ABSTRACTS

G.L. Alfimov, M.E. Lebedev

On regular and singular solutions for equation $u_{xx} + Q(x)u + P(x)u^3 = 0$

Abstract. The paper is devoted to the equation $u_{xx} + Q(x)u + P(x)u^3 = 0$. The equations of such kind have been used to describe stationary modes in the models of Bose-Einstein condensate. It is known that under some conditions for $P(x)$ and $Q(x)$ the "most part" of solutions for such equations are singular, i.e. tend to infinity at some point of real axis. In some situations this fact allows us to apply methods of symbolic dynamics to describe non-singular solutions of this equation and to construct comprehensive classification of these solutions. In the paper we present (i) necessary conditions for existence of singular solutions as well as conditions for their absence; (ii) the results of numerical study of the case when $Q(x)$ is a constant and $P(x)$ is an alternate periodic function. Basing on these results, we formulate a conjecture that all the non-singular solutions of the equation can be coded by bi-infinite sequences of symbols of a countable alphabet.

Keywords: ODE with periodic coefficients, singular solutions, nonlinear Schrodinger equation, stationary modes

R.A. Bashmakov, A.A. Makhota, K.V. Trounov

On absence conditions of unconditional bases of exponents

Abstract. In the classical space $L^2(-\pi, \pi)$ there exists the unconditional basis $\{e^{ikt}\}$ (k are integer). In the work we study the existence of unconditional bases in weighted Hilbert spaces $L^2(I, \exp h)$ of functions square integrable on an interval I of the real axis with the weight $\exp(-h)$, where h is a convex function. We obtain conditions showing that unconditional bases of exponents can exist only in very rare cases.

Keywords: Riesz bases, unconditional bases, series of exponents, Hilbert space, Fourier-Laplace transform.

D.I. Borisov, R.Kh. Karimov, T.F. Sharapov

Initial length scale estimate for waveguides with some random singular potentials

Abstract. In this work we consider three examples of random singular perturbations in multi-dimensional models of waveguides. These perturbations are described by a large potential supported on a set of a small measure, by a compactly supported fast oscillating potential, and by a delta-potential. In all cases we prove initial length scale estimate.

Keywords: random operator, initial length scale estimate, perturbation, small parameter, spectral localization.

V.F. Vil'danova, F.Kh. Mukminov

Täcklind uniqueness classes on noncompact Riemannian manifolds

Abstract. We describe uniqueness classes for solution of the Cauchy problem for the heat equation on a connected noncompact complete Riemannian manifold. For the case of manifolds with boundary, we assume that the solution satisfies the Dirichlet and Neumann conditions on the boundary.

Uniqueness classes are determined by a non-negative function growing no faster than the distance from a fixed point along the geodesics. The classes are similar to uniqueness classes of Täcklind type for the equation on the real line.

Keywords:uniqueness classes, heat equation, Riemannian manifold.

R.A. Gaisin

Regularization of sequences in sense of E.M. Dyn'kin

Abstract. We introduce the notion of strong regularization of positive sequences. We prove an existence criterion of regular in the sense of E.M. Dyn'kin non-quasi-analiticity minorant. The criterion is given in terms on the smallest concave majorant of the logarithm of its trace function. The proof is based on the properties of the Legendre transformation.

Keywords: Carleman class, regular sequences, Legendre transform.

Yu.A. Kordyukov, V.A. Pavlenko

On Lefschetz formulas for flows on foliated manifolds

Abstract. The paper is devoted to the Lefschetz formulas for flows on compact manifolds, preserving a codimension one foliation and having fixed points. We develop an approach to the Lefschetz formulae based on the notion of the regularized trace on some algebra of singular integral operators introduced in a previous paper. The Lefschetz formula is proved in the case when the flow preserves a foliation given by the fibers of a fiber bundle over a circle. For a particular example of a flow on a two-dimensional torus, preserving a Reeb type foliation, we prove an analogue of the McKean-Singer formula for smoothed regularized Lefschetz functions.

Keywords: Lefschetz formula, flow, closed orbits, fixed points, foliated manifold, regularized trace.

A.A. Lishanskii

Existence of hypercyclic subspaces for Toeplitz operators

Abstract. In this work we construct a class of coanalytic Toeplitz operators which have an infinite-dimensional closed subspace, where any non-zero vector is hypercyclic. Namely, if for a function φ which is analytic in the open unit disc $\mathbb{D}$ and continuous in its closure the conditions $\varphi(\mathbb{T}) \cap \mathbb{T} \neq \emptyset$ and $\varphi(\mathbb{D}) \cap \mathbb{T} \neq \emptyset$ are satisfied, then the operator $\varphi(S^*)$ (where S^* is the backward shift operator in the Hardy space) has the required property. The proof is based on an application of a theorem by Gonzalez, Leon-Saavedra and Montes-Rodriguez.

Keywords: Toeplitz operators, hypercyclic operators, essential spectrum, Hardy space.

S.V. Khabirov

Space motions with the linear field of velocity without divergence

Abstract. We obtain the formulae defining all possible motions of continuous medium without divergence and with the linear field of the velocity. The field is either linear w.r.t. the time or the matrix has constant singular numbers.

Keywords: dynamics without divergence, linear field of velocity, singular numbers of the matrix.

V.I. Mogilevskii

On spectral and pseudospectral functions of first-order symmetric systems

Abstract. We consider first-order symmetric system $Jy' - B(t)y = \Delta(t)f(t)$ on an interval $\mathcal{I} = [a, b)$ with the regular endpoint a. A distribution matrix-valued function $\Sigma(s)$, $s \in \mathbb{R}$, is called a pseudospectral function of such a system if the corresponding Fourier transform is a partial isometry with the minimally possible kernel. The main result is a parametrization of all pseudospectral functions of a given system by means of a Nevanlinna boundary parameter τ. Similar parameterizations for regular systems have earlier been obtained by Arov and Dym, Langer and Textorius, A. Sakhnovich.

Keywords: spectrum, operator pencil, regularized traces.