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ONE-PARAMETRIC FAMILY OF POSITIVE SOLUTIONS FOR A CLASS OF NONLINEAR DISCRETE HAMMERSTEIN-VOLTERRA EQUATIONS

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Abstract. In the present work we study a class of of nonlinear discrete Hammerstein-Volterra equations in a post-critical case. We prove the existence of a one-parametric family of positive solutions in space l_1. We describe the set of parameters and establish the monotonic dependence of each solution both in a parameter and a corresponding index.

Keywords: post-criticity condition, iterations, monotonocity, one-parametric family of solutions.

Mathematics Subject Classification: 45GXX, 45G05

1. Introduction

The work is devoted to the study of the following class of nonlinear discrete Hammerstein-Volterra equations:

$$x_n = \sum_{j=n}^{\infty} a_{j-n} h_j(x_j), \quad n = 0, 1, 2, \dots \tag{1.1}$$

for an unknown infinite vector

$$x = (x_0, x_1, \dots, x_n, \dots)^T, \tag{1.2}$$

where T denotes the transposition.

In system (1.1) the sequence of elements $\{a_k\}_{k=0}^{\infty}$ satisfies the following conditions:

- $$a_k \geqslant 0, \quad k = 0, 1, 2, \dots, \quad a_0 = 0, \tag{1.3}$$
- $$\mu \equiv \sum_{k=0}^{\infty} a_k < +\infty, \tag{1.4}$$
- $$\text{(over-criticity condition)} \quad \mu > 1. \tag{1.5}$$

For sequence of measurable real functions $\{h_j(u)\}_{j=0}^{\infty}$ we assume the following "criticity" condition:

$$h_j(0) = 0, \quad j = 0, 1, 2, \dots. \tag{1.6}$$

Apart from an independent mathematical interest, system (1.1) arises in discrete problems of transport theory of nonlinear radiation in spectral lines (see [1]).

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Moreover, system (1.1) is a discrete analogue of the nonlinear Hammerstein-Volterra convolution equation:

$$f(x) = \int_{x}^{\infty} v(t-x)H(t, f(t))dt, \quad x \geqslant 0, \tag{1.7}$$

which arises in various fields of natural sciences, in particular, in physical kinetics (kinetic gas theory), in econometrics (theory of income distribution in one-product economy), in biology (in deterministic models of spatial epidemic distribution or of the distribution of auspicious gene in a population along the line with various nonlinearities in genetic models) (see [2]–[5]). Many interesting works were devoted to the studying of Hammerstein-Volterra equations (see [6]-[9] and the references therein). For instance, in [6]-[7], the following nonlinear discrete Hammerstein system was studied:

$$y_n = \sum_{j=1}^{\infty} a_{nj} f_j(y_j) + g_n, \quad n \in \mathbb{N}, \tag{1.8}$$

where

$$f_j(0) = 0, \quad j \in \mathbb{N},$$

and

$$(f_j(u) - f_j(v))(u - v) \leqslant c_f (u - v)^2, \quad j \in \mathbb{N},$$

for some $c_f > 0$ under the assumption

$$c_f \cdot \mu_0 < 1$$

and μ_0 is the smallest positive number satisfying the inequality

$$\|Ay\|_{l_{2,\tau}} \leqslant \mu_0 (Ay, y), \quad y \in l_{2,\tau}.$$

Here $l_{2,\tau}$ is some weighted space of infinite vectors and $A = (a_{nj})_{n,j=1}^{\infty}$.

In work [8], the following discrete Hammerstein-Volterra system was studied:

$$x_n = \sum_{j=n-N_0}^{n} a_{nj} h_j(x_j), \quad n \in \mathbb{N}, \tag{1.9}$$

for an infinite vector $x = (x_0, x_1, \ldots, x_n, \ldots)^T$. Under certain restrictions for $\{a_{nj}\}_{n,j=1}^{\infty}$ and $\{h_j(u)\}_{j=1}^{\infty}$, the existence of periodic solutions was proved in this work.

The issues of linearization for general nonlinear discrete Volterra equations were discussed in work [9].

It should be noted that condition (1.6) in some sense complicates the situation since it follows immediately from (1.6) that the zero vector satisfies system (1.1).

Here the following issues arise:

1) Under which conditions for $\{h_j(u)\}_{j=0}^{\infty}$, apart from the trivial solution, system (1.1) has a component-wise positive solution?

2) In which space is this solution?

3) Whether the constructed solution possesses the uniqueness property in certain class of infinite vectors with positive coordinates?

4) Whether there exists a one-parametric family of positive solutions?

5) If there exists a one-parametric family of solutions, which is the structure of the corresponding set of parameters?

In the present paper, under certain conditions for the sequence of functions $\{h_j(u)\}_{j=0}^{\infty}$ we prove the existence of one-parametric family of component-wise positive solutions. We establish that each such solution in this family belongs to space l_1. We describe the set of parameters. We prove the monotonous dependence of each solution w.r.t. both the parameter and the corresponding index. In the end of the work we provide particular examples of sequence of functions $\{h_j(u)\}_{j=0}^{\infty}$ satisfying the assumptions of formulated theorem. It should be mentioned that the formulated theorem is constructive since apart from appropriate apriori estimates, its proof involves the method of successive approximations.

We also mention that the approaches developed in the work allows us to continue successfully the studies for constructing one-parametric family of positive solutions in $L_1(0,\infty)$ of the corresponding nonlinear integral equation (1.7).

2. Formulation of theorem

Before we formulate the main result of the present work, we introduce some notations.

We consider the following function defined on segment $[0,1]$:

$$\chi(p)=\sum_{k=0}^{\infty}a_kp^k,\quad p\in[0,1],\tag{2.1}$$

where $\{a_k\}_{k=0}^{\infty}$ satisfy conditions (1.3)–(1.5). It follows from (1.3)–(1.5) that

- $\chi(0)=a_0=0,\quad \chi(1)=\mu>1,\quad \chi\in C[0,1],$ (2.2)
- $\chi(p)\ \uparrow\ \text{in}\ p\ \text{on}\ [0,1].$ (2.3)

Therefore, there exists a unique number $p_0>0$ such that $\chi(p_0)=1$. We fix this number and make the following assumptions for

$$\omega_j(u)\equiv h_j(u)-u,\quad j=0,1,2,\ldots :\tag{2.4}$$

I) there exists a number $\alpha>0$ such that for each fixed $j\in\mathbb{N}\cup\{0\}$ functions $\omega_j(u)\ \uparrow$ in u on $[\alpha p_0^j,+\infty)$,

II) $\omega_j\in C(\Omega_j)$, where $\Omega_j\equiv[\alpha p_0^j,+\infty)$, $\ j=0,1,2,\ldots,$

III) there exists $\sup\limits_{u\geqslant\alpha}\omega_j(u)\equiv\tau_j$, $j=0,1,2,\ldots,$ where $\{\tau_j\}_{j=0}^{\infty}$ is a sequence of positive numbers satisfying the condition

$$\sum_{j=0}^{\infty}j\tau_jp_0^{-j}<+\infty,\tag{2.5}$$

IV) $\omega_j(u)\geqslant 0$, $u\in\Omega_j$, $j=0,1,2,\ldots$

The following theorem holds true.

Theorem 1. *Suppose that sequence* $\{a_k\}_{k=0}^{\infty}$ *satisfies conditions* (1.3)–(1.5)*, while* $\{\omega_j(u)\}_{j=0}^{\infty}$ *possesses the properties* (2.4) *and* $I)-IV)$. *Then system* (1.1) *has a one-parametric family of component-wise positive solutions* $\{x_\gamma\}_{\gamma\in\Pi}$, $x_\gamma=(x_{0,\gamma},x_{1,\gamma},\ldots,x_{n,\gamma},\ldots)^T$, *and*

1) $x_\gamma\in l_1,\quad \forall\gamma\in\Pi\equiv[\alpha,+\infty)$,

2) if $\gamma_1,\gamma_2\in\Pi$ *and* $\gamma_1>\gamma_2$, *then the lower estimates*

$$x_{n,\gamma_1}-x_{n,\gamma_2}\geqslant(\gamma_1-\gamma_2)p_0^n,\quad \forall n\in\mathbb{N}\cup\{0\},\tag{2.6}$$

hold true.

3) if there exists a natural number N_0 *such that for each fixed* $u\geqslant 0$

$$\omega_{j+1}(u)\leqslant\omega_j(u),\quad j=N_0,N_0+1,N_0+2,\ldots,\tag{2.7}$$

then

$$x_{n+1,\gamma}\leqslant x_{n,\gamma},\quad n=N_0,N_0+1,N_0+2,\ldots,\tag{2.8}$$

$\forall\gamma\in\Pi$.

3. Proof of theorem

We begin with an auxiliary Volterra type discrete system

$$y_n = z_n + \sum_{j=n}^{\infty} a_{j-n} y_j, \quad n = 0, 1, 2, \ldots, \tag{3.1}$$

for an unknown infinite vector

$$y = (y_0, y_1, \ldots, y_n \ldots)^T, \tag{3.2}$$

where

$$z_n \equiv \sum_{j=n}^{\infty} a_{j-n} \tau_j, \quad n = 0, 1, 2, \ldots. \tag{3.3}$$

We multiply both sides of the system (3.1) by p_0^{-n} ($n \in \mathbb{N} \cup \{0\}$), and denoting

$$y_n^* \equiv p_0^{-n} y_n, \quad z_n^* \equiv p_0^{-n} z_n, \quad b_n \equiv p_0^n a_n, \quad n = 0, 1, 2, \ldots, \tag{3.4}$$

we arrive at the following system for $y^* = (y_0^*, y_1^*, \ldots, y_n^* \ldots)^T$:

$$y_n^* = z_n^* + \sum_{j=n}^{\infty} b_{j-n} y_j^*, \quad n = 0, 1, 2, \ldots. \tag{3.5}$$

Since $\chi(p_0) = 1$, it follows immediately from (3.4) that

$$\sum_{n=0}^{\infty} b_n = 1. \tag{3.6}$$

In what follows we shall make sure that

- $z^* \in l_1, \quad z^* = (z_0^*, z_1^*, \ldots, z_n^* \ldots)^T, \tag{3.7}$
- $\sum_{n=0}^{\infty} n z_n^* < +\infty. \tag{3.8}$

We observe that (3.7) is obviously implied by (3.8). This is why it is sufficient to prove (3.8). Taking into consideration (3.4) and (2.5), for each $N \in \mathbb{N}$ we estimate a partial sum of series (3.8):

$$\begin{aligned}
\sum_{j=0}^{N} j z_j^* &= \sum_{j=0}^{N} j p_0^{-j} \sum_{i=j}^{\infty} a_{i-j} \tau_i \leqslant \sum_{j=0}^{N} \sum_{i=j}^{\infty} a_{i-j} i p_0^{-i} \tau_i = \sum_{j=0}^{N} \sum_{i=j}^{N} a_{i-j} i p_0^{-i} \tau_i + \sum_{j=0}^{N} \sum_{i=N+1}^{\infty} a_{i-j} i p_0^{-i} \tau_i \\
&= \sum_{i=0}^{N} i p_0^{-i} \tau_i \sum_{j=0}^{i} a_{i-j} + \sum_{i=N+1}^{\infty} i p_0^{-i} \tau_i \sum_{j=0}^{N} a_{i-j} \leqslant \sum_{i=0}^{N} i p_0^{-i} \tau_i \sum_{j=0}^{i} a_{i-j} + \sum_{i=N+1}^{\infty} i p_0^{-i} \tau_i \sum_{j=0}^{i} a_{i-j} \\
&= \sum_{i=0}^{N} i p_0^{-i} \tau_i \sum_{m=0}^{i} a_m + \sum_{i=N+1}^{\infty} i p_0^{-i} \tau_i \sum_{m=0}^{i} a_m \leqslant \mu \left(\sum_{i=0}^{N} i p_0^{-i} \tau_i + \sum_{i=N+1}^{\infty} i p_0^{-i} \tau_i \right) \\
&= \mu \sum_{i=0}^{\infty} i p_0^{-i} \tau_i < +\infty.
\end{aligned}$$

Since $N \in \mathbb{N}$ is arbitrary, and $z_n^* \geqslant 0$, $n \in \mathbb{N} \cup \{0\}$, the obtained estimate implies (3.8).

Thus, we have obtained that free term z^* of system (3.5) and sequence $\{b_n\}_{n=0}^{\infty}$ satisfy respectively conditions (3.8), (3.7) and (3.6). Therefore, the results of work [10, Lm. 4.8] yield that system (3.5) has a component-wise solution in space l_1.

It follows from (3.4) that

$$y_n = p_0^n \cdot y_n^*, \quad n = 0, 1, 2, \ldots, \tag{3.9}$$

is a solution to system (3.1). Since $y^* \in l_1$ and $p_0 \in (0,1)$, by (3.9) we obtain

$$y = (y_0, y_1, \dots, y_n, \dots)^T \in l_1. \tag{3.10}$$

Now for main system (1.1) we introduce the following iterations:

$$x_{n,\gamma}^{(m+1)} = \sum_{j=n}^{\infty} a_{j-n} h_j(x_{j,\gamma}^{(m)}), \quad x_{n,\gamma}^{(0)} = \gamma p_0^n, \quad n = 0,1,2,\dots, \quad m = 0,1,2,\dots, \quad \gamma \in \Pi. \tag{3.11}$$

Let us prove by induction by m that

$$A)\ \ x_{n,\gamma}^{(m)} \ \uparrow \ \text{in} \ m, \quad \forall \gamma \in \Pi, \quad \forall n \in \mathbb{N} \cup \{0\},$$

$$B)\ \ x_{n,\gamma}^{(m)} \leqslant \gamma p_0^n + y_n, \quad \forall m \in \mathbb{N} \cup \{0\}, \quad \forall \gamma \in \Pi, \quad \forall n \in \mathbb{N} \cup \{0\}.$$

We first prove the monotonicity of sequence $\{x_{n,\gamma}^{(m)}\}_{m=0}^{\infty}$ in m. Indeed, by the monotonicity of $\{\omega_j(u)\}_{j=0}^{\infty}$ in u on $[\alpha p_0^j, +\infty)$, $j = 0,1,2,\dots$, in view of Condition IV) of the theorem, thanks to (3.11) we have

$$\begin{aligned} x_{n,\gamma}^{(1)} &= \sum_{j=n}^{\infty} a_{j-n}(x_{j,\gamma}^{(0)} + \omega_j(x_{j,\gamma}^{(0)})) \geqslant \gamma \sum_{j=n}^{\infty} a_{j-n} p_0^j \\ &= \gamma \sum_{i=0}^{\infty} a_i p_0^{n+i} = \gamma p_0^n \chi(p_0) = \gamma p_0^n = x_{n,\gamma}^{(0)}. \end{aligned}$$

Assuming that

$$x_{n,\gamma}^{(m)} \geqslant x_{n,\gamma}^{(m-1)}$$

for some $m \in \mathbb{N}$, $n \in \mathbb{N} \cup \{0\}$, $\gamma \in \Pi$ and taking into account the monotonicity of $\omega_j(u)$ in u, by (3.11) we obtain

$$x_{n,\gamma}^{(m+1)} \geqslant \sum_{j=n}^{\infty} a_{j-n}(x_{j,\gamma}^{(m-1)} + \omega_j(x_{j,\gamma}^{(m-1)})) = x_{n,\gamma}^{(m)}.$$

Let us prove inequalities B). As $m = 0$, it is obvious since $y_n \geqslant 0$, $n = 0,1,2,\dots$ We assume that B) is satisfied for some $m \in \mathbb{N}$. Then, taking into consideration I), III) and IV), by (3.11) we get

$$\begin{aligned} x_{n,\gamma}^{(m+1)} &\leqslant \sum_{j=n}^{\infty} a_{j-n}(\gamma p_0^j + y_j + \omega_j(\gamma p_0^j + y_j)) \leqslant \sum_{j=n}^{\infty} a_{j-n}(\gamma p_0^j + y_j + \omega_j(\gamma + y_j)) \\ &\leqslant \sum_{j=n}^{\infty} a_{j-n}(\gamma p_0^j + y_j + \tau_j) = \gamma \sum_{j=n}^{\infty} a_{j-n} p_0^j + \sum_{j=n}^{\infty} a_{j-n} y_j + z_n = \gamma p_0^n + y_n. \end{aligned}$$

It follows from A) and B) that for each fixed $\gamma \in \Pi$ the sequence of infinite vectors $\{x_\gamma^{(m)}\}_{m=0}^{\infty}$, $x_\gamma^{(m)} = (x_{0,\gamma}^{(m)}, x_{1,\gamma}^{(m)}, \dots, x_{n,\gamma}^{(m)}, \dots)^T$, has a limit if and only if $m \to \infty$: $\lim\limits_{m\to\infty} x_\gamma^{(m)} = x_\gamma$, and in view of Condition II) and the fact

$$\sup_{n \in \mathbb{N}\cup\{0\}} \sum_{j=n}^{\infty} a_{j-n}(x_{j,\gamma} + \omega_j(x_{j,\gamma})) \leqslant \gamma + \sup_{n \in \mathbb{N}\cup\{0\}} y_n < +\infty,$$

the limiting vector satisfies system (1.1). It also follows from A) and B) that

$$\gamma p_0^n \leqslant x_{n,\gamma} \leqslant \gamma p_0^n + y_n, \quad \gamma \in \Pi, \quad n \in \mathbb{N} \cup \{0\}.$$

Let us prove inequality (2.6). In order to do it, by induction in m we first make sure that if $\gamma_1, \gamma_2 \in \Pi$, $\gamma_1 > \gamma_2$, then

$$x_{n,\gamma_1}^{(m)} - x_{n,\gamma_2}^{(m)} \geqslant (\gamma_1 - \gamma_2) p_0^n, \quad n = 0,1,2,\dots, \quad m = 0,1,2,\dots. \tag{3.12}$$

In the case $m = 0$, inequality (3.12) is obviously true since it becomes the identity. Suppose that (3.12) holds true for some $m \in \mathbb{N}$. Then by the monotonicity of $\omega_j(u)$ in u on $[\alpha p_0^j, +\infty)$, $j = 0, 1, 2, \dots$ and $\gamma_i \geqslant \alpha$, $i = 1, 2,$, we get

$$x_{n,\gamma_1}^{(m+1)} - x_{n,\gamma_2}^{(m+1)} = \sum_{j=n}^{\infty} a_{j-n}(x_{j,\gamma_1}^{(m)} - x_{j,\gamma_2}^{(m)} + \omega_j(x_{j,\gamma_1}^{(m)}) - \omega_j(x_{j,\gamma_2}^{(m)})) \geqslant \sum_{j=n}^{\infty} a_{j-n}(x_{j,\gamma_1}^{(m)} - x_{j,\gamma_2}^{(m)})$$

$$\geqslant (\gamma_1 - \gamma_2) \sum_{j=n}^{\infty} a_{j-n} p_0^j = (\gamma_1 - \gamma_2) p_0^n \cdot \chi(p_0) = (\gamma_1 - \gamma_2) p_0^n.$$

Passing to limit as $m \to \infty$ in (3.12), we arrive at (2.6).

To complete the proof of the theorem, it remains to make sure that condition (2.7) implies inequality (2.8).

We first prove that under condition (2.7) we have

$$x_{n+1,\gamma}^{(m)} \leqslant x_{n,\gamma}^{(m)}, \quad n = 0, 1, 2, \dots, \quad m = 0, 1, 2, \dots, \quad \gamma \in \Pi. \tag{3.13}$$

As $m = 0$, it is implied by the following simple inequality:

$$x_{n+1,\gamma}^{(0)} = \gamma p_0^{n+1} \leqslant \gamma p_0^n = x_{n,\gamma}^{(0)}.$$

Suppose that (3.13) is satisfied for some $m \in \mathbb{N}$. Then in view of (2.7), the monotonicity of $\omega_j(u)$ in u on $[\alpha p_0^j, +\infty)$, $j = 0, 1, 2, \dots$, by (3.11) we obtain

$$\begin{aligned} x_{n+1,\gamma}^{(m+1)} - x_{n,\gamma}^{(m+1)} &= \sum_{j=n+1}^{\infty} a_{j-(n+1)}(x_{j,\gamma}^{(m)} + \omega_j(x_{j,\gamma}^{(m)})) - \sum_{j=n}^{\infty} a_{j-n}(x_{j,\gamma}^{(m)} + \omega_j(x_{j,\gamma}^{(m)})) \\ &= \sum_{k=0}^{\infty} a_k(x_{k+n+1,\gamma}^{(m)} + \omega_{k+n+1}(x_{k+n+1,\gamma}^{(m)})) - \sum_{k=0}^{\infty} a_k(x_{k+n,\gamma}^{(m)} + \omega_{k+n}(x_{k+n,\gamma}^{(m)})) \\ &= \sum_{k=0}^{\infty} a_k(x_{k+n+1,\gamma}^{(m)} - x_{k+n,\gamma}^{(m)} + \omega_{k+n+1}(x_{k+n+1,\gamma}^{(m)}) - \omega_{k+n}(x_{k+n,\gamma}^{(m)})) = I_1 + I_2 + I_3, \end{aligned}$$

where

$$I_1 \equiv \sum_{k=0}^{\infty} a_k(x_{k+n+1,\gamma}^{(m)} - x_{k+n,\gamma}^{(m)}) \leqslant 0$$

by the induction assumption,

$$I_2 \equiv \sum_{k=0}^{\infty} a_k(\omega_{k+n+1}(x_{k+n+1,\gamma}^{(m)}) - \omega_{k+n+1}(x_{k+n,\gamma}^{(m)})) \leqslant 0,$$

since $\omega_j(u) \uparrow$ w.r.t. u on $[\alpha p_0^j, +\infty)$, $\;j = 0, 1, 2, \dots$, and by the induction assumption, while

$$I_3 \equiv \sum_{k=0}^{\infty} a_k(\omega_{k+n+1}(x_{k+n,\gamma}^{(m)}) - \omega_{k+n}(x_{k+n,\gamma}^{(m)})) \leqslant 0$$

by condition (2.7).

Therefore,

$$x_{n+1,\gamma}^{(m+1)} \leqslant x_{n,\gamma}^{(m+1)}, \quad n = 0, 1, 2, \dots, \quad \gamma \in \Pi.$$

Passing to the limit as m goes to infinity in (3.13), we arrive at (2.8). It completes the proof of the theorem.

In conclusion we provide some examples of sequence $\{\omega_j(u)\}_{j=0}^{\infty}$ satisfying all the assumptions of the formulated theorem:

$$a)\ \ \omega_j(u) = p_0^{2j}(1-e^{-u}),\quad j=0,1,2,\dots,\quad u\geqslant 0,$$

$$b)\ \ \omega_j(u) = p_0^{2j}\frac{u}{u+c},\quad \forall c>0,\quad j=0,1,2,\dots,\quad u\geqslant 0,$$

$$c)\ \ \omega_j(u) = p_0^{2j}\frac{u^q}{u^q+c},\quad \forall c>0,\quad \forall q>2,\quad j=0,1,2,\dots,\quad u\geqslant 0,$$

$$d)\ \ \omega_j(u) = p_0^{2j}\frac{u+sin^2u}{u+sin^2u+1},\quad j=0,1,2,\dots,\quad u\geqslant 0.$$

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