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OPTIMAL SYSTEM FOR NON-SIMILAR SUBALGEBRAS OF SUM OF TWO IDEALS

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Abstract. We consider a twelve-dimensional Lie algebra L_{12} admitted by the gas dynamic equations with equation of state of a special form. Lie algebra L_{12} is a direct sum of two ideals L_{11} and Y_1. For Lie algebra L_{11} admitted by gas dynamic equations with an arbitrary equation of state, the optimal system of non-similar subalgebras is built up to inner automorphisms. Using the optimal system for Lie algebra L_{11}, in the article we obtain an optimal system for non-similar subalgebras of the sum of two ideals for L_{11} and Y_1 and the rule of construction of such subalgebras.

Keywords: Lie algebra, optimal system, gas dynamics.

Mathematics Subject Classification: 35Q99

1. Introduction

Gas dynamics equations (GDE) read as [1]

$$\begin{gathered} \rho_t + (\vec{u}\cdot\nabla)\rho + \rho\,\mathrm{div}\vec{u} = 0, \\ \vec{u}_t + (\vec{u}\cdot\nabla)\vec{u} + \rho^{-1}\nabla p = 0, \\ S_t + (\vec{u}\cdot\nabla)S = 0. \end{gathered} \tag{1}$$

Here $\vec{u}$ is a velocity, ρ is a density, S is an entropy, p is a pressure. They related by the equation of state $p = f(\rho, S)$.

Equations (1) with a general equation of state admit maximal Lie algebra L_{11}, for which the optimal system of non-similar subalgebras was constructed in [1]. For the Lie algebras admitted by GDE with equation of state of special form [2], the optimal system are not constructed for each algebra. They are not known for Lie algebras $L_{11} \oplus \{Y_1\}$, $L_{11} \oplus \{Y_1, Y_p\}$ and $L_{11} \oplus \{Y_1, Y_p, Y_{p^2}\}$, where $Y_1 = \partial_p$, $Y_p = \rho\,\partial_\rho + p\,\partial_p$, $Y_{p^2} = 2\rho p\,\partial_p + p^2\partial_p$. In the present work we construct an optimal system of non-similar subalgebras for $L_{11} \oplus \{Y_1\}$, which is the sum of two ideals.

Gas dynamics equation with an arbitrary equation of state are invariant under the action of group G_{11}, which is the Galilean group extended by the uniform dilatation:

1^o. $\vec{x}^{\,'} = \vec{x} + \vec{a}$ (space translations),
2^o. $t^{'} = t + a_o$ (time translation),
3^o. $\vec{x}^{\,'} = O\vec{x}, \vec{u}^{\,'} = O\vec{u}, OO^T = 1, \det O = 1$ (rotations),
4^o. $\vec{x}^{\,'} = \vec{x} + t\vec{b}, \vec{u}^{\,'} = \vec{u} + \vec{b}$ (Galilean translations),
5^o. $t^{'} = tc, \vec{x}^{\,'} = c\vec{x}$ (uniform dilatation).

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To group G_{11}, Lie algebra L_{11} corresponds with a basis of operators written in the Cartesian coordinate system

$$\begin{gathered}
X_1 = \partial_x, \quad X_2 = \partial_y, \quad X_3 = \partial_z, \\
X_4 = t\partial_x + \partial_u, \quad X_5 = t\partial_y + \partial_\nu, \quad X_6 = t\partial_z + \partial_w, \\
X_7 = y\partial_z - z\partial_y + \nu\partial_w - w\partial_\nu, \\
X_8 = z\partial_x - x\partial_z + w\partial_u - u\partial_w, \\
X_9 = x\partial_y - y\partial_x + u\partial_\nu - \nu\partial_u, \quad X_{10} = \partial_t, \\
X_{11} = t\partial_t + x\partial_x + y\partial_y + z\partial_z.
\end{gathered} \tag{2}$$

The commutators of basis operators are given in Table 1, where instead of operators X_i, $i = \overline{1, 11}$, we simply write indices i:

Table 1

	1	2	3	4	5	6	7	8	9	10	11
1								−3	2		1
2							3		−1		2
3							−2	1			3
4								−6	5	−1	
5							6		−4	−2	
6							−5	4		−3	
7		−3	2		−6	5		−9	8		
8	3		−1	6		−4	9		−7		
9	−2	1		−5	4		−8	7			
10				1	2	3					10
11	−1	−2	−3							−10	

2. Optimal system of non-similar subalgebras L_{12}

If the equation of state is of general form, then the maximal Lie algebra admitted by equations (1) is L_{11}. For special equations of state there appear additional operators extending the admissible algebra to L_k with k being the dimension of the algebra. Non-isomorphic algebras are given in work [3]. For almost all algebras the optimal systems of non-similar subalgebras were constructed. It remained to construct the optimal systems for Lie algebras being the direct sum of two ideals (3 cases). Here we construct the optimal system of subalgebras for 12-dimensional Lie algebras being isomorphic one to another. These are Lie algebra no. 9: $L_{11} \oplus Y_1$ with the equation of state $p = f(\rho) + g(S)$ and no. 3: $L_{11} \oplus Y_{\overline{p}}$ with the equation of state $\overline{p} = \overline{\rho}\,\overline{f}(\overline{g}(S)\overline{\rho})$, where $Y_1 = \partial_p$, $Y_{\overline{p}} = \overline{\rho}\,\partial_{\overline{\rho}} + \overline{p}\,\partial_{\overline{p}}$. These algebras are equivalent. Indeed, under the change $\rho = \dfrac{\overline{p}}{\overline{\rho}}$, $p = \ln \overline{p}$ we have $Y_1 = Y_{\overline{p}}$, while the operators in L_{11} remain unchanged. By the change, the equation of state implies the identity $\ln(\overline{\rho}\,\overline{f}(\overline{g}(S)\overline{\rho})) = f(\overline{f}(\overline{g}(S)\overline{\rho})) + g(S)$. The change $\tau = \overline{\rho}\,\overline{g}(S)$ yields the identity $-f(\overline{f}(\tau)) + \ln(\tau\overline{f}(\tau)) = \ln(\overline{g}(S)) + g(S)$, where variables τ and S separate. One can assume that both the sides of the identity vanish: $g(S) = -\ln(\overline{g}(S))$; $\ln(\tau\overline{f}(\tau)) = f(\overline{f}(\tau))$. Therefore, functions $\overline{g}$, $\overline{f}$ are determined in terms of functions g, f, i.e. the equations of state are consistent. At that, in system (1) only the first equation changes $D \ln \overline{\rho} = (1 + \rho f'(\rho)) D \ln \rho$, if $f(\rho)$ is variable.

The commutator of Y_1 with all the operators X_i, $i = \overline{1, 11}$, vanishes. Thus, algebra L_{12} is the direct sum of two ideals $L_{12} = L_{11} \oplus \{Y_1\}$. Then we list the subalgebras of various dimensions by means of known subalgebras L_{11} [4, Appendix]. At that we make use of internal automorphisms appearing while solving the initial problem for the linear equation $X_a^{'} = [X^{'}, Y]$, $X^{'}\mid_{a=0} = X$, $Y = X_i, Y_1$, $i = \overline{1, 11}$ [2] in Lie algebra L_{12}, where $X = x^i X_i + y^0 Y_1$, $X' = x^i{}' X_i + y^0{}' Y_1$. All the internal automorphisms are compactly written in Table 2.

Table 2

T	$p_1(x^{'}) = p_1(x) + x^{11}\vec{\alpha_1} - \vec{\alpha_1} \times p_3(x)$
	$p_1(x^{'}) = p_1(x) - x^{10}\vec{\alpha_2}, p_2(x^{'}) = p_2(x) - \vec{\alpha_2} \times p_3(x)$
O	$p_1(x^{'}) = Op_1(x), p_2(x^{'}) = Op_2(x), p_3(x^{'}) = Op_3(x)$
A_{10}	$p_1(x^{'}) = p_1(x) + a_{10}p_2(x), x^{10'} = x^{10} + a_{10}x^{11}$
A_{11}	$p_1(x^{'}) = a_{11}p_1(x), x^{10'} = a_{11}x^{10}$
B_1	$y^{o'} = b_1 y^o$
ε_1	$p_1(x^{'}) = -p_1(x), p_2(x^{'}) = -p_2(x)$
ε_2	$p_2(x^{'}) = -p_2(x), x^{10'} = -x^{10}$

where $\vec{\alpha_1} = (a_1, a_2, a_3)$, $\vec{\alpha_2} = (a_4, a_5, a_6)$, a_{10}, a_{11}, b_1 are the parameters of automorphisms, O is the rotation matrix defined by the rotation angles around one of orthogonal axes, and ε_1, ε_2 are noticed discrete automorphisms.

We deduce the rule of construction of n-dimensional non-similar subalgebras in $L_{12} = L_{11} \oplus Y_1$, where $n = 2, 3, \ldots, 12$, by means of internal automorphisms.

A subalgebra of dimension n in L_{12} is determined by the basis $\alpha_1 Y_1 + Z_1$, $\alpha_2 Y_1 + Z_2$, $\ldots$, $\alpha_n Y_1 + Z_n$. We can assume that $\alpha_1 \neq 0$, i.e., subalgebra in L_{12} essential. Then automorphism B_1 makes $\alpha_1 = 1$. Subtracting operator $Y_1 + Z_1$ multiplying by an appropriate coefficient from the others gives $\alpha_2 = \ldots = \alpha_n = 0$.

Operator Z_1 is defined up to a linear combination of operators Z_2, $\ldots$, Z_n. The commutators of basis operators of subalgebras $Z_1 + Y_1, Z_2, \ldots, Z_n$ are

$$[Z_1 + Y_1, Z_j] = \sum_{k=2}^{n} c_{1j}^k \cdot Z_k = [Z_1, Z_j] \tag{3}$$

since $[Y_1, Z_j] = 0$; $[Z_j, Z_k] = \sum_{l=2}^{n} c_{jk}^l \cdot Z_l$, $k = 2, \ldots, n$.

Thus, $Z_2, \ldots, Z_n$ is an ideal of dimension $n - 1$ in subalgebra $Y_1 + Z_1, Z_2, \ldots, Z_n \subset L_{12}$ and is an ideal in subalgebra $Z_1, \ldots, Z_n \subset L_{11}$ if $Z_1 \neq 0$. Thus, we can list subalgebras in L_{12} as follows. We choose subalgebras $Z_2, \ldots, Z_n$ in the optimal system L_{11}. Then we add operator $Z_1 + Y_1$ to the basis ones, where we subtract a linear combination of operators $Z_2, \ldots, Z_n$ from Z_1. The form of operator Z_1 is specified by calculating the commutator by formula (3). The simplest form for Z_1 is obtained by internal automorphisms preserving operators $Z_2, \ldots, Z_n$.

Remark 1. Once $Z_1 = 0$, it is the trivial subalgebra of L_{12}. We do not add such subalgebras in Table 3.

Let us show how one can calculate a subalgebra in the optimal system for L_{12}. To the subalgebra in L_{11} with the number 3.35, the basis $a1 + 4$, $b3 + 5$, $b2 - 6$, $a^2 + b^2 = 1$, in accordance with the calculation rule we add the operator $Y_1 + x_1 X^1 + \ldots + x_{11} X^{11}$ from which we subtract a linear combination of the operators $a1+4, b3+5, b2-6$. We find the commutators:

$$\begin{aligned}
[a1 + 4, Y_1 + x_1 X^1 + \ldots + x_{11} X^{11}] =& - ax^8 X_3 + ax^9 X_2 + ax^{11} X_1 - x^8 X_6 + x^9 X_5 - x^{10} X_1 \\
=& \lambda_1(a1 + 4) + \mu_1(b3 + 5) + \gamma_1(b2 - 6) \\
[b3 + 5, Y_1 + x_1 X^1 + \ldots + x_{11} X^{11}] =& - bx^7 X_2 + bx^8 X_1 + bx^{11} X_3 + x^7 X_6 - x^9 X_4 - x^{10} X_2 \\
=& \lambda_2(a1 + 4) + \mu_2(b3 + 5) + \gamma_2(b2 - 6) \\
[b2 - 6, Y_1 + x_1 X^1 + \ldots + x_{11} X^{11}] =& bx^7 X_3 - bx^9 X_1 + bx^{11} X_2 + x^7 X_5 - x^8 X_4 + x^{10} X_3 \\
=& \lambda_3(a1 + 4) + \mu_3(b3 + 5) + \gamma_3(b2 - 6).
\end{aligned}$$

Comparing the coefficients at the like basis operators implies the system of equations: $ax^{11} = 0; ax^9 = bx^8; -ax^8 = bx^9; bx^8 = -ax^9; -bx^7 - x^{10} = -bx^7; bx^{11} = 0;\ -bx^9 = -ax^8; x^{10} = 0; a^2 + b^2 = 1$. Its solution reads as $x^8 = x^9 = x^{10} = x^{11} = 0$. Thus, the subalgebra becomes $a1 + 4$, $b3 + 5$, $b2 - 6$, $Y_1 + x^1X_1 + x^2X_2 + x^3X_3 + x^7X_7$. The internal automorphisms simplify the form of the subalgebra.

If $x^7 \neq 0$, then automorphisms T, A_{11} (Table 2) imply $a1 + 4$, $b3 + 5$, $b2 - 6$, $Y_1 + \varepsilon 1 + 7$, $a^2 + b^2 = 1$. We obtain subalgebra 4.42 in Table 3. If $x^7 = 0$, it yields $a1 + 4$, $b3 + 5$, $b2 - 6$, $Y_1 + c1 + d2 + e3$. Automorphism O rotates vectors $\begin{Vmatrix}0\\b\end{Vmatrix}, \begin{Vmatrix}1\\0\end{Vmatrix}, \begin{Vmatrix}b\\0\end{Vmatrix}, \begin{Vmatrix}0\\-1\end{Vmatrix}, \begin{Vmatrix}d\\e\end{Vmatrix}$ simultaneously around axes x^1 and x^4 by the angle φ. Angle φ can be chosen so the coefficient at 3 vanishes ($\tan\varphi = \dfrac{-e}{d}$) after the transformation, while by a change of basis the second and the third operator cast into the original form. As a result we obtain the subalgebra $a1+4$, $b3+5$, $b2-6$, $Y_1 + c1 + d2$, $a^2 + b^2 = 1$, $c^2 + d^2 = 1$ with the number 4.43 in Table 3.

Theorem 1. *All non-similar nontrivial subalgebras of L_{12} are given in Table 3, where r is the dimension of a subalgebra, i is the index of a subalgebra of a given dimension, and in the last two columns we provide the number of subalgebras in L_{11} by which we construct a subalgebra in L_{12}. At that, if we neglect the operator involving Y_1, we obtain a subalgebra of the dimension less by one in the optimal system for L_{11}.*

Table 3

Optimal system of non-similar subalgebras for L_{12}

r	i	Basis of subalgebra	i (L_{11})	
			r–1, i	r, i
2	1	$b4 + c7 + 11$, $Y_1 + a4 + 7$	1.1	2.1
	2	$a4 + 7$, $Y_1 + b4 + 11$	1.2, 1.3	2.1
	3	10, $Y_1 + 7 + a11$	1.10	2.2, 2.5
	4	$4 + 10$, $Y_1 + a1 + 7$	1.9	2.3
	5	$7 + c(4 + 10)$, $Y_1 + a1 + 4 + 10$	1.5	2.3
	6	$1 + 7$, $Y_1 + 10$	1.4	2.4
	7	10, $Y_1 + 1 + 7$	1.10	2.4
	8	$7 + \varepsilon 10$, $Y_1 + 10$; $\varepsilon = 0 \vee 1$	1.3, 1.6	2.5
	9	10, $Y_1 + 7$	1.10	2.5
	10	10, $Y_1 + 11$	1.10	2.6
	11	$b4 + 7 + a11$, $Y_1 + 4$; $a \neq 0$	1.1	2.7
	12	4, $Y_1 + 7 + a11$	1.12	2.7, 2.10
	13	1, $Y_1 + b4 + 7 + a11$; $a \neq 0$	1.13	2.8
	14	$\varepsilon 1 + 7$, $Y_1 + 4$	1.3, 1.4	2.9, 2.10
	15	4, $Y_1 + \varepsilon 1 + 7$	1.12	2.9, 2.10
	16	$\varepsilon 1 + 7$, $Y_1 + 1$	1.3, 1.4	2.11
	17	$a4 + 7$, $Y_1 + 1$, $a \neq 0$	1.2	2.11
	18	$a4 + 7 + 10$, $Y_1 + 1$	1.5	2.12
	19	1, $Y_1 + a4 + 7 + \varepsilon 10$	1.13	2.11, 2.12
	20	$a4 + 11$, $Y_1 + 5$	1.7	2.13, 2.14
	21	4, $Y_1 + a5 + 11$	1.12	2.14
	22	1, $Y_1 + a4 + b5 + 11$	1.13	2.15
	23	10, $Y_1 + 1$	1.10	2.17
	24	1, $Y_1 + 10$	1.13	2.17

	25	$4+10,\ Y_1+a1+3$	1.9	2.18
	26	$3,\ Y_1+4+a6+10$	1.13	2.18
	27	$4+10,\ Y_1+1$	1.9	2.19
	28	$1,\ Y_1+4+10$	1.13	2.19
	29	$a1+c3+5,\ Y_1+b1+d2+6;\ a^2+b^2+(c+d)^2=1$	1.11	2.20
	30	$3+5,\ Y_1+2-6$	1.11	2.21
	31	$5,\ Y_1+6$	1.12	2.22
	32	$3+4,\ Y_1+2$	1.11	2.23
	33	$a1+2,\ Y_1+3+4$	1.13	2.23
	34	$b2+4,\ Y_1+a1+2$	1.11, 1.12	2.24
	35	$a1+2,\ Y_1+4$	1.13	2.24
	36	$3+4,\ Y_1+1$	1.11	2.25
	37	$1,\ Y_1+3+4$	1.13	2.25
	38	$4,\ Y_1+1$	1.12	2.26
	39	$1,\ Y_1+4$	1.13	2.26
	40	$a2+3,\ Y_1+2$	1.13	2.27
3	1	$a4+7,\ b4+11,\ Y_1+4$	2.1	3.3
	2	$10,\ 7+a11,\ Y_1+11$	2.2, 2.5	3.2
	3	$a1+7,\ 4+10,\ Y_1+1$	2.3	3.6
	4	$1+7,\ 10,\ Y_1+1$	2.4	3.5
	5	$7,\ 10,\ Y_1+1$	2.5	3.5
	6	$10,\ 11,\ Y_1+7$	2.6	3.2
	7	$4,\ 7+a11,\ Y_1+11$	2.7, 2.10	3.3
	8	$1,\ b4+7+a11,\ Y_1+c4+d11,\ a\neq 0,\ c^2+d^2=1$	2.8	3.4, 3.12
	9	$4,\ \varepsilon 1+7,\ Y_1+1$	2.9, 2.10	3.13
	10	$1,\ a4+7,\ Y_1+b4+11$	2.11	3.4
	11	$1,\ a4+7,\ Y_1+b4+c10,\ b^2+c^2=1$	2.11	3.5, 3.6, 3.13
	12	$1,\ a4+7+10,\ Y_1+b4+c10,\ b^2+c^2=1$	2.12	3.5, 3.6, 3.18
	13	$4,\ 11,\ Y_1+5$	2.13	3.21
	14	$4,\ 11,\ Y_1+7$	2.13	3.3
	15	$4,\ a5+11,\ Y_1+b5+c6,\ a\neq 0,\ b^2+c^2=1$	2.14	3.21
	16	$1,\ a4+b5+11,\ Y_1+c4+d5+e6,\ c^2+d^2+e^2=1$	2.15, 2.16	3.22, 3.23, 3.24
	17	$1,\ a4+11,\ Y_1+c4+7$	2.15, 2.16	3.4
	18	$1,\ 10,\ Y_1+a2+b4,\ a^2+b^2=1$	2.17	3.28, 3.29, 3.33
	19	$1,\ 10,\ Y_1+a4+7$	2.17	3.5
	20	$3,\ 4+a6+10,\ Y_1+b1+c2+d6,\ b^2+c^2+d^2=1$	2.18	3.27, 3.30, 3.31, 3.32
	21	$1,\ 4+10,\ Y_1+a2+b4,\ a^2+b^2=1$	2.19	3.28, 3.29, 3.31

22	$1,\ 4+10,\ Y_1+b4+7$	2.19	3.6
23	$a1+c3+5,\ b1+d2+6,\ Y_1+e1+f3+\varepsilon 4,\ a^2+b^2+(c+d)^2=1,\ (e^2+f^2=1,\ \varepsilon=0)$	2.20	3.34
24	$3+5,\ 2-6,\ Y_1+a4+7,\ a\neq 0$	2.21	3.9
25	$3+5,\ 2-6,\ Y_1+\varepsilon 1+7$	2.21	3.9
26	$3+5,\ 2-6,\ Y_1+a1+b2,\ a^2+b^2=1$	2.21	3.37, 3.40
27	$3+5,\ 2-6,\ Y_1+a2+4$	2.21	3.34, 3.35
28	$5,\ 6,\ Y_1+a4+11$	2.22	3.21
29	$5,\ 6,\ Y_1+a2+4$	2.22	3.35, 3.36
30	$a1+2,\ 3+4,\ Y_1+b1+c3+d5+e6,\ b^2+c^2+d^2+e^2=1$	2.23	3.37, 3.44
31	$a1+2,\ 4,\ Y_1+b5+c6+11$	2.24	3.22
32	$a1+2,\ 4,\ Y_1+d1+e3+f5+h6,\ d^2+e^2+f^2+h^2=1$	2.24	3.37, 3.38, 3.39, 3.41, 3.42, 3.45
33	$1,\ 3+4,\ Y_1+a5+b6+10$	2.25	3.27, 3.28
34	$1,\ 3+4,\ Y_1+a2+b5+6$	2.25	3.37
35	$1,\ 3+4,\ Y_1+a3+5$	2.25	3.37
36	$1,\ 3+4,\ Y_1+a2+b3,\ a^2+b^2=1$	2.25	3.44
37	$1,\ 4,\ Y_1+7+a10+b11,\ a\cdot b=0$	2.26	3.12, 3.13, 3.18
38	$1,\ 4,\ Y_1+a5+11$	2.26	3.23, 3.24
39	$1,\ 4,\ Y_1+a5+10$	2.26	3.27, 3.29
40	$1,\ 4,\ Y_1+a3+5$	2.26	3.37
41	$1,\ 4,\ Y_1+2$	2.26	3.45
42	$2,\ 3,\ Y_1+a4+7+b11$	2.27	3.14, 3.15, 3.17
43	$2,\ 3,\ Y_1+a4+b5+11$	2.27	3.25, 3.26
44	$2,\ 3,\ Y_1+a4+b5+10$	2.27	3.30, 3.31, 3.32, 3.33
45	$2,\ 3,\ Y_1+4+b5$	2.27	3.42, 3.43
46	$2,\ 3,\ Y_1+5$	2.27	3.45
47	$2,\ 3,\ Y_1+1+b5$	2.27	3.44, 3.46

	48	2, 3, $Y_1 + a4 + 7 + b10$, $b \neq 0$	2.27	3.19, 3.20
4	1	7, 8, 9, $Y_1 + 11$	3.1	4.1
	2	7, 8, 9, $Y_1 + 10$	3.1	4.2
	3	1, $a4 + 7$, $b4 + 11$, $Y_1 + 4$	3.4	4.5
	4	1, 10, $b4 + 7 + a11$, $Y_1 + c4 + d11$, $c^2 + d^2 = 1$	3.5	4.3, 4.7
	5	1, $4 + 10$, $a4 + 7$, $Y_1 + 4$	3.6	4.7
	6	1, 10, $a4 + 11$, $Y_1 + b4 + c7$, $b^2 + c^2 = 1$	3.7	4.3, 4.12
	7	5, 6, $b4 + 7 + a11$, $Y_1 + c4 + d11$, $a \neq 0$, $c^2 + d^2 = 1$	3.8	4.4, 4.15
	8	$3 + 5$, $2 - 6$, $a1 + b4 + 7$, $Y_1 + c1 + d4$, $c^2 + d^2 = 1$	3.9	4.17, 4.20
	9	5, 6, $\varepsilon 1 + a4 + 7$, $Y_1 + b1 + c4$, $b^2 + c^2 = 1$	3.10, 3.11	4.16, 4.18, 4.19
	10	5, 6, $a4 + 7$, $Y_1 + b4 + 11$	3.11	4.4
	11	1, 4, $7 + a11$, $Y_1 + 11$	3.12, 3.13	4.5
	12	1, 4, 7, $Y_1 + 10$	3.13	4.7
	13	2, 3, $b4 + 7 + a11$, $Y_1 + c4 + d11$, $c^2 + d^2 = 1$, $a \neq 0$	3.14	4.6, 4.21
	14	2, 3, $a4 + 7$, $Y_1 + b4 + 11$, $a \neq 0$	3.15	4.6
	15	2, 3, $a4 + 7$, $Y_1 + b1 + c4$, $b^2 + c^2 = 1$, $a \neq 0$	3.15	4.21, 4.22, 4.24
	16	2, 3, $1 + 7$, $Y_1 + a4 + 10$	3.16	4.10, 4.11
	17	2, 3, $1 + 7$, $Y_1 + a1 + b4$, $a^2 + b^2 = 1$	3.16	4.22, 4.24
	18	2, 3, 7, $Y_1 + a4 + 11$	3.17	4.6
	19	2, 3, 7, $Y_1 + a4 + 10$	3.17	4.9, 4.11
	20	2, 3, 7, $Y_1 + a1 + b4$, $a^2 + b^2 = 1$	3.17	4.21, 4.24
	21	1, 4, $7 + 10$, $Y_1 + 10$	3.18	4.7
	22	2, 3, $a4 + 7 + a10$, $Y_1 + b1 + c(4 + 10)$, $b^2 + c^2 = 1$, $a \neq 0$	3.19	4.11, 4.25
	23	2, 3, $7 + 10$, $Y_1 + a1 + b10$, $a^2 + b^2 = 1$	3.20	4.9, 4.10, 4.25
	24	5, 6, $a4 + 11$, $Y_1 + b4 + c7$, $b^2 + c^2 = 1$	3.21	4.4, 4.26
	25	1, $a4 + 5$, $b4 + c6 + 11$, $Y_1 + d4 + e6$, $d^2 + e^2 = 1$	3.22	4.27, 4.28, 4.29
	26	1, 4, $a5 + 11$, $Y_1 + b5 + c6$, $b^2 + c^2 = 1$, $a \neq 0$	3.23	4.29
	27	1, 4, 11, $Y_1 + 7$	3.24	4.5
	28	1, 4, 11, $Y_1 + 5$	3.24	4.29
	29	2, 3, $a4 + b5 + 11$, $Y_1 + c4 + d5 + e6$, $c^2 + d^2 + e^2 = 1$, $b \neq 0$	3.25	4.30, 4.31
	30	2, 3, $a4 + 11$, $Y_1 + b4 + 7$	3.26	4.6
	31	2, 3, $a4 + 11$, $Y_1 + b4 + c5$, $b^2 + c^2 = 1$	3.26	4.30, 4.32
	32	3, $a1 + b2 + 6$, $4 + 10$, $Y_1 + c1 + d2$, $c^2 + d^2 = 1$	3.27	4.35, 4.36

33	$1,\ 2+4,\ 10,\ Y_1+a2+b3,\ a^2+b^2=1$	3.28	4.37, 4.38
34	$1,\ 4,\ 10,\ Y_1+a7+11$	3.29	4.7, 4.12
35	$1,\ 4,\ 10,\ Y_1+7$	3.29	4.7
36	$2,\ 3,\ 4+a5+10,\ Y_1+b1+c5+d6,\ b^2+c^2+d^2=1,\ a\neq 0$	3.30	4.35, 4.39
37	$2,\ 3,\ 5+10,\ Y_1+a1+b5+c6,\ a^2+b^2+c^2=1$	3.31	4.36, 4.37, 4.38, 4.39
38	$2,\ 3,\ \varepsilon 4+10,\ Y_1+a1+7$	3.32, 3.33	4.9, 4.10, 4.11
39	$2,\ 3,\ 4+10,\ Y_1+a1+b5,\ a^2+b^2=1$	3.32	4.35, 4.39
40	$2,\ 3,\ 10,\ Y_1+a1+b5,\ a^2+b^2=1$	3.33	4.37, 4.38, 4.40
41	$-a2+b3+4,\ a1+d2-c3+5,\ -b1+c2+e3+6,\ Y_1+f1+g2+h3,\ f^2+g^2+h^2=1,\ a^2(e-d)^2+b^2e^2+c^2d^2=1$	3.34	4.41
42	$a1+4,\ b3+5,\ b2-6,\ Y_1+\varepsilon 1+7,\ a^2+b^2=1$	3.34, 3.35	4.17, 4.18
43	$a1+4,\ b3+5,\ b2-6,\ Y_1+c1+d2,\ a^2+b^2=1,\ c^2+d^2=1$	3.35	4.41, 4.42
44	$4,\ 5,\ 6,\ Y_1+a7+11$	3.36	4.15, 4.26
45	$4,\ 5,\ 6,\ Y_1+\varepsilon 1+7$	3.36	4.16, 4.18
46	$4,\ 5,\ 6,\ Y_1+1$	3.36	4.43
47	$a1+3,\ b1+5,\ c1+d2+6,\ Y_1+e1+f2+4,\ e^2+f^2=\varepsilon,\ b^2+c^2+d^2=1$	3.37	4.41
48	$a1+3,\ b1+5,\ c1+d2+6,\ Y_1+e1+f2,\ e^2+f^2=1,\ b^2+c^2+d^2=1$	3.37	4.44, 4.47
49	$a1+3,\ 5,\ 6,\ Y_1+b4+11$	3.38	4.27, 4.28, 4.29
50	$a1+3,\ 5,\ 6,\ Y_1+b1+c2+4,\ b^2+c^2=\varepsilon$	3.38	4.41, 4.43
51	$a1+3,\ 5,\ 6,\ Y_1+b1+c2,\ b^2+c^2=1$	3.38	4.46, 4.47
52	$1,\ 3+5,\ a2+6,\ Y_1+b2+c3+4,\ a\neq -1$	3.39	4.41
53	$1,\ 3+5,\ a2+6,\ Y_1+b2+c3,\ b^2+c^2=1$	3.39, 3.40	4.44, 4.47
54	$1,\ 3+5,\ 2-6,\ Y_1+a4+7$	3.40	4.20
55	$1,\ 3+5,\ 2-6,\ Y_1+b2+c3+4$	3.40	4.41, 4.42
56	$1,\ 5,\ 6,\ Y_1+a4+b7+11$	3.41	4.19, 4.28
57	$1,\ 5,\ 6,\ Y_1+b4+7$	3.41	4.19

	58	1, 5, 6, $Y_1 + a2 + b3 + 4$, $a^2 + b^2 = \varepsilon$	3.41	4.41, 4.43
	59	1, 5, 6, $Y_1 + a2 + b3$, $a^2 + b^2 = 1$	3.41	4.47
	60	2, 3, 4, $Y_1 + 7 + b11$, $b \neq 0$	3.42	4.21
	61	2, 3, 4, $Y_1 + b1 + 7$	3.42	4.21, 4.22
	62	2, $a1 + 3$, 4, $Y_1 + \varepsilon 1 + b5 + c6$, $b^2 + c^2 = 1$	3.42	4.44, 4.47
	63	2, $a1 + 3$, 4, $Y_1 + 1$	3.42, 3.43	4.48
	64	2, 3, 4, $Y_1 + a7 + 11$	3.43	4.21
	65	2, 3, 4, $Y_1 + \varepsilon 1 + 7$	3.43	4.21, 4.22
	66	2, 3, 4, $Y_1 + \varepsilon 1 + a5 + b6$, $a^2 + b^2 = 1$	3.43	4.44, 4.47
	67	1, 2, $3 + 4$, $Y_1 + a5 + b6 + 10$	3.44	4.35, 4.36, 4.37
	68	1, 2, $3 + 4$, $Y_1 + a5 + 6$	3.44	4.44, 4.45
	69	1, 2, $3 + 4$, $Y_1 + a3 + 5$	3.44	4.45
	70	1, 2, $3 + 4$, $Y_1 + 3$	3.44	4.48
	71	1, 2, 4, $Y_1 + a5 + b6 + c10 + d11$, $c^2 + d^2 = 1$	3.45	4.30, 4.35, 4.36, 4.38
	72	1, 2, 4, $Y_1 + b5 + 6$	3.45	4.47
	73	1, 2, 4, $Y_1 + a3 + b5$, $a^2 + b^2 = 1$	3.45	4.45, 4.46, 4.48
	74	1, 2, 3, $Y_1 + a4 + 7 + \varepsilon 10 + c11$, $c \neq 0 \Rightarrow \varepsilon = 0$	3.46	4.23, 4.24, 4.25
	75	1, 2, 3, $Y_1 + a4 + 11$	3.46	4.33, 4.34
	76	1, 2, 3, $Y_1 + \varepsilon 4 + 10$	3.46	4.39, 4.40
	77	1, 2, 3, $Y_1 + 4$	3.46	4.48
5	1	7, 8, 9, 10, $Y_1 + 11$	4.2	5.1
	2	1, $a4 + 7$, 10, $b4 + 11$, $Y_1 + 4$	4.3	5.2
	3	5, 6, $a4 + 7$, $b4 + 11$, $Y_1 + 4$	4.4	5.4
	4	2, 3, $a4 + 7$, $b4 + 11$, $Y_1 + 4$	4.6	5.6
	5	1, 4, 10, $7 + a11$, $Y_1 + 11$	4.7	5.2
	6	2, 3, 10, $7 + a11$, $Y_1 + 11$	4.8, 4.9	5.3
	7	2, 3, $\varepsilon 1 + 7$, 10, $Y_1 + 1$	4.9, 4.10	5.8
	8	2, 3, $a1 + 7$, $4 + 10$, $Y_1 + 1$	4.11	5.9
	9	1, 4, 10, 11, $Y_1 + 7$	4.12	5.2
	10	2, 3, 10, $a5 + 11$, $Y_1 + b5 + c6$, $b^2 + c^2 = 1$	4.13, 4.14	5.10
	11	2, 3, 10, 11, $Y_1 + 7$	4.14	5.3

	12	$4,\ 5,\ 6,\ 7+a11,\ Y_1+11,\ a\neq 0$	4.15	5.4
	13	$4,\ 5,\ 6,\ 7,\ Y_1+a1+b11,\ a^2+b^2=1$	4.16	5.4, 5.18
	14	$a1+4,\ 3+5,\ 2-6,\ b1+7,\ Y_1+1$	4.17	5.25
	15	$a1+4,\ 5,\ 6,\ b1+7,\ Y_1+1,\ a^2+b^2=1$	4.18	5.18
	16	$1,\ 5,\ 6,\ b4+7+a11,\ Y_1+c4+d11,\ c^2+d^2=1$	4.19	5.5, 5.18
	17	$1,\ 3+5,\ 2-6,\ a4+7,\ Y_1+4$	4.20	5.25
	18	$2,\ 3,\ 4,\ 7+a11,\ Y_1+11,\ a\neq 0$	4.21	5.6
	19	$2,\ 3,\ 4,\ 7,\ Y_1+b1+c11,\ b^2+c^2=1$	4.21	5.6, 5.24
	20	$2,\ 3,\ 4,\ 1+7,\ Y_1+1$	4.22	5.24
	21	$1,\ 2,\ 3,\ b4+7+a11,\ Y_1+c4+d11,\ c^2+d^2=1,\ a\neq 0$	4.23	5.7, 5.23
	22	$1,\ 2,\ 3,\ a4+7,\ Y_1+b4+c11,\ b^2+c^2=1$	4.24	5.7, 5.24
	23	$1,\ 2,\ 3,\ a4+7,\ Y_1+b4+c10,\ b^2+c^2=1$	4.24	5.8, 5.9, 5.24
	24	$1,\ 2,\ 3,\ a4+7+10,\ Y_1+b4+c10,\ b^2+c^2=1$	4.25	5.8, 5.9, 5.28
	25	$4,\ 5,\ 6,\ 11,\ Y_1+7$	4.26	5.4
	26	$1,\ a4+5,\ 6,\ b4+11,\ Y_1+4,\ a\neq 0$	4.27	5.29
	27	$1,\ 5,\ 6,\ a4+11,\ Y_1+b4+c7,\ b^2+c^2=1$	4.28	5.5, 5.29
	28	$1,\ 4,\ 6,\ a5+11,\ Y_1+5$	4.29	5.29
	29	$2,\ 3,\ a4+6,\ b4+c5+11,\ Y_1+d4+e5,\ d^2+e^2=1$	4.30	5.30, 5.31 5.32
	30	$2,\ 3,\ 4,\ a5+11,\ Y_1+b5+c6,\ b^2+c^2=1,\ a\neq 0$	4.31	5.32
	31	$2,\ 3,\ 4,\ 11,\ Y_1+7$	4.32	5.6
	32	$2,\ 3,\ 4,\ 11,\ Y_1+5$	4.32	5.32
	33	$1,\ 2,\ 3,\ a4+11,\ Y_1+b4+7$	4.33, 4.34	5.7
	34	$1,\ 2,\ 3,\ a4+11,\ Y_1+b4+c5,\ b^2+c^2=1,\ a\neq 0$	4.33	5.33
	35	$1,\ 2,\ 3,\ 11,\ Y_1+4$	4.34	5.34
	36	$2,\ 3,\ a1+5,\ 4+b6+10,\ Y_1+c1+d6,\ c^2+d^2=1$	4.35	5.13, 5.15, 5.17
	37	$2,\ 3,\ a1+5,\ 6+10,\ Y_1+b1+c6,\ b^2+c^2=1$	4.36	5.14, 5.16, 5.17
	38	$2,\ 3,\ 1+5,\ 10,\ Y_1+a1+b6,\ a^2+b^2=1$	4.37	5.12, 5.14, 5.16
	39	$2,\ 3,\ 5,\ 10,\ Y_1+a6+11$	4.38	5.10
	40	$2,\ 3,\ 5,\ 10,\ Y_1+a1+b6,\ a^2+b^2=1$	4.38	5.12, 5.14, 5.16
	41	$1,\ 2,\ 3,\ 4+10,\ Y_1+a4+7$	4.39	5.9
	42	$1,\ 2,\ 3,\ 4+10,\ Y_1+a4+b5,\ a^2+b^2=1$	4.39	5.17
	43	$1,\ 2,\ 3,\ 10,\ Y_1+a4+7+b11$	4.40	5.8
	44	$1,\ 2,\ 3,\ 10,\ Y_1+a4+b11,\ a^2+b^2=1$	4.40	5.11, 5.12
	45	$1,\ a2+b3+4,\ c3+5,\ d2+6,\ Y_1+3,\ a^2+b^2+(c+d)^2=1$	4.41	5.36
	46	$1,\ 4,\ 3+5,\ 2-6,\ Y_1+7$	4.42	5.25
	47	$1,\ 4,\ 3+5,\ 2-6,\ Y_1+a2+b3,\ a^2+b^2=1$	4.42	5.36
	48	$1,\ 4,\ 5,\ 6,\ Y_1+7+a11$	4.43	5.18

	49	1, 4, 5, 6, Y_1+11	4.43	5.29
	50	1, 4, 5, 6, Y_1+2	4.43	5.35
	51	2, $1+a3$, $3+5$, 6, $Y_1+b3+c4$, $b^2+c^2=1$	4.44	5.36, 5.37
	52	2, 3, $1+5$, 6, $Y_1+a4+10$	4.45	5.13, 5.14
	53	2, 3, $1+5$, 6, Y_1+4	4.45	5.36
	54	2, 3, $1+5$, 6, Y_1+1	4.45	5.37
	55	2, 3, 5, 6, $Y_1+a4+b7+11$	4.46	5.19[1], 5.31
	56	2, 3, 5, 6, $Y_1+a4+b7+10$	4.46	5.15, 5.16, 5.26, 5.27
	57	2, 3, 5, 6, Y_1+4+a7	4.46	5.20, 5.35
	58	2, 3, 5, 6, $Y_1+a1+b7$, $a^2+b^2=1$	4.46	5.21, 5.22, 5.37
	59	$1+a3$, 2, 5, 6, $Y_1+b4+11$	4.47	5.30, 5.32
	60	$1+a3$, 2, 5, 6, $Y_1+b3+c4$, $b^2+c^2=1$	4.47	5.35, 5.36, 5.37
	61	1, 2, 3, 4, $Y_1+7+a11$, $a\neq 0$	4.48	5.23
	62	1, 2, 3, 4, $Y_1+7+a10$	4.48	5.24, 5.28
	63	1, 2, 3, 4, $Y_1+a5+b10$, $a^2+b^2=1$	4.48	5.12, 5.17, 5.37
	64	1, 2, 3, 4, $Y_1+a5+11$	4.48	5.33, 5.34
6	1	1, 5, 6, $a4+7$, $b4+11$, Y_1+4	5.5	6.4
	2	1, 2, 3, $a4+7$, $b4+11$, Y_1+4	5.7	6.5
	3	1, 2, 3, 10, $b4+7+a11$, $Y_1+c4+d11$, $c^2+d^2=1$	5.8	6.3, 6.11
	4	1, 2, 3, $a4+7$, $4+10$, Y_1+4	5.9	6.11
	5	2, 3, 5, 10, $a6+11$, Y_1+6	5.10	6.12
	6	1, 2, 3, 10, 11, Y_1+a4+7	5.11	6.3
	7	1, 2, 3, 10, 11, Y_1+4	5.11	6.14
	8	1, 2, 3, 10, $4+a11$, Y_1+b4+7	5.12	6.3, 6.11
	9	1, 2, 3, 10, $4+a11$, $Y_1+b4+c5$, $b^2+c^2=1$	5.12	6.13, 6.14, 6.21
	10	2, 3, $a1+5$, 6, $4+10$, Y_1+1, $a\neq 0$	5.13	6.22
	11	2, 3, $1+5$, 6, 10, Y_1+1	5.14	6.21

[1]in work [4], Appendix, Optimal system of subalgebras of main algebra, in subalgebra 5.19 there is a misprint, see book [1], Appendix, Table 2, subalgebra 5.10.

12	2, 3, 5, 6, $\varepsilon 4+10$, $Y_1+a1+b7$, $a^2+b^2=1$	5.15, 5.16	6.8, 6.9, 6.10, 6.21, 6.22
13	1, 2, 3, 6, $4+10$, $Y_1+a4+b5$, $a^2+b^2=1$	5.17	6.21, 6.22
14	1, 4, 5, 6, $7+a11$, Y_1+11	5.18	6.4
15	2, 3, 5, 6, $7+a11$, $Y_1+b4+c11$, $b^2+c^2=1$	5.19, 5.22	6.6, 6.15, 6.17
16	2, 3, 5, 6, $a4+7$, $Y_1+b4+11$, $a\neq 0$	5.20	6.6
17	2, 3, 5, 6, $a4+7$, $Y_1+b1+c4$, $b^2+c^2=1$, $a\neq 0$	5.20	6.16, 6.17, 6.19
18	2, 3, 5, 6, $1+7$, $Y_1+a4+10$	5.21	6.9, 6.10
19	2, 3, 5, 6, $1+7$, Y_1+4	5.21	6.16
20	2, 3, 5, 6, $1+7$, Y_1+1	5.21	6.19
21	2, 3, 5, 6, 7, $Y_1+4+b10$	5.22	6.10, 6.17
22	2, 3, 5, 6, 7, Y_1+10	5.22	6.8
23	2, 3, 5, 6, 7, Y_1+1	5.22	6.19
24	1, 2, 3, 4, $7+a11$, Y_1+11	5.23, 5.24	6.5
25	1, 2, 3, 4, 7, Y_1+10	5.24	6.11
26	2, 3, 5, 6, $a4+7+a10$, $Y_1+b1+c4+c10$, $b^2+c^2=1$, $a\neq 0$	5.26	6.10, 6.20
27	2, 3, 5, 6, $7+10$, $Y_1+a1+b10$, $a^2+b^2=1$	5.27	6.8, 6.9, 6.20
28	1, 2, 3, 4, $7+10$, Y_1+10	5.28	6.11
29	1, 4, 5, 6, 11, Y_1+7	5.29	6.4
30	2, 3, 5, 6, $b4+11$, Y_1+7	5.30	6.6
31	2, 3, $a4+5$, 6, $b4+11$, Y_1+4	5.30, 5.31	6.23
32	2, 3, 5, 6, $b4+11$, Y_1+c4+7	5.30, 5.31	6.6
33	2, 3, 4, 6, $a5+11$, Y_1+5	5.32	6.23
34	1, 2, 3, 4, 11, Y_1+7	5.33, 5.34	6.5
35	1, 2, 3, 4, $a5+11$, $Y_1+b5+c6$, $b^2+c^2=1$	5.33, 5.34	6.24
36	2, 3, 4, 5, 6, $Y_1+a7+11$	5.35	6.15, 6.23
37	2, 3, 4, 5, 6, $Y_1+a1+b7$, $a^2+b^2=1$	5.35	6.16, 6.17, 6.25
38	2, 3, 4, 5, $1+6$, Y_1+1	5.36	6.25
39	1, 2, 3, 5, 6, $Y_1+a4+b7+11$	5.37	6.18, 6.24

	40	1, 2, 3, 5, 6, $Y_1+a4+b7+c10$, $a^2+b^2+c^2=1$	5.37	6.19, 6.20, 6.21, 6.22, 6.25
7	1	4, 5, 6, 7, 8, 9, Y_1+11	6.1	7.2
	2	1, 2, 3, 7, 8, 9, Y_1+11	6.2	7.1
	3	1, 2, 3, 7, 8, 9, Y_1+10	6.2	7.3
	4	1, 2, 3, $a4+7$, 10, $b4+11$, Y_1+4	6.3	7.5
	5	2, 3, 5, 6, $a4+7$, $b4+11$, Y_1+4	6.6	7.8
	6	2, 3, 5, 6, $7+a11$, 10, Y_1+11	6.7, 6.8	7.4
	7	2, 3, 5, 6, $\varepsilon 1+7$, 10, Y_1+1	6.8, 6.9	7.6
	8	2, 3, 5, 6, $a1+7$, $4+10$, Y_1+1	6.10	7.7
	9	1, 2, 3, 4, $7+a11$, 10, Y_1+11	6.11	7.5
	10	2, 3, 5, 6, 10, 11, Y_1+7	6.12	7.4
	11	1, 2, 3, 4, 10, $a5+11$, $Y_1+b5+c6$, $b^2+c^2=1$, $a\neq 0$	6.13	7.10
	12	1, 2, 3, 4, 10, 11, Y_1+7	6.14	7.5
	13	1, 2, 3, 4, 10, 11, Y_1+5	6.14	7.10
	14	2, 3, 4, 5, 6, $7+a11$, Y_1+11	6.15, 6.17	7.8
	15	2, 3, 4, 5, 6, $1+7$, Y_1+1	6.16	7.12
	16	2, 3, 4, 5, 6, 7, Y_1+1	6.17	7.12
	17	1, 2, 3, 5, 6, $b4+7+a11$, $Y_1+c4+d11$, $c^2+d^2=1$, $a\neq 0$	6.18	7.9, 7.11
	18	1, 2, 3, 5, 6, $a4+7$, $Y_1+b4+11$	6.19	7.9
	19	1, 2, 3, 5, 6, $a4+7+\varepsilon 10$, $Y_1+b4+c10$, $b^2+c^2=1$	6.19, 6.20	7.6, 7.7, 7.12, 7.13
	20	1, 2, 3, 5, 6, 10, $Y_1+a4+b7+c11$, $a^2+b^2+c^2=1$	6.21	7.6, 7.10, 7.14
	21	1, 2, 3, 5, 6, $4+10$, $Y_1+a4+b7$, $a^2+b^2=1$	6.22	7.7, 7.14
	22	2, 3, 4, 5, 6, 11, Y_1+7	6.23	7.8
	23	1, 2, 3, 5, 6, $a4+11$, $Y_1+b4+c7$, $b^2+c^2=1$	6.24	7.9, 7.15
	24	1, 2, 3, 4, 5, 6, $Y_1+a7+11$	6.25	7.11, 7.15
	25	1, 2, 3, 4, 5, 6, $Y_1+a7+b10$, $a^2+b^2=1$	6.25	7.12, 7.13, 7.14
8	1	1, 2, 3, 7, 8, 9, 10, Y_1+11	7.3	8.1
	2	1, 2, 3, 5, 6, 10, $b4+7+a11$, $Y_1+c4+d11$, $c^2+d^2=1$	7.6	8.2, 8.3
	3	1, 2, 3, 5, 6, $a4+7$, $4+10$, Y_1+10	7.7	8.3
	4	1, 2, 3, 5, 6, $a4+7$, $b4+11$, Y_1+4	7.9	8.4
	5	1, 2, 3, 5, 6, 10, $a4+11$, $Y_1+b4+c7$, $b^2+c^2=1$	7.10	8.2, 8.5
	6	1, 2, 3, 4, 5, 6, $7+a11$, Y_1+11	7.11, 7.12	8.4
	7	1, 2, 3, 4, 5, 6, $7+\varepsilon 10$, Y_1+10	7.12, 7.13	8.3
	8	1, 2, 3, 4, 5, 6, 10, $Y_1+a7+b11$, $a^2+b^2=1$	7.14	8.3, 8.5
	9	1, 2, 3, 4, 5, 6, 11, Y_1+7	7.15	8.4
9	1	1, 2, 3, 5, 6, $a4+7$, 10, $b4+11$, Y_1+4	8.2	9.1

	2	1, 2, 3, 4, 5, 6, $7 + a11$, 10, $Y_1 + 11$	8.3	9.1
	3	1, 2, 3, 4, 5, 6, 10, 11, $Y_1 + 7$	8.5	9.1
10	1	1, 2, 3, 4, 5, 6, 7, 8, 9, $Y_1 + 10$	9.2	10.2
	2	1, 2, 3, 4, 5, 6, 7, 8, 9, $Y_1 + 11$	9.2	10.1
11	1	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, $Y_1 + 11$	10.2	11.1

Remark 2. If we cross out Y_1 in the last operator in the basis for L_{12}, we obtain the subalgebra in L_{11} whose index is indicated in the last column.

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