

ABSTRACTS

A.S. Asylgareev

ON APPLYING COMPARISON THEOREMS TO STUDYING STABILITY
WITH PROBABILITY 1 OF STOCHASTIC DIFFERENTIAL EQUATIONS

Abstract. In the work we obtain two results concerning trajectory-wise properties of stochastic differential equations (SDE) with Stratonovich integral. First, we prove comparison theorems for SDE with Stratonovich integral with respect to the standard Wiener process, that is, we obtain the conditions for the coefficients of SDE, under which the solutions of one equation for a fixed trajectory of the Wiener process is always located above or below a solution to another equations for the same trajectory. At that, the drift and diffusion coefficients of the studied equations can be different. Second, on the base of the proved theorems we establish the conditions for the stability with probability 1 for perturbed solutions to scalar SDE with Stratonovich integral with respect to the trivial solution. The stability with probability 1 implies the Lyapunov stability for almost all solutions to SDE. It should be noted that, as a rule, the stability for SDE is treated in weaker sense: stability in probability, p -stability, exponential stability. Employing the formula of passage between Ito integral and Stratonovich integral, which is valid for sufficiently smooth coefficients of SDE, these results can be extended to SDE with Ito integral.

The approach of the work is based on the fact that a solution to SDE can be represented as a deterministic function of a random variable solving, in its turn, a chain of ordinary differential equations with a random right hand side. Since this technique is trajectory-wise, the presented results can be also reformulated for deterministic analogues of SDE, namely, for equations with symmetric integrals.

Keywords: stochastic differential equations, stability with probability 1, comparison theorems, equations with symmetric integral, Wiener process.

A.G. Baskakov, N.B. Uskova

FOURIER METHOD FOR FIRST ORDER DIFFERENTIAL EQUATIONS WITH INVOLUTION
AND FOR GROUPS OF OPERATORS

Abstract. In the paper we study a mixed problem for a first-order differential equation with an involution. It is written with the help of a differential operator with an involution acting in the space functions square integrable on a finite interval. We construct a similarity transform of this operator in an operator being an orthogonal direct sum of an operator of finite rank and operators of rank 1. The method of our study is the method of similar operators. Theorem on similarity serves as the basis for constructing groups of operators, whose generator is the original operator. We write out asymptotic formulae for groups of operators. The constructed group allows us to introduce the notion of a weak solution, and also to describe the weak solutions to the considered problem.

This serves to justify the Fourier method. Almost periodicity of bounded weak solutions is established. The proof of almost periodicity is based on the asymptotic representation of the spectrum of a differential operator with an involution.

Keywords: method of similar operator, spectrum, mixed problem, group of operators, differential operator with involution.

V.I. Kachalov

ON THE HOLOMORPHIC REGULARIZATION OF STRONGLY NONLINEAR SINGULARLY PERTURBED PROBLEMS

Abstract. The method of holomorphic regularization, which is a logical continuation of the method of S.A. Lomova, allows us to construct solutions to nonlinear singularly perturbed initial problems as series in powers of a small parameter converging in the usual sense. The method is based on a generalization of the Poincare decomposition theorem: in the regular case, solutions depend holomorphically on a small parameter, in the singular case the first integrals inherit this dependence. Having arisen in the framework of the regularization method, S.A. Lomov's concept of a pseudo-analytic (pseudoholomorphic) solution of singularly perturbed problems initiated the formation of the analytic theory of singular perturbations. This theory is designed to equalize the rights of regular and singular theories. In the first case, under sufficiently general assumptions, the series obtained in the solution of problems in powers of the small parameter converge in the usual sense, and in the second case they are basically asymptotic. A vivid example of the holomorphic dependence on a parameter of the solution to a differential equation is given by Poincare's decomposition theorem.

In the present paper, the holomorphic regularization method is applied for constructing pseudoholomorphic solutions to a singularly perturbed first order equation and to a second order Tikhonov system.

Keywords: holomorphic regularization, commutation relation, pseudoholomorphic solution, Tikhonov system, passage to the limit.

N.D. Kopachevsky, D.O. Tsvetkov

SMALL MOTIONS OF IDEAL STRATIFIED LIQUID WITH A FREE SURFACE TOTALLY COVERED BY A CRUMBLED ICE

Abstract. Let a rigid immovable vessel be partially filled with an ideal incompressible stratified fluid. We assume that in an equilibrium state the density of a fluid is a function of the vertical variable x_3 , i.e., $\rho_0 = \rho_0(x_3)$. In this case the gravitational field with constant acceleration $\vec{g} = -g\vec{e}_3$ acts on the fluid, here $g > 0$ and \vec{e}_3 is unit vector of the vertical axis Ox_3 , which is directed opposite to \vec{g} . Let Ω be the domain filled with a fluid in equilibrium state, S be rigid wall of the vessel adherent to the fluid, Γ be a free surface completely covered with a crumbled ice. As the crumbled ice we mean that on the free surface float heavy particles of some substance, and that these particles do not interact (or the interaction is small enough to be neglected) as the free surface oscillates. We should note that in foreign publications, such fluids are frequently called liquids with inertial free surfaces. The problem is studied on the base of an approach connected with application of so-called operator matrices theory. To this end, we introduce Hilbert spaces and some

their subspaces, also auxiliary boundary value problems. The initial boundary value problem is reduced to the Cauchy problem for the differential second-order equation in Hilbert space. After a detailed study of the properties of the operator coefficients corresponding to the resulting system of equations, we prove a theorem on the strong solvability of the Cauchy problem obtained on a finite time interval. On this base, we find sufficient conditions for the existence of a strong (with respect to time variable) solution to the initial-boundary value problem describing the evolution of the hydrosystem.

Keywords: stratification effect in ideal fluids, initial boundary value problem, differential equation in Hilbert space, Cauchy problem, strong solution.

M.A. Petrosova, I.V. Tikhonov, V.B. Sherstyukov

ON GROWTH RATE OF COEFFICIENTS IN BERNSTEIN POLYNOMIALS
FOR THE STANDARD MODULUS FUNCTION ON A SYMMETRIC INTERVAL

Abstract. The subject of the paper is closely related to one general direction in the approximation theory, within which the growth rate of the coefficients of algebraic polynomials is studied for uniform approximations of continuous functions. The classical Bernstein polynomials play an important role here. We study in detail a model example of Bernstein polynomials for the standard modulus function on a symmetric interval. The question under consideration is the growth rate of the coefficients in these polynomials with an explicit algebraic representation. It turns out that in the first fifteen polynomials the growth of the coefficients is almost not observed. For the next polynomials the situation changes, and coefficients of exponential growth appear. Our main attention is focused on the behaviour of the maximal coefficient, for which exact exponential asymptotics and corresponding two-sided estimates are established (see Theorem 2). As it follows from the obtained result, the maximal coefficient has growth $2^{n/2}/n^2$, where n is the index of the Bernstein polynomial. It is shown that the coefficients equidistant from the maximal one have a similar growth rate (for details, see Theorem 3). The group of the largest coefficients is located in the central part of the Bernstein polynomials but at the ends the coefficients are sufficiently small. The behavior of the sum of absolute values of all coefficients is also considered. This sum admits an explicit expression that is not computable in the sense of traditional combinatorial identities. On the base of a preliminary recurrence relation, we succeeded to obtain the exact asymptotics for the sum of absolute values of all coefficients and to give the corresponding two-sided estimates (see Theorem 4). The growth rate of the sum is $2^{n/2}/n^{3/2}$. In the end of the paper, we compare this result with a general Roulier estimate and new related problems for research are formulated.

Keywords: standard modulus function, Bernstein polynomials, growth of coefficients.

N.M. Poluboyarova

ON INSTABILITY OF EXTREMALS OF POTENTIAL ENERGY FUNCTIONAL

Abstract. The paper is devoted to studying the stability and instability of extremals of a potential energy functional. A particular case of this functional is the area type functionals. The potential energy functional is the sum of functionals of area type and of volume density of forces. The potential energy functional is constructed in

such way in order to take into consideration the loads on the surface from outside and inside. The stability is defined as the sign-definiteness of the second variation. In this paper we prove the formulae for the first and second variations of the functional. We also prove that the extremal surface can be locally minimal and locally maximal depending on the sign of matrix G . Using the G -capacity and the second variation of the functional, we obtain the conditions for the instability of the extremals of the potential energy functional. This technique was developed in works by V.РЪ. Miklyukov and V.A. Klyachin. For G -parabolic extremal surfaces we prove the degeneracy into the plane. This result is an analogue of the theorems by M. do Carmo and C.K. Peng. By an example of n -dimensional surfaces of revolution we demonstrate the formulae for the first and second variations of the functional. We also prove the criteria of stability and instability for n -dimensional surfaces of revolution. Similar extremal surfaces arise in applications, in physical problems (e.g. soap films, capillary surfaces, magnetic liquids in a gravitational field with a potential), and the properties of extreme surfaces are used in applied problems (e.g. modeling of awning coverings).

Keywords: the variation of functional, extreme surface, area type functional, volumetric power density functional, functional of potential energy, G -capacity, G -parabolicity, the stability.

M.N. Poptsova, I.T. Habibullin

ALGEBRAIC PROPERTIES OF QUASILINEAR TWO-DIMENSIONAL CHAIN RELATED TO INTEGRABILITY

Abstract. In the paper we discuss a classification method for nonlinear integrable equations with three independent variables based on the notion of the integrable reductions. We call an equation integrable if it admits a large class of reductions being Darboux integrable systems of hyperbolic type equations with two independent variables. The most natural and convenient object to be studied in the framework of this scheme is the class of two dimensional lattices generalizing the well-known Toda lattice. In the present article we study the quasilinear lattices of the form

$$u_{n,xy} = \alpha(u_{n+1}, u_n, u_{n-1})u_{n,x}u_{n,y} + \beta(u_{n+1}, u_n, u_{n-1})u_{n,x} + \gamma(u_{n+1}, u_n, u_{n-1})u_{n,y} + \delta(u_{n+1}, u_n, u_{n-1}).$$

We specify the coefficients of the lattice assuming that there exist cutting off conditions which reduce the lattice to a Darboux integrable hyperbolic type system of the arbitrarily high order. Under some extra assumption of nondegeneracy we describe the class of the lattices integrable in the above sense. There are new examples in the obtained list of chains.

Keywords: two-dimensional integrable lattice, x -integral, integrable reduction, cut off condition, open chain, Darboux integrable system, characteristic Lie algebra.

A.R. Khakimova

ON DESCRIPTION OF GENERALIZED INVARIANT MANIFOLDS FOR NONLINEAR EQUATIONS

Abstract. In the paper we discuss the problem on constructing generalized invariant manifolds for nonlinear partial differential equations. A generalized invariant manifold for a given nonlinear equation is a differential connection that is compatible with the linearization of this equation. In fact, this concept generalizes symmetry. Examples of generalized invariant manifolds obtained from symmetries are given in the paper. However, there exist generalized invariant manifolds irreducible to symmetries,

exactly they are of the greatest interest. Such generalized invariant manifolds allow one to construct effectively Lax pairs, recursion operators, and particular solutions to integrable equations. In the work we present the algorithm for constructing a generalized invariant manifold for a given equation. A complete description of generalized invariant manifolds of order $(2, 2)$ is given for the Korteweg–de Vries equation. We describe briefly a method for constructing a Lax pair and a recursion operator by means of the generalized invariant manifolds. As an example, we consider the Korteweg–de Vries equation is considered.

Keywords: Lax pair, higher symmetry, invariant manifold, recursion operator.

A. Bërdëllima

ON A CONJECTURE OF KHABIBULLIN ABOUT A PAIR OF INTEGRAL INEQUALITIES

Abstract. Khabibullin’s conjecture is a statement about a pair of integral inequalities, where one inequality implies the other. They depend on two parameters $n \geq 2, n \in \mathbb{N}$, and $\alpha \in \mathbb{R}_+$. These inequalities were originally introduced by Khabibullin [?] in his survey regarding Paley problem in \mathbb{C}_n and related topics about meromorphic functions. It is possible to express the inequalities in three equivalent forms. The first statement is in terms of logarithmically convex functions, the second statement is in terms of increasing functions, and the third statement is in terms of non-negative functions. In this paper we work solely with the second variant of the hypothesis. It is well established that the conjecture is true whenever $0 \leq \alpha \leq 1/2$ for all n . Several proofs exist in the literature among which one is given by the author [?] and it relates the integral inequalities with the general theory of Laplace transform. But it was not known if the statement was true when $\alpha > 1/2$ until Sharipov [?] showed that the conjecture fails when $\alpha = 2, n = 2$. However the question of whether this conjecture holds for at least some $n \geq 2$ and $\alpha > 1/2$ remained an open problem. In this paper we aim to solve this question. Motivated by Sharipov’s approach, we develop a method of constructing a counterexample for the more general case $n \geq 2$ and $\alpha > 1/2$. By an explicit counterexample we show that Khabibullin’s conjecture does not hold in general.

Keywords: Khabibullin’s conjecture, Khabibullin’s theorem, Khabibullin’s constants, integral inequalities, counterexample, plurisubharmonic function, sharp estimate.

M. Garayev, H. Guediri, H. Sadraoui

NEW CHARACTERIZATIONS OF BLOCH SPACES, BERS-TYPE AND ZYGMUND-TYPE SPACES AND RELATED QUESTIONS

Abstract. In terms of Berezin symbols, we give new characterizations of the Bloch spaces \mathcal{B} and \mathcal{B}_0 , Bers-type and the Zygmund-type spaces of analytic functions on the unit disc \mathbb{D} in the complex plane \mathbb{C} . We discuss some properties of Toeplitz operators on the Bergman space $L_a^2(\mathbb{D})$. We provide a new characterization of certain function space with variable exponents. Namely, given a function $f(z) = \sum_{n=0}^{\infty} \widehat{f}(n)z^n \in$

$\text{Hol}(\mathbb{D})$ with a bounded sequence $\{\widehat{f}(n)\}_{n \geq 0}$ of Taylor coefficients $\widehat{f}(n) = \frac{f^{(n)}(0)}{n!}$, $(n = 0, 1, 2, \dots)$, we have $f \in H_{p(\cdot), q(\cdot), \gamma(\cdot)}$ if and only if

$$\int_0^1 \left(\frac{1}{2\pi} \int_0^{2\pi} \left| \widetilde{D}_{(\widehat{f}(n)e^{int})}(\sqrt{r}) \right|^{p(t)} dt \right)^{\frac{q(t)}{p(t)}} (1-r)^{\frac{\gamma(t)p(t)-q(t)}{p(t)}} dr < +\infty.$$

Here $D_{(a_n)}$ denotes the associate diagonal operator on the Hardy-Hilbert space H^2 defined by the formula $D_{(a_n)}z^n = a_n z^n$ ($n = 0, 1, 2, \dots$).

Keywords: Bers-type space, Zygmund-type space, Bloch spaces, Berezin symbol.

E. Zikkos

A TAYLOR-DIRICHLET SERIES WITH NO SINGULARITIES ON ITS ABCISSA OF CONVERGENCE

Abstract. G. Pólya proved that given a sequence of positive real numbers $\Lambda = \{\lambda_n\}_{n=1}^\infty$ of a density d and satisfying the gap condition $\inf_{n \in \mathbb{N}} (\lambda_{n+1} - \lambda_n) > 0$, the Dirichlet series $\sum_{n=1}^\infty c_n e^{\lambda_n z}$ has at least one singularity in each open interval whose length exceeds $2\pi d$ and lies on the abscissa of convergence. This raises the question whether the same result holds for a Taylor-Dirichlet series of the form

$$g(z) = \sum_{n=1}^{\infty} \left(\sum_{k=0}^{\mu_n-1} c_{n,k} z^k \right) e^{\lambda_n z}, \quad c_{n,k} \in \mathbb{C}$$

when its associated multiplicity-sequence $\Lambda = \{\lambda_n, \mu_n\}_{n=1}^\infty$

$$\{\lambda_n, \mu_n\}_{n=1}^\infty := \left\{ \underbrace{\lambda_1, \lambda_1, \dots, \lambda_1}_{\mu_1\text{-times}}, \underbrace{\lambda_2, \lambda_2, \dots, \lambda_2}_{\mu_2\text{-times}}, \dots, \underbrace{\lambda_k, \lambda_k, \dots, \lambda_k}_{\mu_k\text{-times}}, \dots \right\}$$

has the following two properties:

- (1) Λ has density d , that is, $\sum_{\lambda_n \leq t} \mu_n / t \rightarrow d$ as $t \rightarrow \infty$,
- (2) λ_n satisfy the gap condition $\inf_{n \in \mathbb{N}} (\lambda_{n+1} - \lambda_n) > 0$.

In this article we present a counterexample. We prove that for any non-negative real number d there exists a multiplicity-sequence $\Lambda = \{\lambda_n, \mu_n\}_{n=1}^\infty$ having properties (1) and (2), but with unbounded multiplicities μ_n , such that its Krivosheev characteristic S_Λ is negative. For this Λ , and based on a result obtained by O.A. Krivosheeva, we show that for any $a \in \mathbb{R}$, there exists a Taylor-Dirichlet series $g(z)$ whose abscissa of convergence is the line $\operatorname{Re} z = a$, such that $g(z)$ has no singularities on this line.

Keywords: Taylor-Dirichlet series, Singularities, Fabry-Pólya