

## ABSTRACTS

**Yu.Yu. Bagderina**

SEPARATION OF AN EQUATION IN THE SYSTEM OF TWO SECOND-ORDER ORDINARY  
DIFFERENTIAL EQUATIONS

**Abstract.** We consider projectable type systems of two second-order ordinary differential equations with cubic nonlinearity of the right-hand side in first derivatives. For such systems we obtain criteria of reducibility by local transformation to a system with a separating equation in one of the unknown functions. Applications of the criteria and construction of the corresponding transformation is illustrated by a number of examples.

**Keywords:** second-order equation, decoupling of equations, separation of an equation, submersive system

**A.R. Bikmetov, R.R. Gadyl'shin**

PERTURBATION OF AN ELLIPTIC OPERATOR BY A NARROW POTENTIAL  
IN AN  $n$ -DIMENSIONAL DOMAIN

**Abstract.** We study a discrete spectrum of an elliptic operator of the second order in an  $n$ -dimensional domain,  $n \geq 2$ , perturbed by a potential depending on two parameters, one of the parameters describes the length of the support of the potential and the inverse of the other corresponds to the magnitude of the potential. We give the relation between these parameters, under which the generalized convergence of the perturbed operator to the unperturbed one holds. Under this relation we construct the asymptotics w.r.t. small parameters of the eigenvalues of the perturbed operators.

**Keywords:** Elliptic operator, perturbation, matching of asymptotic expansions

**D.I. Borisov, A.M. Golovina**

ON THE RESOLVENTS OF PERIODIC OPERATORS WITH DISTANT PERTURBATIONS

**Abstract.** We consider distant perturbations for an abstract periodic operator. The unperturbed operator is introduced as a closed operator on the Sobolev space defined on a periodic domain in a multidimensional space. We impose certain condition for the unperturbed operator being a natural generalization of the ellipticity and periodicity conditions for the differential operators. The perturbations are described by abstract relatively bounded operators being localized in a certain sense. We study the case when the distance between the domains, where the perturbations are localized, increases unboundedly. The main obtained result is the explicit representation for the resolvent of the perturbed operator.

**Keywords:** resolvent, periodic operator, distant perturbations.

## I.F. Galikhanov, V.N. Pavlenko

### PERIODIC SOLUTIONS OF THE TELEGRAPH EQUATION WITH A DISCONTINUOUS NONLINEARITY

**Abstract.** We consider telegraph equations with a variable inner energy, discontinuous by phase, and the homogeneous Dirichlet boundary condition. Question of existence of general periodic solutions in the resonant case, when the operator created by a linear part of the equation with the homogeneous Dirichlet boundary condition and the condition of periodicity has a non zero kernel, and nonlinearity appearing in the equation is limited. We obtained an existence theorem for the general periodic solution by means of the topological method. The proof is based on the Leray-Schauder principle for convex compact mappings. The main difference from similar results of other authors is an assumption that there are breaks in the phase variable of the inner energy of the telegraph equation.

**Keywords:** nonlinear telegraph equation, discontinuous nonlinearity, periodic solutions, resonance problem

## R.N. Garifullin

### PHASE SHIFT FOR THE COMMON SOLUTION OF THE KDV AND THE FIFTH ORDER DIFFERENTIAL EQUATION

**Abstract.** We investigate the special solution of Korteweg-de Vries equation. This solution describes the influence of small dispersion to a process of transformation from weak to strong discontinuities in inviscid fluid dynamics. This solution also satisfies the fifth order ordinary differential equation. We construct the asymptotic solution in the Witham zone up to a phase shift. We obtain an equation for phase shift and, using the numerical experiments, we choose the concrete solution of this equation. This solution is a constant function. **Keywords:** phase shift, Korteweg-de Vries equation, nondissipative shock waves.

## A.R. Danilin

### OPTIMAL BOUNDARY CONTROL IN A SMALL CONCAVE DOMAIN

**Abstract.** The paper is devoted to investigation of an asymptotics of a solution of the problem of optimal boundary control [1] in a small concave domain. Construction of an asymptotics of a boundary value problem for an elliptic operator in a small concave domain is considered in [2], and an asymptotics of the distributed control in a small concave domain in [3]. The Asymptotics of boundary control for an operator with a small factor at the higher derivative was considered in [4], [5]. Other problems of control by solutions of boundary value problems of the optimal control containing a small parameter are considered in [6], [7].

**Keywords:** asymptotic, boundary control, matching method, boundary value problems, systems of equations in partial derivatives.

## L.A. Kalyakin

### ASYMPTOTIC ANALYSIS OF THE SURFING ACCELERATION MODEL

**Abstract.** A mathematical model of acceleration of the charge particles by the electromagnetic waves is under investigation. Averaging equations, describing the resonance interaction of the particle with the electromagnetic wave are obtained. We show that each particle leaves the resonance zone under growth of time. The time length of the stay in the resonance depending on the initial data is calculated.

**Keywords:** Nonlinear oscillations, small parameter, perturbation, averaging, adiabatic.

## F.S. Nasyrov, E.V. Yureva

### ON SOLUTIONS OF THE FIRST-ORDER PDE WITH A MULTIDIMENSIONAL SYMMETRIC INTEGRAL AND THEIR MODELLING

**Abstract.** The deterministic analog of the multidimensional Stratonovich integral is constructed. Method of solution of a system of equations with a multidimensional symmetric integral is elaborated. The method of characteristics for solving the Cauchy problem for first-order partial differential equations with a multidimensional symmetric integral is developed. This method reduces solving the initial-value problem of the above equations to solving a system of equations with a multidimensional symmetric integral.

**Keywords:** multidimensional symmetric integral, differential equations system with multidimensional symmetric integral, partial differential equations with multidimensional symmetric integral, the method of characteristics

## B.I. Suleimanov

### THE “QUANTUM” LINEARIZATION OF THE PAINLEVÉ EQUATIONS AS A COMPONENT OF THEIR $L - A$ PAIRS

**Abstract.** The procedure of the “quantum” linearization of the Hamiltonian ordinary differential equations with one degree of freedom is investigated. It is offered to be used for the classification of integrable equations of the Painleve type. For the Hamiltonian  $H = (p^2 + q^2)/2$  and all natural numbers  $n$  the new solutions  $\Psi(\hbar, t, x, n)$  of the non-stationary Schrödinger equation are constructed. The solutions tend to zero at  $x \rightarrow \pm\infty$ . On curves  $x = q_n(\hbar, t)$ , defined by the old Bohr- Zommerfeld rule, the solutions satisfy the relation  $i\hbar\Psi'_x \equiv p_n(\hbar, t)\Psi$ . In this relation  $p_n(\hbar, t) = (q_n(\hbar, t))'_t$  is the classical momentum corresponding to the harmonic  $q_n(\hbar, t)$ .

**Keywords:** quantization, linearization, non-stationary Schrödinger equation, Painleve equations, isomonodrominiS deformations.