

ABSTRACTS

M.S. Akbasheva, A.B. Shabat

THEOREM ON COMMUTATION IN THE PRINCIPAL PART

Abstract. In the present paper we demonstrate how one can use the Poisson bracket in order to build up and to classify commuting pairs of partial differential operators with two independent variables. The commutativity condition is reduced to the simple functional equation with shifts of the arguments for considered operators. The Poisson bracket represents the limiting case of that functional equation in which the shifts are replaced by the corresponding directional derivatives. **Keywords:** Differential operators, commutators and the Poisson bracket, functional equation

S.N. Askhabov

APPROXIMATE SOLUTION OF NONLINEAR EQUATIONS WITH WEIGHTED POTENTIAL TYPE OPERATORS

Abstract. Global theorems on existence, uniqueness and ways of finding solutions are proved in a real space $L_2(-\infty, \infty)$ for different classes of nonlinear integral equations with weighted potential type operators

$$F(x, u(x)) + \int_{-\infty}^{\infty} \frac{[a(x) - a(t)] u(t)}{|x - t|^{1-\alpha}} dt = f(x),$$
$$u(x) + \int_{-\infty}^{\infty} \frac{[a(x) - a(t)] F(t, u(t))}{|x - t|^{1-\alpha}} dt = f(x),$$
$$u(x) + F \left(x, \int_{-\infty}^{\infty} \frac{[a(x) - a(t)] u(t)}{|x - t|^{1-\alpha}} dt \right) = f(x)$$

by means of combining the basic principle of monotone operators theory by Browder-Minty with the Banach contraction mapping principle. It is shown that the solutions can be found by using the Picard successive approximations method and speed estimates of their convergence are proved. The obtained results cover, in particular, the linear integral equations case with potential type kernels of a special form.

Keywords: nonlinear integral equations, potential type operator, monotone operator.

E.F. Akhmerova

ASYMPTOTIC PRESENTATION OF EIGENFUNCTIONS OF A TWO-DIMENSIONAL HARMONIC OSCILLATOR

Abstract. The asymptotics of eigenfunctions of a two-dimensional harmonic oscillator has been obtained all over the space. The need of such presentation arises when studying the spectral characteristics of non compact perturbation of the two-dimensional harmonic oscillator. The absence of exact asymptotic equalities for fundamental systems of solutions to the differential equation complicates the study, because eigenfunctions of the two-dimensional harmonic oscillator are represented in the form of a product of normalized eigenfunctions of the one-dimensional harmonic oscillator. The usage of standard solutions helps to solve the problem.

Keywords: harmonic oscillator, eigenfunctions, trace formulas, eigenvalue asymptotics.

S.P. Balandin, I.Yu. Cherdantzev

THE MATRIX ANALOGS OF THE FIRST PAINLEVÉ EQUATIONS

Abstract. Earlier, Balandin and Sokolov obtained non-Abelian analogs of the first and second transcendent Painlevé equations and studied whether they possess the Painlevé property. In the present paper the integrability of generalizations of such equations are studied using the Painlevé–Kowalevskaya test. As a result, integrability criteria for generalized non-Abelian analogs of the first and second Painlevé equations are obtained. An important role in finding these criteria is played by the eigenvalue problems for the corresponding linear operators. The obtained results are in agreement with the earlier investigations of special cases of our equations.

Keywords: integrability, Painlevé test, matrix equations.

A.G. Barseghyan

ON SOLUTION OF A TWO KERNEL EQUATION REPRESENTED BY EXPONENTS

Abstract. The integral equation with two kernels

$$f(x) = g(x) + \int_0^{\infty} K_1(x-t)f(t)dt + \int_{-\infty}^0 K_2(x-t)f(t)dt, \quad -\infty < x < +\infty,$$

where the kernel functions $K_{1,2}(x) \in L$, is considered on the whole line. The present paper is devoted to solvability of the equation, investigation of properties of solutions and description of their structure. It is assumed that the kernel functions $K_m \geq 0$ are even and represented by exponentials as a mixture of the two-sided Laplace distributions:

$$K_m(x) = \int_a^b e^{-|x|s} d\sigma_m(s) \geq 0, \quad m = 1, 2.$$

Here $\sigma_{1,2}$ are nondecreasing functions on $(a, b) \subset (0, \infty)$ such that

$$0 < \lambda_1 \leq 1, \quad 0 < \lambda_2 < 1, \quad \text{где} \quad \lambda_i = \int_{-\infty}^{\infty} K_i(x)dx = 2 \int_a^b \frac{1}{s} d\sigma_i(s), \quad i = 1, 2.$$

Keywords: the basic solution, Ambartsumian equation, Laplace transform, system of integral equations.

A.A. Vyshinskiy

APPROXIMATE STUDY OF MULTIPARAMETER BIFURCATIONS IN MODELS OF POPULATION DYNAMICS

Abstract. The paper presents a new general scheme for qualitative and approximate investigation of basic scenarios of local bifurcations in models of population dynamics. Necessary and sufficient conditions for bifurcation of the equilibrium state and the Andronov-Hopf systems for population dynamics are given.

Keywords: plate deflection, critical forces, bifurcation points, asymptotic formulas, equilibrium state.

V.F. Gilimshina, F.Kh. Mukminov

ON DECAY RATE OF SOLUTION TO DEGENERATING LINEAR PARABOLIC EQUATIONS

Abstract. Existence and uniqueness of the solution to a linear degenerating parabolic equation is established in unbounded domains by the method of Galerkin's approximations. The first and the third boundary-value conditions are considered. The upper estimate of the solution decay rate is established when $x \rightarrow \infty$ in view of the influence of higher-order coefficients of the equation. The upper estimate of the decay rate of the solution $t \rightarrow \infty$ depending on the geometry of the unbounded domain is proved as well.

Keywords: degenerating parabolic equation, decay rate of solution, upper estimates, existence of solution.

V.M.Kartak, V.V.Kartak

COMBINATORIAL COMPLEXITY OF A CERTAIN 1-DIMENSIONAL CUTTING STOCK PROBLEM

Abstract. The classical Cutting Stock Problem (1dCSP) is considered. It is known that 1CSP is at least NP-hard. In the present paper a combinatorial algorithm for its solution based on the Branch and Bound Method is described. We estimate the complexity of this algorithm presented for a class of problems that is called compact. The most difficult examples to solve by combinatorial algorithms are identified. This result is consistent with experimental data and could be used to generate difficult test problems, as well as for predicting the time of the algorithm.

Keywords: Cutting Stock Problem, Branch and Bound Method, Integer Programming, Combinatorial complexity.

L.M. Kozhevnikova, A.A. Leontiev

SOLUTION ESTIMATES FOR AN ANISOTROPIC DOUBLY NONLINEAR PARABOLIC EQUATION

Abstract. The first mixed problem with the Dirichlet homogeneous boundary-value condition and a finite initial function is considered for a certain class of second-order anisotropic doubly nonlinear parabolic equations in a cylindrical domain $D = (0, \infty) \times \Omega$. Upper estimates characterizing the dependence of the decay rate of the solution to the problem on geometry of an unbounded domain $\Omega \subset \mathbb{R}_n$, $n \geq 3$ are established when $t \rightarrow \infty$. Existence of strong solutions is proved by the method of Galerkin's approximations. The method of their construction for the modelling isotropic equation has been earlier offered by F.Kh. Mukminov, E.R. Andriyanova. The estimate of the admissible decay rate of the solution on an unbounded domain has been obtained on the basis of Galerkin's approximations. It proves the accuracy of the upper estimate.

Keywords: anisotropic equation, doubly nonlinear parabolic equations, existence of strong solution, decay rate of solution.

K.G. Malyutin, N.M. Sadik

THE INDICATOR OF A DELTA-SUBHARMONIC FUNCTION IN A HALF-PLANE

Abstract. Delta-subharmonic functions of a completely regular growth in the upper half-plane have been introduced in the joint work of the authors, published in Reports of the Russian Academy of Sciences (2001). In this work, criteria whether a delta-subharmonic function in the upper half-plane belongs to a class of functions of a completely regular growth have been obtained on the basis of the theory of Fourier coefficients of delta-subharmonic functions in the half-plane developed in the beginning of this century by the first author of the present article. The present paper is a natural continuation of this research. The concept of the indicator of a delta-subharmonic function of a completely regular growth in the upper half-plane is introduced. It is proved that the indicator of a delta-subharmonic function of a completely regular growth in the upper half-plane belongs to a class $L_p[0, \pi]$ ($1 < p \leq 2$). The proof is based on the lemma about Polya peaks and the Hausdorff-Young theorem.

Keywords: delta-subharmonic functions of a completely regular growth in the upper half-plane, Fourier coefficients, the indicator, Polya peaks, Hausdorff-Young theorem.

S.I. Mitrokhin

ON SPECTRAL PROPERTIES OF A DIFFERENTIAL OPERATOR WITH SUMMABLE COEFFICIENTS WITH A RETARDED ARGUMENT

Abstract. The paper considers spectral properties of differential operators of the sixth order with a retarded argument. It is supposed that coefficients of the operator are summable functions on a segment. One can study 36 kinds of boundary-valued conditions simultaneously by one method. The asymptotics of eigenvalues of the differential operator is also calculated.

Keywords: differential operator, summable coefficients, retarded argument, asymptotics of eigenvalues, asymptotics of solutions.

R.D. Murtazina

NONLINEAR HYPERBOLIC EQUATIONS WITH CHARACTERISTIC RING OF DIMENSION 3

Abstract. The paper provides a method of classification of Darboux integrable nonlinear hyperbolic equations $u_{xy} = f(u, u_x, u_y)$ based on investigation of the characteristic pairs of Lie rings. Constructive conditions on the right-hand side f of the equation with the characteristic ring of the dimension three are obtained. These equations possess second-order integrals. In particular, a list of equations satisfying the constructive conditions is given for the equation $u_{xy} = \varphi(u)\psi(u_1)h(\bar{u}_1)$. Formulae of x - and y -integrals are given for these equations.

Keywords: integrals, characteristic ring, vector fields

V.V. Napalkov (Jr.)

AN EQUIVALENT INTEGRAL NORM IN A DUAL SPACE

Abstract. In the present paper, the problem of describing a dual space in terms of the Hilbert transform is considered. We establish the necessary and sufficient conditions for the space $\tilde{B}_2(G, \mu)$ to possess an integral norm equivalent to the initial one. We find the form of this norm. Using the general result of this work, we specify the recent result of the author and R.S. Yulmukhametov. The method suggested in the paper is based on the theory of orthosimilar systems. This method can be used to solve the problem of describing a dual space in terms of the Fourier–Laplace transform and in terms of others complete system of functions.

Keywords: Hilbert transform, reproducing kernel, orthosimilar system, wavelet transform, integral frames

A.Yu. Trynin

DIFFERENTIAL PROPERTIES OF ZEROS OF EIGENFUNCTIONS OF THE STURM-LIOUVILLE PROBLEM

Abstract. Differential relations for functionals that set the k -th zero of the n -th eigenfunction in correspondence with the summable potential q of the Sturm-Liouville problem are obtained in terms of the Gato differentials.

Keywords: Differential properties, eigenfunction, Sturm-Liouville problem

A.B. Yakhshimuratov

INTEGRATING THE KORTEWEG-DE VRIES EQUATION WITH A SPECIAL FREE TERM IN THE CLASS OF PERIODIC FUNCTIONS

Abstract. In the present paper we use the method of the inverse spectral problem to integrate the Korteweg-de Vries equation with a free term independent of the spatial variable in the class of periodic functions.

Keywords: Sturm-Liouville operator, inverse spectral problem, the Dubrovin-Trubowitz system of equations, the Korteweg-de Vries equation.