

ABSTRACTS

A.G. Baskakov, E.E. Dikarev

SPECTRAL THEORY OF FUNCTIONS IN STUDYING PARTIAL DIFFERENTIAL OPERATORS

Abstract. The work is devoted to studying the spectral properties of differential operators with constant coefficients defined on subspaces of bounded continuous functions. Our main methods are spectral theory of Banach modules, theory of functions, abstract harmonic analysis and theory of representations, which were developed and described in detail in the monograph by A. G. Baskakov «Harmonic Analysis in Banach Modules and the Spectral Theory of Linear Operators», Voronezh, VSU Publishing House, 2016. We introduce the algebra of polynomials by means of which we define differential operators. We also introduce closed subspaces of the space of bounded continuous functions called homogeneous function spaces, which play an important role in the analysis. An important class of spectrally homogeneous spaces is introduced as well. We obtain results relating the zero set of a polynomial with the properties of kernels and images of differential operators defined by these polynomials. We define the notion of a regular at infinity polynomial (ellipticity-type conditions) and we provide important examples of partial differential operators defined by such polynomials. The conditions of invertibility of such differential operators are obtained. In particular, we obtain criteria of invertibility in spectrally homogeneous spaces and spaces of periodic function. We get a result on coincidence of the spectrum of a differential operator with the image of polynomial generating this operator in spectrally homogeneous spaces. Conditions of compactness of the resolvent of partial differential operators defined by polynomials regular at the infinity are obtained.

Keywords: partial differential operator, regular polynomial, Beurling spectrum of function, spectrum of operator, Banach module, kernel and image of linear operator, invertibility of operator

B.V. Vynnyts'kyi, V.L. Sharan, I.B. Sheparovych

ON AN INTERPOLATION PROBLEM IN THE CLASS OF FUNCTIONS OF EXPONENTIAL TYPE
IN A HALF-PLANE

Abstract. The conditions of solvability of the interpolation problem $f(n) = d_n$, $n \in \mathbb{N}$ in the class of entire functions satisfying the condition $|f(z)| \leq e^{\pi|Imz| + o(|z|)}$, $z \rightarrow \infty$ are well known. In the presented paper we study the interpolation problem $f(\lambda_n) = d_n$ in the class of exponential type functions in the half-plane. We find sufficient solvability conditions for the considerate problem are found. In particular, a sufficient part of Carleson's interpolation theorem is generalized and an analogue of a classic interpolation condition is found in a form

$$\sum_{j=k}^{\infty} \operatorname{Re} \left(-\xi_j \frac{\lambda_k^2 - 1}{\lambda_k + \lambda_j} \right) \leq c_3, \quad \xi_j := \frac{\operatorname{Re} \lambda_j}{1 + |\lambda_j|^2}.$$

The necessity of sufficient conditions is also discussed. The results are applied to studying a problem on splitting and searching an analogue of the identity

$2 \cos z = \exp(-iz) + \exp(iz)$ for each function of exponential type in the half-plane. We prove that each holomorphic in the right-hand half-plane function f obeying the estimate $|f(z)| \leq O(\exp(\sigma|\operatorname{Im} z|))$ can be represented in the form $f = f_1 + f_2$ and the functions f_1 and f_2 holomorphic in the right-hand half-plane satisfy conditions $|f_1(z)| \leq O(\exp(|z|h_-(\varphi)))$ and $|f_2(z)| \leq O(\exp(|z|h_+(\varphi)))$, where $\sigma \in [0; +\infty)$, $z = re^{i\varphi}$,

$$h_+(\varphi) = \begin{cases} \sigma|\sin \varphi|, \varphi \in [0; \pi/2], \\ 0, \varphi \in [-\pi/2; 0], \end{cases}, \quad h_-(\varphi) = \begin{cases} 0, \varphi \in [0; \pi/2], \\ \sigma|\sin \varphi|, \varphi \in [-\pi/2; 0]. \end{cases}$$

The paper uses methods from Carleson's works also the works by P. Jones, K. Kazaryan, K. Malyutin and other mathematicians.

Keywords: holomorphic functions of exponential type in the half-plane, interpolation, splitting of holomorphic functions

M.V. Dontsova

SOLVABILITY OF CAUCHY PROBLEM FOR A SYSTEM OF FIRST ORDER QUASILINEAR EQUATIONS WITH RIGHT-HAND SIDES $f_1 = a_2u(t, x) + b_2(t)v(t, x)$, $f_2 = g_2v(t, x)$

Abstract. We consider a Cauchy problem for a system of two first order quasilinear differential equations with right-hand sides $f_1 = a_2u(t, x) + b_2(t)v(t, x)$, $f_2 = g_2v(t, x)$. We study the solvability of the Cauchy problem on the base of the additional argument method. We obtain the sufficient conditions for the existence and uniqueness of a local solution to the Cauchy problem in terms of the original coordinates coordinates for a system of two first order quasilinear differential equations with right-hand sides $f_1 = a_2u(t, x) + b_2(t)v(t, x)$, $f_2 = g_2v(t, x)$, under which the solution has the same smoothness in x as the initial functions in the Cauchy problem. A theorem on the local existence and uniqueness of a solution to the Cauchy problem is formulated and its proof is provided.

The theorem on the local existence and uniqueness of a solution to the Cauchy problem for a system of two first order quasilinear differential equations with right-hand sides $f_1 = a_2u(t, x) + b_2(t)v(t, x)$, $f_2 = g_2v(t, x)$ is proved by the additional argument method. We obtain the sufficient conditions of the existence and uniqueness of a nonlocal solution to the Cauchy problem in terms of the initial coordinates for a system of two first order quasilinear differential equations with right-hand sides $f_1 = a_2u(t, x) + b_2(t)v(t, x)$, $f_2 = g_2v(t, x)$. A theorem on the nonlocal existence and uniqueness of the solution of the Cauchy problem is formulated and the proof is given. The proof of the nonlocal solvability of the Cauchy problem for a system of two quasilinear first order partial differential equations with right-hand sides $f_1 = a_2u(t, x) + b_2(t)v(t, x)$, $f_2 = g_2v(t, x)$ is based on global estimates.

Keywords: first order partial differential equations, Cauchy problem, additional argument method, global estimates.

S.B. Klimentov

ON ISOMORPHISM OF SOME FUNCTIONAL SPACES UNDER ACTION OF INTEGRO-DIFFERENTIAL OPERATORS

Abstract. In the paper we consider representations of «the second kind» for solutions to the linear general uniform first order elliptic system in the unit circle $D = \{z : |z| \leq 1\}$ written in terms of complex functions:

$$\mathcal{D}w \equiv \partial_{\bar{z}}w + q_1(z)\partial_z w + q_2(z)\partial_{\bar{z}}\bar{w} + A(z)w + B(z)\bar{w} = R(z),$$

where $w = w(z) = u(z) + iv(z)$ is the sought complex function, $q_1(z)$ and $q_2(z)$ are given measurable complex functions satisfying the uniform ellipticity condition of the system:

$$|q_1(z)| + |q_2(z)| \leq q_0 = \text{const} < 1, \quad z \in \bar{D},$$

and $A(z), B(z), R(z) \in L_p(\bar{D})$, $p > 2$, are also given complex functions.

The representation of the second kind is based on the well-known Pompeiu's formula: if $w \in W_p^1(\bar{D})$, $p > 2$, then

$$w(z) = \frac{1}{2\pi i} \int_{\Gamma} \frac{w(\zeta)}{\zeta - z} d\zeta - \frac{1}{\pi} \iint_D \frac{\partial w}{\partial \bar{z}} \cdot \frac{d\xi d\eta}{\zeta - z},$$

where $w(z) \in W_p^1(\bar{D})$, $p > 2$. Then for the solution $w(z)$ we can write the representation

$$\Omega(w) = \frac{1}{2\pi i} \int_{\Gamma} \frac{w(\zeta)}{\zeta - z} d\zeta + TR(z)$$

where

$$\Omega(w) \equiv w(z) + T(q_1(z)\partial_z w + q_2(z)\partial_{\bar{z}}\bar{w} + A(z)w + B(z)\bar{w}).$$

Under appropriate assumptions about on coefficients we prove that Ω is the isomorphism of the spaces $C_\alpha^k(\bar{D})$ and $W_p^k(\bar{D})$, $k \geq 1$, $0 < \alpha < 1$, $p > 2$. These results develop and complete B.V. Boyarsky's works, where representations of «the first kind» were obtained. Also this work complete author's results on representations of «the second kind» with more difficult operators. As an implication of the properties of the operator Ω , we obtain the following a priori estimates for the norms $\|w\|_{C_\alpha^{k+1}(\bar{D})}$ and $\|w\|_{W_p^k(\bar{D})}$.

Keywords: general elliptic first order system, representation of the second kind.

R.Ch. Kulaev, A.B. Shabat

CONSERVATION LAWS FOR VOLTERRA CHAIN WITH INITIAL STEP-LIKE CONDITION

Abstract. In the present work we study a system of equations in the Volterra chain with initial step-like condition. The solutions to the Cauchy problem are sought in the class of positive functions. The nature of the problem is in some sense close to the problem on collapse of a discontinuity for the Korteweg-de-Vries equation. We show that the solution to the Cauchy problem for the Volterra chain can be constructed as a Taylor series. For bounded initial conditions, we obtain estimates implying that the convergence series exceeds zero. We formulate a local existence and uniqueness theorem for the solution to the Cauchy problem with bounded initial conditions.

We consider a special condition of the break of the Volterra chain: $b_n b_{n+1} = 1$, $n \geq N \geq 2$. For the closure of the chain we provide specified estimates for its solutions. We prove that under break, the solutions to the chain are defined for all

positive time. We also establish two conservation laws for the closed chain. One of the laws follows the break condition, while the other is implied by the Lagrange property.

Keywords: Volterra chain, Langmuir chain, integrable systems, conservation laws, problem on collapse of an initial discontinuity.

A.I. Rubinshtein

ON BARY–STECHKIN THEOREM

Abstract. In the beginning of the last century, N.N. Luzin proved almost everywhere convergence of an improper integral representing the function \bar{f} conjugated to a 2π -periodic summable with a square function $f(x)$. A few years later I.I. Privalov proved a similar fact for a summable function. V.I. Smirnov showed that if \bar{f} is summable, then its Fourier series is conjugate to the Fourier series for $f(x)$. It is easy to see that if $f(x) \in \text{Lip } \alpha$, $0 < \alpha < 1$, then $\bar{f}(x) \in \text{Lip } \alpha$. The Hilbert transformation for $f(x)$ differs from $\bar{f}(x)$ by a bounded function and has a simpler kernel. It is easy to show that the Hilbert transformation of $f(x) \in \text{Lip } \alpha$, $0 < \alpha < 1$, also belongs to $\text{Lip } \alpha$. In 1956 N.K. Bari and S.B. Stechkin found the necessary and sufficient condition on the modulus of continuity $f(x)$ for the function $\bar{f}(x)$ to have the same modulus of continuity. In 2016, the author introduced the concept of conjugate function as Hilbert transformation for functions defined on a dyadic group. In the present paper we show an analogue of the Bari–Stechkin (and Privalov) theorem fails that for a conjugated in this sense function.

Keywords: dyadic group, conjugate function, modulus of continuity, Bari–Stechkin theorem.

S.N. Sidorov

INVERSE PROBLEMS FOR A DEGENERATE MIXED PARABOLIC-HYPERBOLIC EQUATION ON FINDING TIME-DEPENDING FACTORS IN RIGHT HAND SIDES

Abstract. For an equation of a mixed parabolic-hyperbolic type with a degenerate hyperbolic part in a rectangular area, we consider direct and inverse problems on determining time-dependent factors in the right hand sides. As a preliminary, we study a direct initial boundary problem for this equation. By means of the method of spectral analysis we establish the uniqueness criterion for the solution and the solution is constructed as a sum over the system of the eigenfunctions of the corresponding one-dimensional Sturm-Liouville spectral problem. In justifying the convergence of the series, the problem of small denominators arise. Because of this, we prove the estimates for the distance from the zero to the small denominators with a corresponding asymptotics. These estimates allow us to justify the convergence of the constructed series in the class of regular solutions of this equation. On the base of the solution to the direct problem, we formulate and study three inverse problems on finding time-dependent factors in the right hand side only by the parabolic or hyperbolic part of the equation, and also as the factors in the both sides of the equation are unknown. Using the formula of solution to the direct initial boundary problem, the solution of inverse problems is equivalently reduced to the solvability of loaded integral equations. By means of the theory of integral equations, the corresponding theorems of uniqueness and the existence of solutions of the stated inverse problems are proved. At, the solutions of inverse problems are constructed explicitly, as sums of orthogonal series.

Keywords: equation of mixed parabolic-hyperbolic type, initial boundary value problem, inverse problems, uniqueness, existence, series, small denominators, integral equations.

S.V. Khabirov

SIMPLE PARTIALLY INVARIANT SOLUTIONS

Abstract. The continuous medium models of hydrodynamic type admit 11th dimensional Lie algebra of Galilei group extended by uniform dilatation of all independent variables. All subalgebras of this Lie algebra are listed up to inner automorphisms. We consider invariant submodels for subalgebras of the small dimensions from 1 to 3. For 4-th dimensional subalgebras, the invariant solutions are the simple solutions depending on finite numbers constants. We formulate a problem on finding partial invariant solutions of the minimal rank. For all 48 types of 4th dimensional subalgebras we calculate the bases of point invariants in terms of the variables convenient for further calculations. This allows us to consider simplest partially invariant solutions of rank 1 and defect 1. In addition, both regular and irregular partially invariant submodels are obtained.

We consider three of the 4-th dimensional subalgebras producing regular partially invariant solutions in the Cartesian, cylindrical and spherical coordinates, respectively. We obtain a solution depending on an arbitrary function of two variables in Cartesian coordinates. In the cylindrical coordinates, a submodel is reduced to a first order ordinary differential equation. In the spherical coordinates, we generalize invariant solutions of spherical vortex constructed by a rotation group.

We consider two of 4-th dimensional subalgebras producing irregular partially invariant solutions. The arising overdetermined systems are reduced into an involution. The compatibility conditions give a series of exact solutions depending on arbitrary functions, so-called simple waves. We obtain solutions with a level surface of invariant functions in the form of a moving plane with the constant normal but a varying velocity. For stationary motions with a rotation, we obtain the series of exact solutions depending on arbitrary functions.

Keywords: system of hydrodynamic type, invariants of subalgebra, partial invariant solution, simple solutions.

A.I. Bandura, O.B. Skaskiv

EXHAUSTION BY BALLS AND ENTIRE FUNCTIONS OF BOUNDED \mathbf{L} -INDEX IN JOINT VARIABLES

Abstract. For entire functions of several complex variables, we prove criteria of boundedness of \mathbf{L} -index in joint variables. Here $\mathbf{L} : \mathbb{C}^n \rightarrow \mathbb{R}_+^n$ is a continuous vector function. The criteria describe local behavior of partial derivatives of entire function on sphere in an n -dimensional complex space. Our main result provides an upper bound for maximal absolute value of partial derivatives of entire function on the sphere in terms of the absolute value of the function at the center of the sphere multiplied by some constant. This constant depends only on the radius of sphere and is independent of the location of its center. Some of the obtained results are new even for entire functions with a bounded index in joint variables, i.e., $\mathbf{L}(z) \equiv 1$, because we use an exhaustion of \mathbb{C}^n by balls instead an exhaustion of \mathbb{C}^n by polydiscs. The ball exhaustion is based on Cauchy's integral formula for a ball. Also we weaken sufficient conditions of index boundedness in our main result by replacing an universal

quantifier by an existential quantifier. The polydisc analogues of the obtained results are fundamental in theory of entire functions of bounded index in joint variables. They are used for estimating the maximal absolute value by the minimal absolute value, for estimating partial logarithmic derivatives and distribution of zeroes.

Keywords: entire function, ball, bounded L-index in joint variables, maximum modulus, partial derivative, Cauchy’s integral formula, geometric exhaustion.

Subhasis Das

ON ZEROS OF POLYNOMIAL

Abstract. For a given polynomial

$$P(z) = z^n + a_{n-1}z^{n-1} + a_{n-2}z^{n-2} + \dots + a_1z + a_0$$

with real or complex coefficients, the Cauchy bound

$$|z| < 1 + A, \quad A = \max_{0 \leq j \leq n-1} |a_j|$$

does not reflect the fact that for A tending to zero, all the zeros of $P(z)$ approach the origin $z = 0$. Moreover, Guggenheimer (1964) generalized the Cauchy bound by using a lacunary type polynomial

$$p(z) = z^n + a_{n-p}z^{n-p} + a_{n-p-1}z^{n-p-1} + \dots + a_1z + a_0, \quad 0 < p < n.$$

In this paper we obtain new results related with above facts. Our first result is the best possible. For the case as A tends to zero, it reflects the fact that all the zeros of $P(z)$ approach the origin $z = 0$; it also sharpens the result obtained by Guggenheimer. The rest of the related results concern zero-free bounds giving some important corollaries. In many cases the new bounds are much better than other well-known bounds.

Keywords: zeroes, region, Cauchy bound, Lacunary type polynomials.

Ashok Rathod

CHARACTERISTIC FUNCTION AND DEFICIENCY OF ALGEBROID FUNCTIONS ON ANNULI

Abstract In this paper, we develop the value distribution theory for meromorphic functions with maximal deficiency sum for algebroid functions on annuli and we study the relationship between the deficiency of algebroid function on annuli and that of their derivatives. Let $W(z)$ be an ν -valued algebroid function on the annulus $\mathbb{A} \left(\frac{1}{R_0}, R_0 \right)$ ($1 < R_0 \leq +\infty$) with maximal deficiency sum and the order of $W(z)$ is finite. Then

- i. $\limsup_{r \rightarrow \infty} \frac{T_0(r, W')}{T_0(r, W)} = 2 - \delta_0(\infty, W) - \theta_0(\infty, W)$;
- ii. $\limsup_{r \rightarrow \infty} \frac{N_0(r, \frac{1}{W'})}{T_0(r, W')} = 0$;
- iii. $\frac{1 - \delta_0(\infty, W)}{2 - \delta_0(\infty, W)} \leq K_0(W') \leq \frac{2(1 - \delta_0(\infty, W))}{2 - \delta_0(\infty, W)}$,

where

$$K_0(W') = \limsup_{r \rightarrow \infty} \frac{N_0(r, W') + N_0(r, \frac{1}{W'})}{T_0(r, W')}.$$

Keywords : Nevanlinna Theory, maximal deficiency sum, algebroid functions, the annuli, etc.

Gurmeet Singh, Gagandeep Singh, Gurcharanjit Singh

A NEW SUBCLASS OF UNIVALENT FUNCTIONS

Abstract. Complex analysis is an old and vulnerable subject. Geometric function theory is a branch of complex analysis that deals and studies the geometric properties of the analytic functions. The geometric function theory studies the classes of analytic functions in a domain lying in the complex plane C subject to various conditions. The cornerstone of the Geometric function theory is the theory of univalent and multivalent functions which is considered as one of the active fields of the current research. Most of this field is concerned with the class S of functions analytic and univalent in the unit disc $E = \{z : |z| < 1\}$. One of the most famous problem in this field was Bieberbach Conjecture. For many years this problem stood as a challenge to the mathematicians and inspired the development of many new techniques in complex analysis. In the course of tackling Bieberbach Conjecture, new classes of analytic and univalent functions such as classes of convex and starlike functions were defined and some nice properties of these classes were widely studied. In the present study, we introduce an interesting subclass of analytic and close-to-convex functions in the open unit disc E . For functions belonging to this class, we derive several properties such as coefficient estimates, distortion theorems, inclusion relation, radius of convexity and Fekete-Szegö Problem. The various results presented here would generalize some known results.

Keywords: Subordination, univalent functions, starlike functions, close-to-convex functions, coefficient estimates, Fekete-Szegö problem