devoted to studying symmetry properties of a nonlinear anomalous diffusion equation involving a Riemann-Liouville fractional derivative with respect to the time. We resolve a problem on group classification with respect to the diffusion coefficient treated as a function of the unknown. We show that for an arbitrary function, the equation admits a seven-dimensional Lie algebra of infinitesimal operators corresponding to the groups of translations, rotations and dilations. In contrast to the symmetries of the equations with integer derivatives, no translation in time is admitted. Moreover, the coefficients of the group of dilations are different. If the coefficient is power, the admissible algebra is enlarged to a eight-dimensional one by an additional operator in the group of dilatations. For two specific

values of the exponent in the power, the algebra can be further enlarged to a nine-dimensional one or to a eleven-dimensional one and at that, additional admissible operators correspond to various projective transformations. For the obtained Lie algebras of symmetries with dimensions from seven to nine, we construct optimal systems of subalgebras and provide ansatzes for corresponding invariant solutions of various ranks. We provide also general forms of writing invariant solutions convenient for the reduction as the fractional Riemann-Liouville derivative is present. We make a symmetry reduction on subalgebras allowing one to find invariant solutions of rank one. We provide corresponding reduced ordinary fractional differential equations.

Keywords: fractional derivatives, symmetry reduction, optimal system of subalgebras, nonlinear fractional diffusion equation.

F.N. Garif'yanov, E.V. Strezhneva

On applications of summary equation induced by quadrilateral

Abstract. Let D be an arbitrary quadrilateral. On this quadrilateral, we consider a linear summary four-elements equation with the class of solutions holomorphic outside D and vanishing at infinity. Their boundary the values satisfy the Hölder condition on each compact set containing no peaks. If the peaks are present, at them, at most logarithmic singularities are admitted. The free term is holomorphic on D its boundary value satisfies the Hölder condition. It is not assume to admit an analytic continuation through some segment of the boundary, that is, the solution and the free term belong to different classes of holomorphic functions. In order to regularize this equation on the boundary of the quadrilateral, we introduce a piece-wise linear Carleman translation mapping each side into itself by changing the orientation. This translation is discontinuous at the vertices and has fixed points at the centers of the side. The solution can be represented as a Cauchy type integral over a boundary with an unknown density invariant with respect to the shift on one pair of adjacent sides and anti-invariant on the other pair. We show that the regularization is equivalent. In some particular cases the obtained Fredholm equation is solvable. As an example, we choose an quadrilateral with a straight angle. We construct a system of entire functions of a completely regular growth biorthogonal to the system of powers with a piece-wise quasi-polynomial weight.

Keywords: equivalent regularization, biorthogonal systems of analytic functions, moment problem for entire functions of exponential type.

I.A. Kaliev, G.S. Sabitova

Neumann boundary value problem for system of equations of non-equilibrium sorption

Abstract.

Filtration of liquids and gases containing associated (dissolved, suspended) solids in porous media is accompanied by diffusion of these substances and mass transfer between the liquid (gas) and solid phases. In this work, we study the system of equations modeling the process of a non-equilibrium sorption. We prove a existence and uniqueness theorem for the solution to a multi-dimensional Neumann initial-boundary value problem in the Hölder classes of functions. We obtain a maximum

principle, which plays an important role in the proof of the theorem. The uniqueness of the solution follows this principle. The existence of a solution to the problem is shown by Schauder fixed point theorem for a completely continuous operator; we describe a corresponding operator. We obtain estimates ensuring the complete continuity of the constructed operator and the mapping of some closed set of functions into itself over a small period of time. Then we obtain the estimates allowing us to continue the solution up to arbitrary finite time.

Keywords: modeling of process of non-equilibrium sorption, Neumann initial boundary value problem, global unique solvability.

D.S. Klimentov

Stochastic analogue of fundamental theorem of surface theory for surfaces with bounded distortion and positive curvature

Abstract. In this paper, we prove a stochastic analogue of Gauss–Peterson–Codazzi equations and provide a stochastic analogue of the fundamental theorem in the theory of surfaces for surfaces of a bounded distortion and a positive curvature. In 1956, I.Ya. Bakelman derived the Gauss–Peterson–Codazzi equations for surfaces of bounded distortion, that is, for the surfaces defined by functions with continuous first derivatives and square summable square generalized second derivatives in the sense of Sobolev. In 1988, Yu.E. Borovsky proved that the Gauss–Peterson–Codazzi equations (derived by I.Ya. Bakelman) uniquely determine the surface of a limited curvature.

The aim of this paper is to present the results of I.Ya. Bakelman and Borovsky Y.E. in terms of the theory of random processes in the case of a surface of a positive bounded distortion and a positive curvature.

By means of two fundamental forms of the surface, we construct two random processes and derive a system of equations relating the characteristics (transition functions) of these processes. The resulting system is a stochastic analogue of the system of Gauss–Peterson–Codazzi equations and is a criterion determining uniquely the surface up to a motion. The generators of random processes are second order operators generated by the fundamental forms of the surface. For instance, if the surface metrics is given by the expression $I = ds^2 = g_{ij}dx^i dx^j$, then the generator of the corresponding process is $A = g^{ij}\partial_i\partial_j$. We establish a relationship between the transition functions of the random process and the generator coefficients. The obtained expressions are substituted into the generalized Gauss вЂ“ Peterson – Codazzi equations, which leads us to the desired result.

Keywords: surface of bounded distortion, curvature, random process, transition function of random process, Kolmogorov equation.

D.F. Kuznetsov

Expansion of iterated Stratonovich stochastic integrals based on generalized multiple Fourier series

Abstract. The article is devoted to expansions of iterated Stratonovich stochastic integrals of multiplicities 1-4 on the base of the method of generalized multiple Fourier series. We prove the mean-square convergence of expansions in the case of Legendre polynomials as well as in the case of trigonometric functions. The considered expansions contain only one passage to the limit in contrast to its existing analogues. This property is very convenient for the mean-square approximation of iterated stochastic integrals. It is well-known that a prospective approach to numerical solving of Itô stochastic differential equations being adequate mathematical models of dynamical systems of various physical nature is one based on stochastic analogue of Taylor formula for the solutions to these equations. The iterated stochastic Stratonovich integrals are parts of so-called Taylor-Stratonovich expansion being one of the aforementioned stochastic analogues of Taylor formula. This is why the results of the paper can be applied to constructing strong numerical methods of convergence orders 1.0, 1.5 and 2.0 for Itô stochastic differential equations. The method of generalized multiple Fourier series does not require the integration interval for iterated stochastic Stratonovich integrals. This feature is essential since the mentioned integration interval is small playing a role of the integration step in numerical methods for Itô stochastic differential equations.

Keywords: iterated Stratonovich stochastic integral, multiple Fourier series, Legendre polynomial, expansion, mean-square convergence.

Sh.A. Muranov

On estimates for oscillatory integrals with phase depending on parameters

Abstract. We consider estimates for the Fourier transforms of measures supported on analytic hypersurfaces involving a damping factor. As a damper we naturally take a power of the Gaussian curvature of the surface. It is known if the exponent in this power is a sufficiently large positive number, then the Fourier transform of the corresponding measure have an optimal decay. C.D.Sogge and E.M.Stein formulated a problem on a minimal power of the Gaussian curvature ensuring an optimal decay for the Fourier transform. In the paper we resolve the problem by C.D.Sogge and E.M.Stein on an optimal decay for the Fourier transform with a damping factor of a particular class of families of analytic surfaces in the three-dimensional Euclidean space. We note that the power we provide is sharp not only for the families of analytic hypersurfaces but also for a fixed analytic hypersurface. The proof of main result is based on the methods of the theory of analytic functions, more precisely, on the statements like a preparation Weierstrass theorem. As D.M. Oberlin showed, similar statements fail for infinitely differentiable hypersurfaces.

Keywords: oscillating integrals, Fourier transform, dumping factor, maximal operator.

A.B. Khasanov, F.R. Tursunov

Of Cauchy problem for Laplace equation

Abstract. The paper is devoted to studying the continuation of a solution and estimates of the stability in the Cauchy problem for the Laplace equation in a domain G by its known values on the smooth part S of the boundary∂G. The considered issue is among the problems of mathematical physics, in which there is no continuous dependence of solutions on the initial data. While solving applied problems, one needs to find not only an approximate solution, but also its derivative. In the work, given the Cauchy data on a part of the boundary, by means of Carleman function, we recover not only a harmonic function, but also its derivatives. If the Carleman function is constructed, then by employing the Green function, one can find explicitly the regularized solution. We show that an effective construction of the Carleman function is equivalent to the constructing of the regularized solution to the Cauchy problem. We suppose that the solutions of the problem exists and is continuously differentiable in a closed domain with exact given Cauchy data. In this case we establish an explicit formula for continuation of the solution and its derivative as well as a regularization formula for the case, when instead of Cauchy initial data, their continuous approximate are prescribed with a given error in the uniform metrics. We obtain stability estimates for the solution to the Cauchy problem in the classical sense.

Keywords: Cauchy problem, ill-posed problems, Carleman function, regularized solutions, regularization, continuation formulae.

A.Y. Abdelwanis

On triple derivations of partially ordered sets

Abstract. In this paper, as a generalization of derivation on a partially ordered set, the notion of a triple derivation is presented and studied on a partially ordered set. We study some fundamental properties of the triple derivation on partially ordered sets. Moreover, some examples of triple derivations on a partially ordered set are given. Furthermore, it is shown that the image of an ideal under triple derivation is an ideal under some conditions. Also, the set of fixed points under triple derivation is an ideal under certain conditions. We establish a series of further results of the following nature. Let $(P, \leqslant)$ be a partially ordered set.
1. If d, s are triple derivations on P, then $d = s$ if and only if $\mathrm{Fix}_d(P) = \mathrm{Fix}_s(P)$.
2. If d is a triple derivation on P, then, for all $x \in P$;$\mathrm{Fix}_d(P) \cap l(x) = l(d(x))$.
3. If d and s are two triple derivations on P, then d and s commute.
4. If d and s are two triple derivations on P, then $d \leqslant s$ if and only if $sd = d$.
In the end, the properties of ideals and operations related to triple derivations are examined.

Keywords: triple derivation, fixed point, ideal, partially ordered set.

M. Benallia, M. Moussai

Realization of homogeneous v Triebel-Lizorkin spaces with $p = \infty$ and characterizations via differences

Abstract. In this paper, via the decomposition of Littlewood-Paley, the homogeneous Triebel-Lizorkin space $\dot{F}^s_{\infty,q}$ is defined on $\mathbb{R}$ by distributions modulo polynomials in the sense that $\|f\| = 0$ ($\|\cdot\|$ the quasi-seminorm in $\dot{F}^s_{\infty,q}$) if and only if f is a polynomial on $\mathbb{R}$. We consider this space as a set of "true" distributions and we are lead to examine the convergence of the Littlewood-Paley sequence of each element in $\dot{F}^s_{\infty,q}$. First we use the realizations and then we obtain the realized space $\dot{\widetilde{F}}^s_{\infty,q}$ of $\dot{F}^s_{\infty,q}$.

Our approach is as follows. We first study the commuting translations and dilations of realizations in $\dot{F}^s_{\infty,q}$, and employing distributions vanishing at infinity in the weak sense, we construct $\dot{\widetilde{F}}^s_{\infty,q}$. Then, as another possible definition of $\dot{F}^s_{\infty,q}$, in the case $s > 0$, we make use of the differences and describe $\dot{\widetilde{F}}^s_{\infty,q}$ as $s > \max(n/q - n, 0)$.

Keywords: Triebel-Lizorkin spaces, Littlewood-Paley decomposition, realizations.

M.C. Dağlı

Some relations for universal Bernoulli polynomials

Abstract. In this paper, we consider a generalization of the Bernoulli polynomials, which we call universal Bernoulli polynomials. They are related to the Lazard universal formal group. The corresponding numbers by construction coincide with the universal Bernoulli numbers. They turn out to have an important role in complex cobordism theory. They also obey generalizations of the celebrated Kummer and Clausen–von Staudt congruences.

We derive a formula on the integral of products of higher-order universal Bernoulli polynomials. As an application of this formula, the Laplace transform of periodic universal Bernoulli polynomials is presented. Moreover, we obtain the Fourier series expansion of higher-order universal Bernoulli function.

Keywords: Bernoulli polynomials and numbers, formal group, integrals, Fourier series.

S.N. Lakaev, M. Darus, S.T. Dustov

Threshold phenomenon for a family of the generalized Friedrichs models with the perturbation of rank one

Abstract. In this work we consider a family $H_\mu(p)$, $\mu > 0$, $p \in \mathbb{T}^3$, of the generalized Friedrichs models with the perturbation of rank one. This system describes a system of two particles moving on the three dimensional lattice $\mathbb{Z}^3$ and interacting via a pair of local repulsive potentials. One of the reasons to consider such family of the generalized Friedrichs models is that this family generalizes and involves some important behaviors of the Hamiltonians for systems of both bosons and fermions on lattices. In the work, we study the existence or absence of the eigenvalues of the operator $H_\mu(p)$ located outside the essential spectrum depending on the values of $\mu > 0$ and $p \in U_\delta(p_0) \subset \mathbb{T}^3$. We prove a analytic dependence on the parameters for such eigenvalue and an associated eigenfunction and the latter is found in a certain explicit form. We prove the existence of coupling constant threshold $\mu = \mu(p) > 0$

for the operator $H_\mu(p)$, $p \in U_\delta(p_0)$, namely, we show that the operator $H_\mu(p)$ has no eigenvalue for all $0 < \mu < \mu(p)$ and there exists a unique eigenvalue $z(\mu, p)$ for each $\mu > \mu(p)$ and this eigenvalue is located above the threshold $z = M(p)$. We find necessary and sufficient conditions allowing us to determine whether the threshold $z = M(p)$ is an eigenvalue or a virtual level or a regular point in the essential spectrum of the operator $H_\mu(p)$, $p \in U_\delta(p_0)$.

Keywords: coupling constant threshold, repulsive potential, eigenvalue, generalized Friedrichs model, regular point.

H.A. Wahash, M.S. Abdo, S.K. Panchal

Fractional integrodifferential equations with nonlocal conditions and generalized Hilfer fractional derivative

Abstract. We study some basic properties of the qualitative theory such as existence, uniqueness, and stability of solutions to the first-order of weighted Cauchy-type problem for nonlinear fractional integro-differential equation with nonlocal conditions involving a general form of Hilfer fractional derivative. The fractional integral and derivative of different orders are involved in the given problem and the classical integral is involved in nonlinear terms. We establish the equivalence between the weighted Cauchy-type problem and its mixed type integral equation by employing various tools and properties of fractional calculus in weighted spaces of continuous functions. The Krasnoselskii's fixed point theorem and the Banach fixed point theorem are used to obtain the existence and uniqueness of solutions of a given problem, and also the results of nonlinear analysis such as Arzila-Ascoli theorem and some special functions like Gamma function, Beta function, and Mittag-Leffler function serves as tools in our analysis. Further, the generalized Gronwall inequality is used to obtain the Ulam-Hyers, generalized Ulam-Hyers, Ulam-Hyers-Rassias, and generalized Ulam-Hyers-Rassias stability of solutions of the weighted Cauchy-type problem. In the end, we provide two examples demonstrating our main results.

Keywords: fractional integro-differential equations, nonlocal conditions ,$\psi-$Hilfer fractional derivative, existence and Ulam-Hyers stability, fixed point theorem.