ABSTRACTS

S. Baizaev, M.A. Rakhimova

Some functional equations in Schwartz space and their applications

In the paper we consider functional equations of form

$$(B + r^2 E)u(z) = 0,$$

where B is a constant complex $n \times n$ matrix, E is the unit $n \times n$ matrix, z is a complex variable, $r = |z|$, $u(z)$ is the sought generalized vector function. For this equation, we study the existence of non-trivial solutions and the manifold of all solutions in the functional space $D' = D'(C, C^n)$ of generalized vector function and in the space $S' = S'(C, C^n)$ of tempered distribution. We also study the existence of solutions growing at most polynomially at infinity.

Such study is motivated by the problem on finding the solutions in S' for elliptic systems of first order elliptic equations. Here an important role is played by the statement on the structure of generalized functions with supports located in a circumference. This statement provides an explicit representation of generalized functions supported in a circumference and this representation consists of a linear combinations of Cartesian product of generalized periodic functions and δ-function and its derivatives. The process of finding all solutions to this equation in the space D' consists of three stages. At the first stage, by reducing the matrix to the normal Jordan form, we split this equation into one-dimensional equations. At the second stage we prove that if the matrix B has non negative and zero eigenvalues, that is, $\sigma(B) \cap (-\infty, 0] = \varnothing$, where $\sigma(B)$ is the spectrum of the matrix B, then in the space D', this equation has only trivial solution. At the third stage, in the case $\sigma(B) \cap (-\infty, 0] \neq \varnothing$, we find all solutions to this equation in the space D'. Subject to the eigenvalues of the matrix B, the set of all solutions to this equation in the space D' is either zero or depends on finitely many arbitrary generalized 2π-periodic functions of one variable and finitely many arbitrary constants. The number of these functions and constants depend on the order of the solution; the order is prescribed. As an application, we find solutions in the space S', in particular, polynomially growing solutions to elliptic systems of partial differential equations and to overdetermined systems. The results obtained in the work can be employed in studying the problems on solutions defined on the entire complex plane or a half-plane and in studying more general linear multi-dimensional elliptic systems and overdetermined systems of partial differential equations.

Keywords: functional equations, Schwarz spaces, generalized functions supported in a circumference.

E.I. Galakhov, O.A. Salieva

Unsolvability conditions for some inequalities and systems with functional parameters and singular coefficients on boundary

Abstract. We consider the problem on nonexistence of positive solutions for some nonlinear elliptic inequalities in a bounded domain. At that, the principal parts of the considered inequalities are $p(x)$-Laplacians with variable exponents. The lower terms of the considered inequalities can depend both on the unknown function and its gradient. We assume that the coefficients at the lower terms have singularities at the boundary. To the best of the authors' knowledge, the conditions for nonexistence of solutions to inequalities with variable exponents were not considered before.

We obtain the sufficient conditions for nonexistence of positive solutions in terms of the exponent $p(x)$, of the order of the singularities and of parameters in the problem. To prove the obtained conditions, we employ an original modification of the nonlinear capacity method proposed by S.I. Pokhozhaev. The method is based on a special choice of test functions in the generalized formulation of the problem and on algebraic transformations of the obtained expression. This allows us to obtain asymptotically sharp apriori estimates for the solutions leading to a contradiction under a certain choice of the parameters. This implies the absence of the solutions. We generalize the obtained results for the case of nonlinear systems with similar conditions for the operators and coefficients.

Keywords: elliptic inequalities, variable exponents, nonexistence of solutions, singular coefficients.

N.I. Gusarova, S.A. Murtazina, M.F. Fazlytdinov, M.G. Yumagulov

Operator methods for calculating Lyapunov values in problems on local bifurcations of dynamical systems

Abstract. In the work we consider basic scenarios of local bifurcations of dynamical systems. We study the systems described by autonomous differential equations, discrete equations, as well as by non-autonomous periodic equations. We provide new formulae for calculating Lyapunov values. The formulae are obtained on the basis of a general operator approach for studying local bifurcations and they do not assume passing to normal forms and using the theorems on a central manifold. This method allows us to obtain new bifurcation formulae for studying main scenarios of local bifurcations. In the work we show how these bifurcation formulae lead one to new formulae for calculation Lyapunov values in problems on bifurcation of equilibria, in Andronov-Hopf problems, in problems of doubling period, in problems on forced oscillations, etc.

In the paper, the main attention is paid to obtain the first and the second Lyapunov value. The proposed approach allows us obtain Lyapunov values of higher order. As an application of the obtained formulae, in the paper we analyze basic scenarios of local bifurcations. We consider the problems on the direction of bifurcations, on stability of emerging solutions, on leading asymptotics for the solutions, etc. As an example, we calculate the Lyapunov values for Andronov-Hopf bifurcation in Langford system and for the problems on doubling period in Henon model.

Keywords: dynamical systems, bifurcation, Lyapunov values, equilibrium, stability.

Sh.Kh. Ishkina

Combinatorial bounds of overfitting for threshold classifiers

Abstract. Estimating the generalization ability is a fundamental objective of statistical learning theory. However, accurate and computationally efficient bounds are still unknown even for many very simple cases. In this paper, we study an one-dimensional threshold decision rules. We employ the combinatorial theory of overfitting based on a single probabilistic assumption that all partitions of a set of objects into an observed training sample and a hidden test sample are of equal probability. We propose a polynomial algorithm for computing both probability of overfitting and complete cross-validation. The algorithm exploits the recurrent calculation of the number of admissible paths while walking on a three-dimensional net between two prescribed points with restrictions of special form. We compare the obtain sharp estimate of the generalized ability and demonstrate that the known upper bound are too overstated and they can not be applied for practical problems.

Keywords: computational learning theory, empirical risk minimization, combinatorial theory of overfitting, probability of overfitting, complete cross-validation, generalization ability, threshold classifier, computational complexity.

A.A. Kononova

On measures generating orthogonal polynomials with similar asymptotic behavior of the ratio at infinity

Abstract. We consider the influence of the measure perturbations on the asymptotic behavior of the ratio of orthogonal polynomials. We suppose the absoputely continuous part of the measure is supported on finitely many Jordan curves. The weight function satisfies the modified Szegö condition.

The singular part of the measure consists of finitely many point masses outside the polynomial convex hall of the support of the absolutely continuous part of the measure. We study the stability of asymptotics of the ratio in the following sense: $\frac{P_{\nu,n}(z)}{P_{\nu,n+1}(z)} - \frac{P_{\mu,n}(z)}{P_{\mu+1}(z)} \to 0$, $n \to \infty$.

The problem is a generalization of the problem on compactness of the perturbation of Jacobi operator generated by the perturbation of its spectral measure. We find a condition necessary (or necessary and sufficient under some additional restriction) for the stability of the asymptotical behavior of the corresponding orthogonal polynomials is found. One of the main tools in the study are the Riemann theta-functions.

Keywords: orthogonal polynomials, multivalued functions.

A.A. Makhota

On completeness of exponential systems in convex domain

Abstract. We work is devoted to studying the completeness of the systems of exponentials in the space of functions analytic in a convex domain. The problem on the completeness of the systems of the exponentials in various functional spaces is classical and was studied by many mathematicians, for instance, by B.Ya. Levin, A.F. Leontiev, A.M. Sedletskii, B.N. Khabibullin, R.S. Yulmukhametov, and others.

We prove that the completeness of the system of exponentials in the space of functions analytic in a convex domain is equivalent to the completeness of the system of exponentials in the space of functions analytic in a circle with the radius depending on the properties of a given convex domain. We also consider an example by choosing an ellipse as the convex domain. Here we find the values of the support function and the radius of the corresponding circle.

Keywords: completeness of a system, convex domain, entire function, Fourier-Laplace transform.

R.B. Salimov

Behavior of singular integral with Hilbert kernel at weak continuity point of density

Abstract. We consider the singular integral with the Hilbert kernel

$$I(\gamma_0)=\int\limits_0^{2\pi}\varphi(\gamma)\operatorname{ctg}\frac{\gamma-\gamma_0}{2}\,d\gamma,$$

whose density $\varphi(\gamma)$ is a continuous in $[0,2\pi]$ function, $\gamma_0 \in [0,2\pi]$, $\varphi(0)=\varphi(2\pi)$, and the integral is treated in the sense of its principal value. We assume that in the vicinity of a fixed point $\gamma=c$, $c\in(c^-,c^+)\subset[0,2\pi]$, $c^+-c^-<1$, the density $\varphi(\gamma)$ satisfies the representation $\varphi(\gamma)=\frac{\Phi(\gamma)}{\left(-\ln\sin^2\frac{\gamma-c}{2}\right)^\beta}$, $\gamma\in(c^-,c^+)$, where $\Phi(\gamma)$ is a given continuous in $[c^-,c]$, $[c,c^+]$ function with not necessarily coinciding one-sided limits $\Phi(c-0)$ and $\Phi(c+0)$, β is a given number, and $\beta>1$. We suppose that the representations $\Phi(\gamma)-\Phi(c\pm0)=\frac{\chi(\gamma)}{\left(-\ln\sin^2\frac{\gamma-c}{2}\right)^\delta}$, $\chi'(\gamma)=\frac{\nu(\gamma)}{\left(-\ln\sin^2\frac{\gamma-c}{2}\right)\operatorname{tg}\frac{\gamma-c}{2}}$, hold, where $\delta>0$ is a given number, $\chi(\gamma)$, $\nu(\gamma)$ are given functions continuous in each of the intervals $[c^-,c]$, $[c,c^+]$, $\nu(c\pm0)=0$, $\Phi(c+0)$ is taken as $\gamma>c$, $\Phi(c-0)$ is taken as $\gamma<c$.

We prove that under the above conditions the representation

$$I(\gamma_0)-I(c)=\frac{\Phi(c-0)-\Phi(c+0)}{(\beta-1)\left(-\ln\sin^2\frac{\gamma_0-c}{2}\right)^{\beta-1}}$$

$$-\frac{U(c+0)-U(c-0)}{\tilde\beta(\tilde\beta-1)\left(-\ln\sin^2\frac{\gamma_0-c}{2}\right)^{\tilde\beta-1}}+o\left(\frac{1}{\left(-\ln\sin^2\frac{\gamma_0-c}{2}\right)^{\tilde\beta-1}}\right)+O\left(\frac{1}{\left(-\ln\sin^2\frac{\gamma_0-c}{2}\right)^{\beta}}\right),$$

holds as $\gamma_0\to c$. Here $\tilde\beta=\beta+\delta$, $\beta>1$, $\delta>0$, $U(c+0)-U(c-0)=\tilde\beta\big(\chi(c+0)-\chi(c-0)\big)$. We also consider the case $\beta=1$. A distinguishing feature of the paper is that while studying the behavior of the considered singular integral in the vicinity of the weak continuity point of its density, we need the Hölder condition no for the density neither for a component of the density. This feature allowed us to extend the range of possible applications of our results.

Keywords: singular integral, Hilbert kernel, Hölder condition, weak continuity.

I.A. Shakirov

On two-sided estimate for norm of Fourier operator

Abstract. In the work we study the behavior of Lebesgue constant L_n of the Fourier operator defined in the space of continuous 2π-periodic functions. The known integral representations expressed in terms of the improper integrals are too cumbersome. They are complicated both for theoretical and practical purposes.

We obtain a new integral representation for L_n as a sum of Riemann integrals defined on bounded converging domains. We establish equivalent integral representations and provide strict two-sided estimates for their components. Then we provide a two-sided estimate for the Lebesgue constant. We solve completely the problem on the upper bound of the constant L_n. We improve its known lower bound.

Keywords: partial sums of Fourier series, norm of Fourier operator, Lebesgue constant, asymptotic formula, estimate for Lebesgue constant, extremal problem.

M. Saidani, B. Belaïdi

On The Growth of Solutions of Some Higher Order Linear Differential Equations With Meromorphic Coefficients

Abstract. In this paper, by using the value distribution theory, we study the growth and the oscillation of meromorphic solutions of the linear differential equation

$$f^{(k)} + \left(A_{k-1,1}(z)e^{P_{k-1}(z)} + A_{k-1,2}(z)e^{Q_{k-1}(z)}\right) f^{(k-1)} + \cdots + \left(A_{0,1}(z)e^{P_0(z)} + A_{0,2}(z)e^{Q_0(z)}\right) f = F(z),$$

where $A_{j,i}(z)\,(\not\equiv 0)$ $(j = 0, \ldots, k-1)$, $F(z)$ are meromorphic functions of a finite order, and $P_j(z), Q_j(z)$ $(j = 0, 1, \ldots, k-1; i = 1, 2)$ are polynomials with degree $n \geqslant 1$. Under some conditions, we prove that as $F \equiv 0$, each meromorphic solution $f \not\equiv 0$ with poles of uniformly bounded multiplicity is of infinite order and satisfies $\rho_2(f) = n$ and as $F \not\equiv 0$, there exists at most one exceptional solution f_0 of a finite order, and all other transcendental meromorphic solutions f with poles of uniformly bounded multiplicities satisfy $\overline{\lambda}(f) = \lambda(f) = \rho(f) = +\infty$ and $\overline{\lambda}_2(f) = \lambda_2(f) = \rho_2(f) \leqslant \max\{n, \rho(F)\}$. Our results extend the previous results due Zhan and Xiao.

Keywords: Order of growth, hyper-order, exponent of convergence of zero sequence, differential equation, meromorphic function.