ABSTRACTS

F.G. Avkhadiev

Estimates of Hardy-Rellich constants for polyharmonic operators and their generalizations

Abstract. We prove the lower bounds for the functions introduced as the maximal constants in the Hardy and Rellich type inequalities for polyharmonic operator of order m in domains in a Euclidean space. In the proofs we employ essentially the known integral inequality by O.A. Ladyzhenskaya and its generalizations. For the convex domains we establish two generalizations of the known results obtained in the paper M.P. Owen, Proc. Royal Soc. Edinburgh, 1999 and in the book A.A. Balinsky, W.D. Evans, R.T. Lewis, The Analysis and Geometry of Hardy's Inequality, Springer, 2015. In particular, we obtain a new proof of the theorem by M.P. Owen for polyharmonic operators in convex domains. For the case of arbitrary domains we prove universal lower estimates for the constants in the inequalities for mth order polyharmonic operators by using the products of m different constants in Hardy type inequalities. This allows us to obtain explicit lower estimates for the constants in Rellich type inequalities for the dimension two and three. In the last section of the paper we discuss two open problems. One of them is similar to the problem by E.B. Davies on the upper bounds for the Hardy constants. The other problem concerns the comparison of the constants in Hardy and Rellich type inequalities for the operators defined in three-dimensional domains.

Keywords: polyharmonic operator, Hardy inequality, Rellich inequality, convex domain.

G.G. Braichev

Two-sided estimates of the relative growth of functions and their derivatives

Abstract. We give an extended presentation of a talk given at the International mathematical conference on theory of functions dedicated to centenary of corresponding member of AS USSR A.F. Leont'ev. We propose a new method for obtaining uniform two-sided estimates for the fraction of the derivatives of two real functions on the base of the information of two-sided estimates for the functions themselves. At that, one of the functions possesses certain properties and serves as a reference for measuring a growth introducing some scale. The other function, whose growth is compared with that of the reference function, is convex, increases unboundedly or decays to zero on a certain interval. The method is also applicable to some class of functions concave on an interval. We consider examples of applications of the obtained results to the behavior of entire functions.

Keywords: monotone function, convex function, relative growth of two functions, uniform upper and lower estimates, entire function.

A.M. Gaisin, G.A. Gaisina

ESTIMATES FOR GROWTH AND DECAY OF FUNCTIONS IN MACINTYRE–EFGRAFOV KIND THEOREMS

Abstract. In the paper we obtain two results on the behavior of Dirichlet series on a real axis.

First of them concerns the lower bound for the sumo of the Dirichlet series on the system of segments $[\alpha, \alpha + \delta]$. Here the parameters $\alpha > 0$, $\delta > 0$ are such that $\alpha \uparrow +\infty$, $\delta \downarrow 0$. The needed asymptotic estimates is established by means of a method based on some inequalities for extremal functions in the appropriate non-quasi-analytic Carleman class. This approach turns out to be more effective than the known traditional ways for obtaining similar estimates.

The second result specifies essentially the known theorem by M.A. Evgrafov on existence of a bounded on $\mathbb{R}$ Dirichlet series. According to Macintyre, the sum of this series tends to zero on $\mathbb{R}$. We prove a spectific estimate for the decay rate of the function in an Macintyre-Evgrafov type example.

Keywords: Dirichlet series, gap-power series, asymptotic behavior.

O.A. Ivanova, S.N. Melikhov, Yu.N. Melikhov

ON THE COMMUTANT OF DIFFERETIATION AND TRANSLATION OPERATORS IN WEIGHTED SPACES OF ENTIRE FUNCTIONS

Abstract. We describe linear continuous operators acting in a countable inductive limit E of weighted Fréchet spaces of entire functions of many complex variables and commuting in these spaces with systems of operators of partial differentiation and translation. Under the made assumptions the commutants of the systems of differentiation and translation operators coincide. They consist of convolution operators defined by a linear continuous functional on E. At that we do not assume that the set of the polynomials is dense in E. In the space E' topologically dual to E, we naturally introduce the multiplication. Under this multiplication, the algebra E' is isomorphic to the aforementioned commutant with the usual mulitplication, which the composition of the operators. This isomorphism is also topological if E' is equipped by a weak topology, while the commutant is equipped by the weak operator topology. This implies that the set of the polynomials of the differentiation operators is dense in the commutant with topology of pointwise convergence. We also studied the possibility of representing an operator in the commutant as an infinite order differential operator with constant coefficients. We prove the immediate continuity of linear operators commuting with all differentiation operators in a weighted (LF)-space of entire functions isomorphic via Fourier-Laplace transform to the space of infinitely differentiable functions compactly supported in a multi-dimensional real space.

Keywords: differentiation operator, translation operator, commutant, weighted space of entire functions.

K.P. Isaev, K.V. Trounov, R.S. Yulmukhametov

REPRESENTATION OF FUNCTIONS IN LOCALLY CONVEX SUBSPACES OF $A^\infty(D)$ BY SERIES OF EXPONENTIALS

Abstract. Let D be a bounded convex domain in the complex plane, $\mathcal{M}_0 = (M_n)_{n=1}^\infty$ be a convex sequence of positive numbers satisfying the "non-quasi-analyticity" condition:

$$\sum_n \frac{M_n}{M_{n+1}} < \infty,$$

$\mathcal{M}_k = (M_{n+k})_{n=1}^\infty$, $k = 0, 1, 2, 3, \ldots$ be the sequences obtained from the initial ones by removing first k terms. For each sequence $\mathcal{M}_0 = (M_n)_{n=1}^\infty$ we consider the Banach space $H(\mathcal{M}_0, D)$ of functions analytic in a bounded convex domain D with the norm:

$$\|f\|^2 = \sup_n \frac{1}{M_n^2} \sup_{z \in D} |f^{(n)}(z)|^2.$$

In the work we study locally convex subspaces in the space of analytic functions in D infinitely differentiable in $\overline{D}$ obtained as the inductive limit of the spaces $H(\mathcal{M}_k, D)$. We prove that for each convex domain there exists a system of exponentials $e^{\lambda_n z}$, $n \in \mathbb{N}$, such that each function in the inductive limit $f \in \lim \mathrm{ind}\, H(\mathcal{M}_k, D) := \mathcal{H}(\mathcal{M}_0, D)$ is represented as the series over this system of exponentials and the series converges in the topology of $\mathcal{H}(\mathcal{M}_0, D)$. The main tool for constructing the systems of exponentials is entire functions with a prescribed asymptotic behavior. The characteristic functions L with more sharp asymptotic estimates allow us to represent analytic functions by means of the series of the exponentials in the spaces with a finer topology. In the work we construct entire functions with gentle asymptotic estimates. In addition, we obtain lower bounds for the derivatives of these functions at zeroes.

Keywords: analytic functions, entire functions, subharmonic functions, series of exponentials.

L.A. Kalyakin

ADIABATIC APPROXIMATION IN A RESONANCE CAPTURE PROBLEM

Abstract. By means of the averaging method, we analyze two model problems on capture into a resonance that leads us to the adiabatic approximation in the leading term in the asymptotics. The main aim is approximate (by using a small parameter) description of the capture in a resonance domain. This domain is in the phase plane and it consists of the initial points for the resonance solutions with an unboundedly increasing energy. The capture domain depends on an additional parameter involved in the equation. We show that the adiabatic approximation fails as the capture domain becomes narrow. In this case we have to modify substantially the averaging method. As a result, a system of nonlinear differential equations arises for the leading term in the asymptotics and this system is not always integrable.

Keywords: nonlinear oscillations, small parameter, asymptotics, capture into a resonance, adiabatic approximation.

N.N. Konechnaya, K.A. Mirzoev

Asymptotics of solutions to a class of linear differential equations

Abstract. In the paper we find the leading term of the asymptotics at infinity for some fundamental system of solutions to a class of linear differential equations of arbitrary order $\tau y = \lambda y$, where λ is a fixed complex number. At that we consider a special class of Shin-Zettl type and τy is a quasi-differential expression generated by the matrix in this class. The conditions we assume for the primitives of the coefficients of the quasi-differential expression τy, that is, for the entries of the corresponding matrix, are not related with their smoothness but just ensures a certain power growth of these primitives at infinity. Thus, the coefficients of the expression τy can also oscillate. In particular, this includes a wide class of differential equations of arbitrary even or odd order with coefficients-distributions of finite order. Employing the known definition of two quasi-differential expressions with non-smooth coefficients, in the work we propose a method for obtaining asymptotic formulae for the fundamental system of solutions to the considered equation in the case when the left hand side of this equations is represented as a product of two quasi-differential expressions.

The obtained results are applied for the spectral analysis of the corresponding singular differential operators. In particular, assuming that the quasi-differential expression τy is symmetric, by the known scheme we define the minimal closed symmetric operator generated by this expression in the space of Lebesgue square-integrable on $[1, +\infty)$ functions (in the Hilbert space $\mathcal{L}^2[1, +\infty)$) and we calculate the deficiency indices for this operator.

Keywords: Quasi-derivative, quasi-differential expression, the main term of asymptotic of the fundamental system of solutions, minimal closed symmetric differential operator, deficiency numbers.

A.S. Krivosheyev, A.F. Kuzhaev

On Leontiev-Levin theorem

Abstract. In this work we study the relations between different densities of a positive sequence and related quantities. More precisely, in the work we consider the upper density, the maximal density introduced by G. Polya, the logarithmic block-density, which seems to be introduced first by L.A. Rubel. In particular, there were obtained relations between the maximal density and a quantity being very close to the logarithmic block-density. The results of these studies are applied for generalizing the classical statement obtained independent by A.F. Leont'ev B.Ya. Levin on the completeness in a convex domain of a system of exponential monomials with positive exponents; we generalized this statement for the exponents with no density. We find out that for the aforementioned result, one can weaken the condition of the measurability of the sequence (that is, the existence of a density) and replace it by the identity of upper and maximal densities. Namely, we obtain a condition under which there holds the criterion of the completeness of the system of exponential monomials in convex domains. It should be noted that this criterion holds in rather wide class of convex domains, for instance, having vertical and horizontal symmetry axes. The main role in solving this issues was played by the results of the studies by L.A. Rubel and P. Malliavin on relation between the growth of an entire function

of exponential type along the imaginary axis and the logarithmic block-density of its positive zeroes. These results were applied by these authors for studying the completeness of the system of exponentials in a horizontal strip.

Keywords: density of sequence, entire function, completeness, convex domain.

O.A. Krivosheeva

Invariant subspaces with zero density spectrum

Abstract. In the paper we show that each analytic solution of a homogeneous convolution equation with the characteristic function of a minimal exponential type is represented by a series of exponential polynomials in the domain, where this solution exists. This series converges absolutely and uniformly on compact subsets of this domain. It is known that if the characteristic function is of a minimal exponential type, the density of its zero set is equal to zero. This is in the work we consider the sequences of exponents having zero density. We provide a simple description of the space of the coefficients for the aforementioned series. Moreover, we provide a complete description of all possible system of functions constructed by rather small groups, for which the representation by the series of exponential polynomials holds.

Keywords: A series of exponential monomials, relatively small clusters, a basis, a convex domai

I.Kh. Musin

On a Hilbert space of entire functions

Abstract. We consider the Hilbert space F_φ^2 of entire functions of n variables constructed by means of a convex function φ in $\mathbb{C}^n$ depending on the absolute value of the variable and growing at infinity faster than $a|z|$ for each $a > 0$. We study the problem on describing the dual space in terms of the Laplace transform of the functionals. Under certain conditions for the weight function φ we obtain the description of the Laplace transform of linear continuous functionals on F_φ^2. The proof of the main result is based on using new properties of Young-Fenchel transform and one result on the asymptotics of the multi-dimensional Laplace integral established by R.A. Bashmakov, K.P. Isaev, R.S. Yulmukhametov.

Keywords: Hilbert space, Laplace transform, entire functions, convex functions, Young-Fenchel transform.

V.Yu. Novokshenov

Discrete integrable equations and special functions

Abstract. A generic scheme based on the matrix Riemann-Hilbert problem theory is proposed for constructing classical special functions satisfying difference equations. These functions comprise gamma- and zeta functions, as well as orthogonal polynomials with corresponding recurrence relations. We show that all difference equations are the compatibility conditions of certain Lax pair coming from the Riemann-Hilbert problem. At that, the integral representations for solutions to the classical Riemann-Hilbert problem on duality of analytic functions on a contour in

the complex plane are generalized for the case of discrete measures, that is, for infinite sequences of points in the complex plane. We establish that such generalization allows one to treat a series of nonlinear difference equations integrable in the sense of solitons theory.

The solutions to the mentioned Riemann-Hilbert problems allows us o reproduce analytic properties of classical special functions described in handbooks and to describe a series of new functions pretending to be special. For instance, this is true for difference Painlevé equations. We provide the example of applying a difference second type Painlevé equation to the representation problem of a symmetric group.

Keywords. Special functions, Riemann-Hilbert problem, isomonodromy deformations, orthogonal polynomials, recurrent relations, gamma function, Riemann zeta function, discrete Painlevé equations, asymptotic expansions, integral representations

A.B. Shabat, M.Kh. Efendiev

On applications of Faa di Bruno

Abstract. Two new modifications of the classical Faa-di-Bruno formula are constructed and applications of the obtained formulas in the theory of integrability of nonlinear partial differential equations

In the work we construct two modifications of the classical Faa-di-Bruno formula. We consider the applications of these formulae in the integrability theory for nonlinear partial differential equations. We discuss the problem on integration by parts in the formal Gelfand-Olver-Sanders.

Keywords: Faa-di-Bruno formula, differential polynomials, integrability conditions.

Yu.Z. Shaygardanov

Asymptotics in a parameter of the solution to an elliptic boundary value problem in the vicinity of the outer touching of the characteristics to the limit equation

Abstract. In a bounded domain $Q \subset \mathbb{R}^3$ with a smooth boundary Γ we consider the boundary value problem

$$\varepsilon Au - \frac{\partial u}{\partial x_3} = f(x), \quad u|_\Gamma = 0.$$

Here A is a second order elliptic operator, ε is a small parameter. The limiting equation, as $\varepsilon = 0$, is the first order equation. Its characteristics are straight lines parallel to the axis Ox_3. For the domain $\overline{Q}$ we assume that the characteristic either intersects Γ at two points or touches Γ from outside. The set of touching point forms a closed smooth curve. In the paper we construct the asymptotics as $\varepsilon \to 0$ for the solutions to the studied problem in the vicinity of this curve. For constructing the asymptotics we employ the method of matching asymptotic expansions.

Keywords: small parameter, asymptotic, elliptic equation.

F.A. Shamoyan

Analytic functions with smooth absolute value of boundary data

Abstract. Let f be an analytic function in the unit circle D continuous up to its boundary Γ, $f(z) \neq 0$, $z \in D$. Assume that on Γ, the function f has a modulus of continuity $\omega(|f|, \delta)$. In the paper we establish the estimate $\omega(f, \delta) \leqslant A\omega(|f|, \sqrt{\delta})$, where A is a some non-negative number, and we prove that this estimate is sharp. Moreover, in the paper we establish a multi-dimensional analogue of the mentioned result. In the proof of the main theorem, an essential role is played by a theorem of Hardy-Littlewood type on Hölder classes of analytic functions in the unit circle.

Keywords: analytic function, modulus of continuity, factorization, outer function.

R.N. Garifullin, R.I. Yamilov

On integrability of a discrete analogue of Kaup-Kupershmidt equation

Abstract. We study a new example of the equation obtained as a result of a recent generalized symmetry classification of differential-difference equations defined on five points of an one-dimensional lattice. We establish that in the continuous limit this new equation turns into the well-known Kaup-Kupershmidt equation. We also prove its integrability by constructing an $L - A$ pair and conservation laws. Moreover, we present a possibly new scheme for constructing conservation laws from $L - A$ pairs.

We show that this new differential-difference equation is similar by its properties to the discrete Sawada-Kotera equation studied earlier. Their continuous limits, namely the Kaup-Kupershmidt and Sawada-Kotera equations, play the main role in the classification of fifth order evolutionary equations made by V.G. Drinfel'd, S.I. Svinolupov and V.V. Sokolov.

Keywords: differential-difference equation, integrability, Lax pair, conservation law.

F. Haslinger

Pauli operators and the $\overline{\partial}$-Neumann problem

Abstract. We apply methods from complex analysis, in particular the $\overline{\partial}$-Neumann operator, to study spectral properties of Pauli operators. For this purpose we consider the weighted $\overline{\partial}$-complex on $\mathbb{C}^n$ with a plurisubharmonic weight function. The Pauli operators appear at the beginning and at the end of the weighted $\overline{\partial}$-complex. We use the spectral properties of the corresponding $\overline{\partial}$-Neumann operator to answer the question when the Pauli operators are with compact resolvent. It is also of importance to know whether the related Bergman space of entire functions is of infinite dimension. The main results are formulated in terms of the properties of the Levi matrix of the weight function. If the weight function is decoupled, one gets additional informations. Finally, we point out that a corresponding Dirac operator fails to be with compact resolvent.

Keywords: $\overline{\partial}$-Neumann problem, Pauli operators, Schrödinger operators, compactness.

A. Sukhov

Levi-flat world: a survey of local theory

Abstract. This expository paper concerns local properties of Levi-flat real analytic manifolds with singularities. Levi-flat manifolds arise naturally in Complex Geometry and Foliation Theory. In many cases (global) compact Levi-flat manifolds without singularities do not exist. These global obstructions make natural the study of Levi-flat objects with singularities because they always exist. The present expository paper deals with some recent results on local geometry of Levi-flat singularities. One of the main questions concerns an extension of the Levi foliation as a holomorphic foliation to a full neighborhood of singularity. It turns out that in general such extension does not exist. Nevertheless, the Levi foliation always extends as a holomorphic web (a foliation with branching) near a non-dicritical singularity. We also present an efficient criterion characterizing these singularities.

Keywords: CR structure, Levi-flat manifold.