

## ABSTRACTS

A.M. Abylayeva, A.O. Baiarystanov

COMPACTNESS CRITERION FOR FRACTIONAL INTEGRATION OPERATOR  
OF INFINITESIMAL ORDER

**Abstract.** We obtain necessary and sufficient conditions of compactness for the operator

$$Kf(x) = \int_0^x \ln \frac{x}{x-s} \frac{f(s)}{s} ds$$

from  $L_{p,v}$  in  $L_{q,u}$  at  $1 < p \leq q < \infty$  and  $v(x) = x^{-\gamma}$ ,  $\gamma > 0$ , where  $L_{q,u}$  is the set of all measurable on  $(0, \infty)$  functions  $f$  with finite norm  $\|uf\|_q$ .

**Keywords:** compactness, fractional integration operator, Riemann-Liouville operator, singular operator, adjoint operator, Holder inequality, weighted inequalities.

A.K. Bazzaev

FINITE-DIFFERENCE SCHEMES FOR DIFFUSION EQUATION OF FRACTIONAL ORDER WITH  
THIRD TYPE BOUNDARY CONDITIONS IN MULTIDIMENSIONAL DOMAIN

**Abstract.** We consider finite difference schemes for diffusion equation of fractional order in a multidimensional field with third type boundary conditions. We prove the stability and convergence of difference schemes for considered problem.

**Keywords:** finite-difference schemes, diffusion equation of fractional order, apriori estimate, maximum principle, third type boundary conditions, stability and convergence of finite-difference scheme

Yu.N. Drozhzhinov, B.I. Zavalov

GENERALIZED FUNCTIONS ASYMPTOTICALLY HOMOGENEOUS WITH RESPECT TO  
ONE-PARAMETRIC GROUP AT ORIGIN

**Abstract.** In the work we obtain a complete description of generalized functions asymptotically homogeneous at origin w.r.t. a multiplicative one-parametric group of transformations so that the real parts of all the eigenvalues of infinitesimal matrix are positive including the case of critical orders. The obtained results are applied for constructing homogeneous solutions to differential equations whose symbols are quasi-homogeneous polynomials w.r.t. this group in a non-critical case.

**Keywords:** generalized functions, homogeneous functions, quasi-asymptotics, partial differential equations.

**Kh.K. Ishkin**

ON ANALYTIC PROPERTIES OF WEYL FUNCTION OF STURM – LIOUVILLE OPERATOR WITH A DECAYING COMPLEX POTENTIAL

**Abstract.** We study the spectral properties of the operator  $L_\beta$  associated with the quadratic form  $\mathcal{L}_\beta = \int_0^\infty (|y'|^2 - \beta x^{-\gamma}|y|^2)dx$  with the domain  $Q_0 = \{y \in W_2^1(0, +\infty) : y(0) = 0\}$ ,  $0 < \gamma < 2$ ,  $\beta \in \mathbf{C}$ , as well as of the perturbed operator  $M_\beta = L_\beta + W$ . Under the assumption  $(1 + x^{\gamma/2})W \in L^1(0, +\infty)$  we prove the existence of finite quantum defect of the discrete spectrum that was established earlier by L.A. Sakhnovich as  $\beta > 0$ ,  $\gamma = 1$  and for real  $W$  satisfying a more strict decaying condition at infinity. The main result of the paper is the proof of necessity (with some reservations) of the sufficient conditions for  $W(x)$  obtained earlier by Kh.Kh. Murtazin under which the Weyl function of the operator  $M_\beta$  possesses an analytic continuation on some angle from non-physical sheet.

**Keywords:** spectral instability, localization of spectrum, quantum defect, Weyl function, Darboux transformation.

**K.A. Kirillov, M.V. Noskov**

A VERSION OF DISCRETE HAAR TRANSFORM WITH NODES OF  $\Pi_0$ -GRIDS

**Abstract.** A version of the two-dimensional discrete Haar transform with  $2^D$  nodes forming  $\Pi_0$ -grid associated with the triangular partial sums of Fourier – Haar series of a given function is proposed. Due to the structure the of  $\Pi_0$ -grids, the computation of coefficients of this discrete transform is based on a cubature formula with  $2^D$  nodes being exact for Haar polynomials of degree at most  $D$ , owing to that all the coefficients  $A_{m_1, m_2}^{(j_1, j_2)}$  of the constructed transform coincide with the Fourier – Haar coefficients  $c_{m_1, m_2}^{(j_1, j_2)}$  for Haar polynomials of degree at most  $D - \max\{m_1, m_2\}$  ( $0 \leq m_1 + m_2 \leq d$ , where  $d \leq D$ ). The standard two-dimensional discrete Haar transform with  $2^D$  nodes does not possess this property.

**Keywords:** cubature formulas exact for Haar polynomials, discrete Haar transform,  $\Pi_0$ -grids

**L.M. Kozhevnikova, A.A. Leontiev**

DECAY OF SOLUTION OF ANISOTROPIC DOUBLY NONLINEAR PARABOLIC EQUATION IN UNBOUNDED DOMAINS

**Abstract.** This work is devoted to a class of parabolic equations with double nonlinearity whose representative is a model equation

$$(|u|^{k-2}u)_t = \sum_{\alpha=1}^n (|u_{x_\alpha}|^{p_\alpha-2}u_{x_\alpha})_{x_\alpha}, \quad p_n \geq \dots \geq p_1 > k, \quad k \in (1, 2).$$

For the solution of the first mixed problem in a cylindrical domain  $D = (0, \infty) \times \Omega$ ,  $\Omega \subset \mathbb{R}_n$ ,  $n \geq 2$  with homogeneous Dirichlet boundary condition and compactly supported initial function precise estimates the rate of decay as  $t \rightarrow \infty$  are established. Earlier these results were obtained by the authors for  $k \geq 2$ . The case  $k \in (1, 2)$  differs by the method of constructing Galerkin’s approximations that for an isotropic model equation was proposed by E.R. Andriyanova and F.Kh. Mukminov.

**Keywords:** anisotropic equation, doubly nonlinear parabolic equations, existence of strong solution, decay rate of solution.

### A.G. Losev, E.A. Mazepa

ON ASYMPTOTIC BEHAVIOR OF POSITIVE SOLUTIONS OF SOME QUASILINEAR INEQUALITIES ON MODEL RIEMANNIAN MANIFOLDS

**Abstract.** In the paper we study asymptotic behavior of positive solutions to some quasilinear elliptic inequalities on spherically symmetric noncompact (model) Riemannian manifolds. In particular, we find conditions under which Liouville type theorems on absence of nontrivial solutions hold true, as well as the conditions of existence and cardinality of the set of positive solutions of the studied inequalities on the Riemannian manifolds. The results generalize similar results obtained previously by Y. Naito and H. Usami for the Euclidean space  $\mathbf{R}^n$ .

**Keywords:** quasilinear elliptic inequality, Liouville type theorem, model Riemannian manifolds

### N.Kh. Mamatova, A.R. Hayotov, Kh.M. Shadimetov

CONSTRUCTION OF OPTIMAL GRID INTERPOLATION FORMULAS IN SOBOLEV SPACE  $\widetilde{L}_2^m(H)$  OF PERIODIC FUNCTION OF  $n$  VARIABLES BY SOBOLEV METHOD

**Abstract.** In the present work we consider the problem of constructing optimal grid interpolation formulas in the space  $\widetilde{L}_2^m(H)$  of periodic function of  $n$  variables. We find the coefficients of grid interpolation formulas.

**Keywords:** Sobolev space, optimal interpolation formula, properties of the discrete analogue of the operator  $\Delta^m$ , optimal coefficients.

### M.M. Matyoqubov, A.B. Yakhshimuratov

INTEGRATION OF HIGHER KORTEWEG-DE VRIES EQUATION WITH A SELF-CONSISTENT SOURCE IN CLASS OF PERIODIC FUNCTIONS

**Abstract.** In the present the inverse spectral problem of Sturm-Liouville operator is applied for integrating higher Korteweg-de Vries equation with a self-consistent source in class of periodic functions

**Keywords:** Sturm-Liouville operator, inverse spectral problem, eigenvalue, eigenfunction, Korteweg-de Vries equation.

### S.N. Mishin

ON GROWTH CHARACTERISTICS OF OPERATOR-VALUED FUNCTIONS

**Abstract.** In the work Liouville theorem and the concept of order and type of entire function are generalized to the case of operator-valued function with values in the space  $\text{Lec}(\mathbf{H}_1, \mathbf{H})$  of all linear continuous operators acting from a locally convex space  $\mathbf{H}_1$  to a locally convex space  $\mathbf{H}$  with equicontinuous bornology. We find the formulae expressing the order and type of operator-valued function in terms of characteristics of the sequence of coefficients. Some properties of order and type of operator-valued function are established.

**Keywords:** locally convex space, order and type of sequence of operators, order and type of entire function, equicontinuous bornology, convergence by bornology, operator-valued function.

### S.V. Khabirov

#### REDUCTIONS OF PARTIALLY INVARIANT SOLUTIONS OF RANK 1 DEFECT 2 FIVE-DIMENSIONAL OVERALGEBRA OF CONICAL SUBALGEBRA

**Abstract.** Conic flows are the invariant rank 1 solutions of the gasdynamics equations on the three-dimensional subalgebra defined by the rotation operators, translation by time and uniform dilatation. The generalization of the conic flows are partially invariant solutions of rank 1 defect 2 on the five-dimensional overalgebra of conic subalgebra extended by the operators of space translations noncommuting with rotation. We prove that that the extensions of conic flows are reduced either to function-invariant plane stationary solutions or to a double wave of isobaric motions or to the simple wave.

**Keywords:** gas dynamics, conic flows, partially invariant solutions.