

ABSTRACTS

A.I. Abdalnagimov, A.S. Krivosheyev

REPRESENTATION OF ANALYTIC FUNCTIONS

Abstract. In this paper we consider exponential series with complex exponents, whose real and imaginary parts are integer. We prove that each function analytical in the vicinity of the closure of a bounded convex domain in the complex plain can be expanded into the above mentioned series and this series converges absolutely inside this domain and uniformly on compact subsets. The result is based on constructing a regular subset with a prescribed angular density of the sequence of all complex numbers, whose real and imaginary parts are integer.

Keywords: analytic function, exponential series, regular set, density of sequence.

G.L. Alfimov, P.P.Kizin

ON SOLUTIONS OF CAUCHY PROBLEM FOR EQUATION $u_{xx} + Q(x)u - P(u) = 0$
WITHOUT SINGULARITIES IN A GIVEN INTERVAL

Abstract. The paper is devoted to Cauchy problem for equation $u_{xx} + Q(x)u - P(u) = 0$, where $Q(x)$ is a π -periodic function. It is known that for a wide class of the nonlinearities $P(u)$ the “most part” of solutions of Cauchy problem for this equation are singular, i.e., they tend to infinity at some finite point of real axis. Earlier in the case $P(u) = u^3$ this fact allowed us to propose an approach for a complete description of solutions to this equations bounded on the entire line. One of the ingredients in this approach is the studying of the set \mathcal{U}_L^+ introduced as the set of the points (u_*, u'_*) in the initial data plane, for which the solutions to the Cauchy problem $u(0) = u_*$, $u_x(0) = u'_*$ is not singular in the segment $[0; L]$. In the present work we prove a series of statements on the set \mathcal{U}_L^+ and on their base, we classify all possible type of the geometry of such sets. The presented results of the numerical calculations are in a good agreement with theoretical statements.

Keywords: ODE with periodic coefficients, singular solutions, nonlinear Schrodinger equation.

M.Sh. Badakhov, A.B. Shabat

DARBOUX TRANSFORMATIONS IN THE INVERSE SCATTERING PROBLEM

Abstract. As it is know, Darboux transformations play a key role in applications in ISP. In the paper we expose the theory of such transformations for the Schrödinger equations with compactly supported potential, which are not necessarily smooth. We study a new class of transformations connected with the zeroes of the reflection coefficient located at conjugated points in the complex plane.

Keywords: Inverse Problem, Schrödinger equation, compactly supported potentials, Darboux transformation.

R.N. Garifullin

ON SIMULTANEOUS SOLUTION OF THE KdV EQUATION AND A FIFTH-ORDER DIFFERENTIAL EQUATION

Abstract. In the paper we consider an universal solution to the KdV equation. This solution also satisfies a fifth order ordinary differential equation. We pose the problem on studying the behavior of this solution as $t \rightarrow \infty$. For large time, the asymptotic solution has different structure depending on the slow variable $s = x^2/t$. We construct the asymptotic solution in the domains $s < -3/4$, $-3/4 < s < 5/24$ and in the vicinity of the point $s = -3/4$. It is shown that a slow modulation of solution's parameters in the vicinity of the point $s = -3/4$ is described by a solution to Painlevé IV equation.

Keywords: asymptotics, matching of asymptotic expansions, Korteweg-de Vries equation, non-dissipative shock waves.

D.B. Davletov, D.V. Kozhevnikov

THE PROBLEM OF STEKLOV TYPE IN A HALF-CYLINDER WITH A SMALL CAVITY

Abstract. In the work we consider a Steklov type problem for the Laplace operator in n -dimensional cylinder with a small cavity. On the lateral surfaces one of three classic boundary conditions is imposed, the boundary of the cavity is subject to the Dirichlet condition, while on the base of the cylinder we impose the spectral Steklov condition. We prove the convergence theorems for the eigenvalues of this problems as the small parameter, the diameter of the cavity, tends to zero. We construct and justify the complete asymptotic expansions in the small parameter converging both to a simple or a double eigenvalue of the limiting problem, which is the problem without the cavity.

Keywords: half-cylinder, Steklov problem, eigenvalue, singular perturbation, small cavity, convergence, asymptotics.

K.P. Isaev, A.V. Lutsenko, R.S. Yulmukhametov

ON UNCONDITIONAL EXPONENTIAL BASES IN WEAK WEIGHTED SPACES ON SEGMENT

Abstract. We show that the existence of unconditional exponential bases is not determined by the growth characteristics of weight function. In order to do this, we construct examples of convex weights with arbitrarily slow growth near the boundary such that unconditional exponential bases do not exist in the corresponding space.

Keywords: Hilbert spaces, entire functions, unconditional exponential bases, Riesz bases.

M. Kostić, V.E. Fedorov

DEGENERATE FRACTIONAL DIFFERENTIAL EQUATIONS IN LOCALLY CONVEX SPACES
WITH A σ -REGULAR PAIR OF OPERATORS

Abstract. We consider a degenerate fractional order differential equation $D_t^\alpha Lu(t) = Mu(t)$ in a Hausdorff sequentially complete locally convex space is considered. Under the p -regularity of the operator pair (L, M) , we find the phase space of the equation and the family of its resolving operators. We show that the identity image of the latter coincides with the phase space. We prove an unique solvability theorem and obtain the form of the solution to the Cauchy problem for the corresponding inhomogeneous equation. We give an example of application the obtained abstract results to studying the solvability of the initial boundary value problems for the partial differential equations involving entire functions on an unbounded operator in a Banach space, which is a specially constructed Fréchet space. It allows us to consider, for instance, a periodic in a spatial variable x problem for the equation with a shift along x and with a fractional order derivative with respect to time t .

Keywords: fractional differential equation, degenerate evolution equation, locally convex space, σ -regular pair of operators, phase space, solution operator.

S.Yu. Lukashchuk

SYMMETRY REDUCTION AND INVARIANT SOLUTIONS
FOR NONLINEAR FRACTIONAL DIFFUSION EQUATION WITH A SOURCE TERM

Abstract. We consider a problem on constructing invariant solutions to a nonlinear fractional differential equations of anomalous diffusion with a source. On the base of an earlier made group classification of the considered equation, for each case in the classification we construct the optimal systems of one-dimensional subalgebras of Lie algebras of infinitesimal operators of the point transformations group admitted by the equation. For each one-dimensional subalgebra of each optimal system we find the corresponding form of the invariant solution and made the symmetry reduction to an ordinary differential equation. We prove that there are three different types of the reduction equations (factor equations): a second order ordinary differential equation integrated by quadratures and two ordinary nonlinear fractional differential equations. For particular cases of the latter we find exact solutions.

Keywords: fractional diffusion equation, symmetry, optimal system of subalgebras, symmetry reduction, invariant solution.

I.Kh. Musin

PERTURBATION OF A SURJECTIVE CONVOLUTION OPERATOR

Abstract. Let $\mu \in \mathcal{E}'(\mathbb{R}^n)$ be a compactly supported distribution such that its support is a convex set with a non-empty interior. Let X_2 be a convex domain in \mathbb{R}^n , $X_1 = X_2 + \text{supp } \mu$. Let the convolution operator $A : \mathcal{E}(X_1) \rightarrow \mathcal{E}(X_2)$ acting by the rule $(Af)(x) = (\mu * f)(x)$ is surjective. We obtain a sufficient condition for a linear continuous operator $B : \mathcal{E}(X_1) \rightarrow \mathcal{E}(X_2)$ ensuring the surjectivity of the operator $A + B$.

Keywords: convolution operator, distribution, Fourier-Laplace transform, entire functions.

A.V. Nekludov

ON SOLUTIONS OF SECOND ORDER ELLIPTIC EQUATIONS IN CYLINDRICAL DOMAINS

Abstract. In a semi-infinite cylinder, we consider a second order elliptic equation with a lower order term. On the lateral boundary of the cylinder we impose the homogeneous Neumann condition. We show that each bounded solution tends to a constant at infinity and once the lower order term does not decay too fast, this constant vanishes. We establish that for a sufficiently fast decay of the lower order term, we have a trichotomy of the solutions as for the equation without the lower order term: the solution tends to a general non-zero constant or grows linearly or grows exponentially. The decay conditions for the lower order term are formulated in an integral form.

Keywords: Elliptic equation, Neumann boundary value condition, unbounded domain, low order term, asymptotic behavior of solutions, trichotomy of solutions.

Yu.Kh. KhasanovON ABSOLUTE CESÁRO SUMMABILITY OF FOURIER SERIES
FOR ALMOST-PERIODIC FUNCTIONS WITH LIMITING POINTS AT ZERO

Abstract. In the paper we establish some tests for absolute Cesáro summability of the Fourier series for almost-periodic in the Bezikovich sense. We consider the case, when the Fourier exponents have a limiting point at zero and as a structure characteristics of the studied function we use a high order averaging modulus.

Keywords: absolute summability, almost-periodic function, Fourier series, Fourier exponents, limiting point at zero, averaging module.

A.Ya. Khrystiyany, A.A. KondratyukMODULO-LOXODROMIC MEROMORPHIC FUNCTIONS IN $\mathbb{C} \setminus \{0\}$

Abstract. We introduce modulo-loxodromic functions and study their representations, zeroes and poles distribution. We also show that each modulo-loxodromic meromorphic function in $\mathbb{C} \setminus \{0\}$ is Julia exceptional.

Keywords: loxodromic meromorphic function, modulo-loxodromic function, Julia exceptional function.