ABSTRACTS

R.A. Atnagulova, I.Z. Golubchik

NEW SOLUTIONS OF YANG-BAXTER EQUATION WITH THE SQUARE

Abstract. The paper is devoted to the Yang-Baxter equation with the square, that is, to the equation

$$R([R(a), b] - [R(b), a]) = R^2([a, b]) + [R(a), R(b)],$$

where $a, b \in g$, g - is a Lie algebra, and R is a linear operator on the vector space g. Two series of operators R, satisfying this equation are constructed. In the construction we use Lie subalgebras in the matrix algebra, complementary to the subspace of matrices with zero last row.

Keywords: The Yang-Baxter equation with the square, integrable differential equations, complementary subalgebras in the algebra of Laurent series.

A.V. Zhiber, R.D. Murtazina, I.T. Habibullin, A.B. Shabat

CHARACTERISTIC LIE RINGS AND INTEGRABLE MODELS IN MATHEMATICAL PHYSICS

Abstract. Review is devoted to a systematic exposition of the algebraic approach to the study of nonlinear integrable partial differential equations and their discrete analogues, based on the concept of the characteristic vector field. A special attention is paid to the Darboux integrable equations. The problem of constructing higher symmetries of the equations, as well as their particular and general solutions is discussed. In particular, it is shown that the partial differential equation of hyperbolic type is integrated in quadratures if and only if its characteristic Lie rings in both directions are of finite dimension. For the hyperbolic type equations integrable by the inverse scattering method, the characteristic rings are of minimal growth. The possible applications of the concept of characteristic Lie rings to the systems of differential equations of hyperbolic type with more than two characteristic directions, to the equations of evolution type, and to ordinary differential equations are discussed.
Keywords: characteristic vector field, symmetry, Darboux integrability.

M.N. Kuznetsova

NONLINEAR HYPERBOLIC DIFFERENTIAL EQUATIONS RELATED WITH THE KLEIN-GORDON EQUATION BY DIFFERENTIAL SUBSTITUTIONS

Abstract. We present a complete classification of nonlinear hyperbolic differential equations in two independent variables $u_{xy} = f(u, u_x, u_y)$ reduced to the Klein-Gordon equation $v_{xy} = F(v)$ by differential substitutions of the special form $v = \varphi(u, u_x)$.
Keywords: nonlinear hyperbolic equations, differential substitutions, the Klein-Gordon equation.

A.G. Meshkov, V.V. Sokolov

Integrable evolution equations with the constant separant

Abstract. The survey contains results of classification for integrable one-field evolution equations of orders 2, 3 and 5 with the constant separant. The classification is based on neccesary integrability conditions that follow from the existence of the formal recursion operator for integrable equations. Recursion formulas for the whole infinite sequence of these conditions are presented for the first time. The most of the classification statements can be found in papers by S.I. Svinilupov and V.V. Sokolov but the proofs never been published before. The result concerning the fifth order equations is stronger then obtained before.

Keywords: evolution differential equation, integrability, higher symmetry, conservation law, classification

A.U. Sakieva

Characteristic Lie ring of the Zhiber-Shabat-Tzitzeica equation

Abstract. A complete description of the characteristic Lie ring for the Zhiber-Shabat-Tzitzeica equation is given. For the linear space of multiple commutators of arbitrary order a basis is constructed. It is proved that the characteristic Lie ring is a ring of slow growth.

Keywords: Lie ring, nonlinear hyperbolic equation, integral, vector field.

S.Ya. Startsev

Darboux integrable differential-difference equations admitting a firts-order integral

Abstract. We obtain a classification of Liouville-type differential-difference equations that admit a first-order integral with respect to one of the characteristics. This classification gives us the complete description of difference substitutions which are applicable to wide classes of differential-difference evolution equations. The classification also allows us to construct the complete list of Darboux integrable differential-difference equations which admit both a second-order integral with respect to one of the characteristics and a non-point invertible transformation with respect to the same characteristic.

Keywords: Darboux integrability, differential-difference equations, difference substititions.

R.N. Garifullin and R.I. Yamilov

Examples of Darboux Integrable Discrete Equations Possessing First Integrals of an Arbitrarily High Minimal Order

Abstract. We consider a discrete equation, defined on the two-dimensional square lattice, which is linearizable, namely, of the Burgers type and depends on a parameter α. For any natural number N we choose α so that the equation becomes Darboux integrable and the minimal orders of its first integrals in both directions are greater or equal than N.

Keywords: discrete equation, Darboux integrability, first integral.