ABSTRACTS

A.M. Bikchentaev

Renormalizations of measurable operator ideal spaces affiliated to a semifinite von Neumann algebra

Abstract. This work is devoted to non-commutative analogues of classical methods of constructing functional spaces. Let a von Neumann algebra $\mathcal{M}$ of operators acts in a Hilbert space $\mathcal{H}$, τ is a faithful normal semifinite trace $\mathcal{M}$. Let $\widetilde{\mathcal{M}}$ be an $*$-algebra of τ-measurable operators, $|X| = \sqrt{X^*X}$ for $X \in \widetilde{\mathcal{M}}$. A lineal $\mathcal{E}$ in $\widetilde{\mathcal{M}}$ is called ideal space on $(\mathcal{M}, \tau)$, if
1) the belonging $X \in \mathcal{E}$ implies $X^* \in \mathcal{E}$;
2) the belongings $X \in \mathcal{E}$, $Y \in \widetilde{\mathcal{M}}$ and the inequality $|Y| \leqslant |X|$ imply that $Y \in \mathcal{E}$.

Let $\mathcal{E}$, $\mathcal{F}$ be ideal spaces on $(\mathcal{M}, \tau)$. We propose a method of constructing a mapping $\tilde{\rho}\colon \mathcal{E} \to [0, +\infty]$ with nice properties by employing a mapping ρ on a positive cone $\mathcal{E}^+$. At that, if $\mathcal{E} = \mathcal{M}$ and $\rho = \tau$, then $\tilde{\rho}(X) = \tau(|X|)$ and if the trace τ is finite, then $\tilde{\rho}(X) = \|X\|_1$ for all $X \in \mathcal{M}$. We study the case as $\tilde{\rho}(X)$ is equivalent to the original mapping $\rho(|X|)$. Employing mappings on $\mathcal{E}$ and $\mathcal{F}$, we construct a new mapping with nice properties on the sum $\mathcal{E} + \mathcal{F}$. We provide examples of such mappings. The results are new also for $*$-algebra $\mathcal{M} = \mathcal{B}(\mathcal{H})$ of all bounded linear operators in $\mathcal{H}$ equipped with a canonical trace $\tau = \mathrm{tr}$.
Keywords: Hilbert space, linear operator, von Neumann algebra, normal trace, measurable operators, ideal space, renormalization.

I.N. Braeutigam, D.M. Polyakov

Asymptotics of the eigenvalues of infinite block matrices

Abstract. The paper is devoted to determining the asymptotic behavior of eigenvalues, which is one of topical direction in studying operators generated by tridiagonal infinite block matrices in Hilbert spaces of infinite sequences with complex coordinates or, in other words, to discrete Sturm-Liouville operators. In the work we consider a class of non-self-adjoint operators with discrete operators being a sum of a self-adjoint operator serving as an unperturbed operator and a perturbation, which is an operator compact relatively the unperturbed operator. In order to study the asymptotic behavior of eigenvalues, in the paper we develop an adapted scheme of abstract method of similar operators. The main idea of this approach is that by means of the similarity operator, the studying of spectral properties of the original operator is reduced to studying the spectral properties of an operator of a simple structure. Employing this scheme, we write out the formulae for the asymptotics of arithmetical means of the eigenvalues of the considered class of the operators. We note that such approach differs essentially from those employed before. The obtained general result is applied for determining eigenvalues of particular operators. Namely, we provide asymptotics for the eigenvalues of symmetric and non-symmetric tridiagonal infinite matrices in the scalar case, the asymptotics for arithmetical means of the eigenvalues

of block matrices with power behavior of eigenvalues of unperturbed operator and generalized Jacobi matrices with various number of non-zero secondary diagonals.

Keywords: infinite tridiagonal block matrices, Jacobi matrices, the method of similar operators, eigenvalues, spectrum.

N.I. Zhukova

Graphs of totally geodesic foliations on pseudo-Riemannian manifolds

Abstract. We study totally geodesic foliations (M, F) of arbitrary codimension q on n-dimensional pseudo-Riemannian manifolds, for which the induced metrics on leaves do not degenerate. We assume that the q-dimensional orthogonal distribution $\mathfrak{M}$ to (M, F) is an Ehresmann connection for this foliation. Since the usual graph $G(F)$ is not Hausdorff manifold in general, we study the graph $G_{\mathfrak{M}}(F)$ of the foliation with an Ehresmann connection $\mathfrak{M}$ introduced early by the author. This graph is always a Hausdorff manifold. We prove that on the graph $G_{\mathfrak{M}}(F)$, a pseudo-Riemannian metric is defined, with respect to which the induced foliation and the simple foliations formed by the fibers of the canonical projections are totally geodesic. We show that the leaves of the induced foliation on the graph are nondegenerately reducible pseudo-Riemannian manifolds and their structure is described. The application to parallel foliations on nondegenerately reducible pseudo-Riemannian manifolds is considered. We also show that each foliation defined by the suspension of a homomorphism of the fundamental group of a pseudo-Riemannian manifold belongs to the considered class of foliations.

Keywords: totally geodesic foliation, pseudo-Riemaniann manifold, graph of a foliation, Ehresmann connection for a foliation.

O.A. Krivosheeva, A.S. Krivosheev, A.I. Rafikov

Lower bounds for entire functions

Abstract. We study lower bounds for entire functions of proximate order and of completely regular growth. We introduce the notion of the index condensation for sequences of complex numbers of proximate order. This notion generalizes that of the index of condensation for sequences of order one. We also introduce a properly balanced set, which is a properly distributed set with zero condensation index. We show that a regular set is properly balanced and we prove that the properly balanced property of the zero set of an entire function is a necessary and sufficient condition for the existence of family of pairwise disjoint circles with the centers at its zeros and with relatively small radii. Outside these circles, the absolute value of the function admits lower bounds asymptotically coinciding with its upper bounds everywhere in the plane. Thus, we show that the notion of a properly balanced set naturally generalizes the notion of a regular set in the case of arbitrary sequences including multiples sequences. A method for constructing of an exceptional set consisting of circles with centers at zeros of entire function is also provided. In some cases, we can make the sum of the radii of these circles arbitrarily small.

Keywords: Entire function, proximate order, completely regular growth, properly balanced set, regular set.

L.B. Mironova

On class of integral equations with partial integrals and its applications

Abstract. We prove the existence and uniqueness of the solution to one class of systems of integral equations with partial integrals. Equations with partial integrals are equations containing an unknown function in the integrands of integrals of different dimension. The feature of the considered class of integral equations is that the equations involve integrals with both variables and constant upper integration limits. We first prove the unique solvability theorem for integral equations in the three-dimensional space. A similar statement is proved for equations with an arbitrary many independent variables. Some applications of the obtained result are provided. For a hyperbolic system with dominant derivatives of the second order with three independent variables, we prove the existence and uniqueness of the solution of the main characteristic problem. The main characteristic problem for the system of equations with dominant derivatives of the second order can be considered as an analogue of the Goursat problem for a hyperbolic system with no multiple characteristics. The solution of this problem is constructed explicitly in terms of the Riemann matrix. The Riemann matrix is defined as the solution of the Volterra system of integral equations. The problem with boundary conditions on five sides of the characteristic parallelepiped for this system of equations with dominant derivatives of the second order is formulated. By reducing the problem to a system of equations with partial integrals and basing on our results, we prove the existence and uniqueness of the solution to this problem.

Keywords: integral equation with partial integrals, problem with conditions on the characteristics.

B.Kh. Turmetov

Green function for analogue of Robin problem for polyharmonic equation

Abstract. We propose a method of constructing the Green function for some boundary value problems for a polyharmonic equation in a multi-dimensional unit ball. The considered problem are analogues of the Robin problem for an inhomogeneous polyharmonic equation. For studying the solvability of these problems first in the class of smooth in a ball functions, we first provide the properties of integral-differential operators. Then, employing these properties, the considered problems are reduced to an equivalent Dirichlet problem with a special right hand side. Using then known statements on the Dirichlet problem, for the main problems we prove the unique solvability theorems. We also obtain integral representations for solutions of these problems via the solutions of the Dirichlet problem. Employing the explicit form of the Green function, we find an integral representation of the Dirichlet problem with a special right hand side. The obtained integral representation then is used to construct the Green function for analogues of Robin problems. We also provide an approach for constructing the Green function for other main problems. In order to do this, we study the differential properties of the fundamental solution of the polyharmonic operator. The obtained properties of the fundamental solutions are applied for studying the properties of the Green function for the Dirichlet problem. We construct the representations of the Green function for analogues of the Robin

problem. While finding the Green functions for these problems, we employ essentially the form of the Green function for the Dirichlet problem for the polyhgarmonic equation. Namely, the Green function of these problems is represented as the sum of the Green function for the Dirichlet problem and some integral term. The obtained results are in agreement with the known results for the Laplace operator.

Keywords: polyharmonic equation, boundary value problem, Dirichlet problem, analogue of Robin problem, Green function, integral representation.

H. El-Azhar, K. Idrissi, E.H. Zerouali

Weak positive matrices and hyponormal weighted shifts

Abstract. In the paper we study k-positive matrices, that is, the class of Hankel matrices, for which the $(k+1)\times(k+1)$-block-matrices are positive semi-definite. This notion is intimately related to a k-hyponormal weighted shift and to Stieltjes moment sequences. Using elementary determinant techniques, we prove that for a k-positive matrix, a $k\times k$-block-matrix has non zero determinant if and only if all $k\times k$-block matrices have non zero determinant. We provide several applications of our main result. First, we extend the Curto-Stampfly propagation phenomena for for 2-hyponormal weighted shift W_α stating that if $\alpha_k=\alpha_{k+1}$ for some $n\geqslant 1$, then for all $n\geqslant 1, \alpha_n=\alpha_k$, to k-hyponormal weighted shifts to higher order. Second, we apply this result to characterize a recursively generated weighted shift. Finally, we study the invariance of k-hyponormal weighted shifts under one rank perturbation. A special attention is paid to calculating the invariance interval of 2-hyponormal weighted shift; here explicit formulae are provided.

Keywords: Subnormal operators, k-hyponormal operators, k-positive matrices, weighted shifts, perturbation, moment problem.

R.N. Garifullin, R.I. Yamilov

On series of Darboux integrable discrete equations on square lattice

Abstract. We present a series of Darboux integrable discrete equations on a square lattice. The equations in the series are numbered by natural numbers M. All the equations possess a first order first integral in one of directions of the two-dimensional lattice. The minimal order of a first integral in the other direction is equal to $3M$ for an equation with the number M. First integrals in the second direction are defined by a simple general formula depending on the number M.

In the cases $M=1,2,3$ we show that the equations are integrable by quadrature. More precisely, we construct their general solutions in terms of the discrete integrals.

We also construct a modified series of Darboux integrable discrete equations having the first integrals of the minimal orders 2 and $3M-1$ in different directions, where M is the equation number in series. Both first integrals are not obvious in this case.

A few similar series of integrable equations were known before; however, they were of Burgers or sine-Gordon type. A similar series of the continuous hyperbolic type equations was discussed by A.V. Zhiber and V.V. Sokolov in 2001.

Keywords: discrete quad-equation, Darboux integrability, general solution.

M.N. Kuznetsova

Classification of a subclass of quasilinear two-dimensional lattices by means of characteristic algebras

Abstract. We consider a classification problem of integrable cases of the Toda type two-dimensional lattices $u_{n,xy} = f(u_{n+1}, u_n, u_{n-1}, u_{n,x}, u_{n,y})$. The function $f = f(x_1, x_2, \cdots x_5)$ is assumed to be analytic in a domain $D \subset \mathbb{C}^5$. The sought function $u_n = u_n(x, y)$ depends on real x, y and integer n. Equations with three independent variables are complicated objects for study and especially for classification. It is commonly accepted that for a given equation, the existence of a large class of integrable reductions indicates integrability. Our classification algorithm is based on this observation. We say that a constraint $u_0 = \varphi(x, y)$ defines a degenerate cutting off condition for the lattice if it divides this lattice into two independent semi-infinite lattices defined on the intervals $-\infty < n < 0$ and $0 < n < +\infty$, respectively. We call a lattice integrable if there exist cutting off boundary conditions allowing us to reduce the lattice to an infinite number of hyperbolic type systems integrable in the sense of Darboux. Namely, we require that lattice is reduced to a finite system of such kind by imposing degenerate cutting off conditions at two different points $n = N_1$, $n = N_2$ for arbitrary pair of integers N_1, N_2. Recall that a system of hyperbolic equations is called Darboux integrable if it admits a complete set of integrals in both characteristic directions. An effective criterion of the Darboux integrability of the system is connected with properties of an associated algebraic structures. More precisely, the characteristic Lie-Rinehart algebras assigned to both characteristic directions have to be of a finite dimension. Since the obtained hyperbolic system is of a very specific form, the characteristic algebras are effectively studied. Here we focus on a subclass of quasilinear lattices of the form

$$u_{n,xy} = p(u_{n-1}, u_n, u_{n+1})u_{n,x} + r(u_{n-1}, u_n, u_{n+1})u_{n,y} + q(u_{n-1}, u_n, u_{n+1}).$$

Keywords: two-dimensional lattice, integrable reduction, characteristic Lie algebra, degenerate cutting off condition, Darboux integrable system, x-integral.