ABSTRACTS

E.D. Avdonina and N.H. Ibragimov

Equivalence group analysis and nonlinear self-adjointness of the generalized Kompaneets equation

Abstract. Equivalence group analysis is applied to the Kompaneets equation. We compute the equivalence Lie algebra for the corresponding *generalized* Kompaneets equation. We also show that the generalized Kompaneets equation is nonlinearly self-adjoint.

The principle of an *a priori* use of symmetries gives a possibility to use the equivalence algebra in order to approximate the Kompaneets equation by an equation having a wider class of symmetries. Using an additional symmetry of the approximating equation and the nonlinear self-adjointness, one can construct new group invariant solutions and conservation laws.

Keywords: Kompaneets equation, Generalized Kompaneets equation, Equivalence algebra, Nonlinear self-adjointness, Invariant solution.

G.G. Amosov, A.D. Baranov, V.V. Kapustin

Applications of model spaces to construction of cocyclic perturbations of a semigroup of shifts on a semiaxis

Abstract. We describe a construction of cocyclic perturbations of the semigroup of shifts on the half-line by means of theory of model spaces. It is shown that one can choose an inner function that determines the model space so that the elements of the perturbed semigroup have a prescribed spectral type and differ from the elements of the initial semigroup by operators from the Schatten–von Neumann class $\mathfrak{S}_p$, $p > 1$. The case of the trace class $\mathfrak{S}_1$ perturbations is considered separately.

Keywords: semigroup of shifts, inner function, Schatten–von Neumann classes.

G.G. Braichev

Exact estimates of types of entire functions of an order $\rho \in (0; 1)$ with zeroes on the ray

Abstract. This paper is a detailed account of the author's report made during VI Ufa international conference "Complex analysis and differential equations", devoted to the 70-th anniversary of Corresponding member of RAS V.V. Napalkov. Sharp lower estimates of an entire function type of a finite order with respect to such well-known characteristics of the distribution of its zeros as the density (conventional and average), step and lacunarity index. The solution of one new extremal problem is also given.

Keywords: type of an entire function, the upper and lower (average) density of zeros, step and lacunarity index of a sequence of zeros.

A.M. Gaisin, Zh.G. Rakhmatullina

Iterations of entire transcendental functions with a regular behavior of the modulus minimum

Abstract. In the paper the Fatou set of an entire transcendental function is considered, i.e. the largest open set of the complex plane, where the family of iterations of the given function forms a normal family according to Montel. The entire function is assumed to be of an infinite lower order. The pair of conditions on the indices of the series providing that every component of the Fatou set is bounded is found. This pair of conditions is optimal in a certain sense and is stronger than the Fejér gap condition. The result under stronger sufficient conditions was proved earlier by Yu. Wang and Zh. Rakhmatullina.

Keywords: entire functions, Fejér gaps, iterations of functions, Fatou set

N.P. Girya, S.Yu. Favorov

A periodicity criterium for quasipolynomials

Abstract. We consider functions from the Δ class, which was introduced by M.G. Krein and B.Ja. Levin in 1949. Δ is a class of almost periodic entire functions of an exponential type with zeros belonging to a horizontal strip of a finite width. In particular, the class contains all finite exponential sums with pure imaginary exponents. Another description of the class Δ is analytic continuations to the complex plane of almost periodic functions on the real axis with a bounded spectrum such that the infimum and the supremum of the spectrum belong to the spectrum too.

It is proved that any function from the class Δ with a discrete set of differences of its zeros is a finite product of shifts of the function $\sin \omega z$ up to a factor $C \exp\{i\beta z\}$ with real β.

Keywords: Almost periodic function, entire function of an exponential type, zero set, discrete set.

M. Gürses, A.V. Zhiber, I.T. Habibullin

Characteristic Lie rings of differential equations

Abstract. The characteristic Lie rings corresponding to hyperbolic type equations are considered. Possible applications of this concept to the problem of integrable classification of systems of the hyperbolic type partial differential equations with more than two characteristic destinations, evolutionary type equations and ordinary differential equations are briefly discussed. The widely known models of mathematical physics as well as the system of „n"-wave equations, the Korteweg-de Vries equation, the Burgers equation and the first Painlevé equation are considered as illustrative examples.

Keywords: characteristic vector fields, characteristic ring, evolution equations, system of „n"-wave equations.

K.P. Isaev, K.V. Trunov

ON THE DISTRIBUTION OF INDICATORS OF UNCONDITIONAL EXPONENTIAL BASES IN SPACES WITH A POWER WEIGHT

Abstract. In the present paper we consider the existence of unconditional exponential bases in a space of locally integrable functions on a bounded interval of the real number line I satisfying

$$\|f\| := \sqrt{\int_I |f(t)|^2 e^{-2h(t)}\, dt} < \infty,$$

where $h(t)$ is a convex function on this interval. The lower estimate was obtained for the frequency of indicators of unconditional bases of exponentials when $I = (-1; 1)$, $h(t) = -\alpha \ln(1 - |t|)$, $\alpha > 0$.

Keywords: series of exponents, unconditional bases, Riesz bases, power weights, Hilbert space.

A.A. Kasatkin

SYMMETRY PROPERTIES FOR SYSTEMS OF TWO ORDINARY FRACTIONAL DIFEFERENTIAL EQUATIONS

Abstract. Lie point symmetries of two systems of ordinary fractional differential equations with the Riemann-Liouville derivatives are considered. Infinite algebra L of equivalence transformation operators is constructed. It is shown that all admitted operators generate some subalgebra in L and classification of systems with respect to point symmetries can be based on the optimal system of subalgebras. The optimal system of one-dimensional L subalgebras and the complete normalized optimal system for its finite-dimensional part L_6 are constructed.

Keywords: fractional derivatives, symmetries, group classification, optimal system of subalgebras

V.E. Kim

EIGENFUNCTIONS OF ANNIHILATION OPERATORS ASSOCIATED WITH WIGNER'S COMMUTATION RELATIONS

Abstract. We consider linear continuous operators acting on the space of all entire functions with the uniform convergence topology and satisfying Wigner's commutation relations. These operators are closely connected with the Dunkl generalized convolution operators. We study the problem of description of eigenfunctions of these operators. It is shown that under some conditions the eigenfunctions of the operator under study can be described by Dunkl generalized translates of entire functions belonging to it's kernel. We also discuss the problem of completeness of the systems of eigenfunctions.

Keywords: commuation relations, Dunkl operator, eigenfunctions, entire functions.

A.S. Krivosheyev

An almost exponential sequence of exponential polynomials

Abstract. A special sequence of exponential polynomials, whose exponentials are divided into relatively small groups, is studied in the article. It is proved that this sequence is almost an exponential sequence for each convex domain of a complex plane. By means of this result necessary and sufficient conditions for the considered sequence to be a basis in a closed and invariant under differentiation subspace of the space of analytic functions in a convex domain are obtained. Two methods of description of the whole class of bases in an invariant subspace, whose elements are exponential polynomials, are given.

Keywords: exponential polynomial, invariant subspace, analytic function, convex domain, basis.

V.I. Kuzovatov

On some families of complex lines sufficient for holomorphic extension of functions

Abstract. The present article is based on the result related to the holomorphic extension of functions. Functions with the one-dimensional holomorphic extension property along families of complex lines are discussed. Real analytic functions given on the boundary of a bounded domain D in $\mathbb{C}^n$, $n > 1$ with the one - dimensional holomorphic extension property along families of complex lines are considered. The existence of holomorphic extensions of these functions to D is studied depending on the kind of the domain and location of the families of complex lines.

Keywords: real analytic function, holomorphic extension, functions with the one - dimensional holomorphic extension property

M.P. Mahola, P.V. Filevych

The angular distribution of zeros of random analytic functions

Abstract. It is proved, that for the majority (in the sense of probability measure) of functions f, analytic in the unit disk with unbounded Nevanlinna characteristic $T_f(r)$, and for all $\alpha < \beta \leqslant \alpha + 2\pi$ the relation

$$N_f(r, \alpha, \beta, 0) \sim \frac{\beta - \alpha}{2\pi} T_f(r), \quad r \to 1,$$

holds, where $N_f(r, \alpha, \beta, 0)$ is the integrated counting functions of zeros of f in the sector $\{z \in \mathbb{C} : \ 0 < |z| \leqslant r, \ \alpha \leqslant \arg_\alpha z < \beta\}$. The analogous proposition is obtained for entire functions under some conditions on their growth.

Keywords: analytic function, random analytic function, distribution of zeros, counting function, integrated counting function, Nevanlinna characteristic.

M.I. Musin

On a space of entire functions decreasing rapidly on a real line

Abstract. A space of entire functions decreasing rapidly on a real line is introduced. It contains the space of the Fourier-Laplace transforms of infinitely differentiable functions on a real line with a compact support as a proper subspace. The Fourier-Laplace transform of functions of this space is studied. Equivalent description in terms of estimates of derivatives of functions on a real line is obtained for the considered space.

Keywords: The Fourier-Laplace transform, entire functions, Paley-Wiener type theorem.

A.Y. Timofeev

Boundary problem for the generalized Cauchy–Riemann equation in spaces, described by the modulus of continuity

Abstract. The article is devoted to the Dirichlet problem in the unit disk G for $\partial_{\bar{z}}w + b(z)\overline{w} = 0$, $\operatorname{Re} w = g$ on ∂G, $\operatorname{Im} w = h$ at the point $z_0 = 1$, where g is a given Lipschitz continuous function. The coefficient b belongs to a subspace of $L_2(G)$ which is not contained in $L_q(G)$, $q > 2$ in the general case. Thus, I. Vekua's theory is not applicable in this case. The article shows that, as well as in the case of Dirichlet's problem for holomorphic functions, there appears a "logarithmic effect". The solution outside the point $z = 0$ satisfies the Lipschits conditions with logarithmic factors. The existence of a continuous solution of the problem in $\overline{G}$ is proved

Keywords: generalized Cauchy–Riemann equation; Dirichlet problem; modulus of continuity; Tikhonov's fixed point theorem

R.A. Sharipov

Perfect cuboids and irreducible polynomials

Abstract. The problem of constructing a perfect cuboid is related to a certain class of univariate polynomials with three integer parameters a, b, and u. Their irreducibility over the ring of integers under certain restrictions for a, b, and u would mean the non-existence of perfect cuboids. This irreducibility is conjectured and then verified numerically for approximately $10\,000$ instances of a, b, and u.

Keywords: an Euler cuboid, a perfect cuboid, irreducible polynomials.

O.V. Sherstyukova

On extremal type of an entire function of order less than unity with zeros of prescribed densities and step

Abstract. Sharp lower estimate for the type of an entire function of the order $\rho \in (0;1)$ with respect to densities and step of its zeros located on the ray is proved.

Keywords: type of an entire function, the upper, lower densities and step of zeros.

O.A. Zolota

On the asymptotic behavior of the Cauchy-Stieltjes integral in the polydisc

Abstract. In the paper the asymptotic behavior of the Cauchy-Stieltjes integral of a complex-valued Borel measure on the skeleton of the polydisc is described. The main result holds outside a set of zero ω-capacity. It generalizes the theorem for the one-dimensional case.

Keywords: modulus of continuity, Cauchy-Stieltjes integral, polydisc, set of zero ω-capacity, non-tangential limit.