



Arlen Michailovich Ilyin
(for the 80th anniversary)

On January 8th, 2012 Arlen Michailovich Ilyin celebrated his 80th anniversary. Arlen Michailovich Ilyin is the academician of the RAS, the Laureate of the State Prize of the Russian Federation and an outstanding Russian mathematician. He is the author of the already classical results on theory of differential equations and asymptotic methods in problems of mathematical physics.

Arlen Michailovich was born in Leningrad but during his life he changed places of living rather often. He spent his childhood in Ulan-Ude, Batumi, Moscow, during the war he was evacuated to Kurgan region. After that he again returned to Moscow, where he studied and worked, then he moved to the Urals: Sverdlovsk, Ufa, Ekaterinburg, Chelyabinsk.

Starting with the 8th form, A.M. Ilyin studied in the famous 59th Moscow school, and finished it with honours in 1949, then he entered MSU, th mechanical department.

He wrote his diploma paper devoted to strongly non degenerate elliptic equations under supervision of O.A. Oleynik. It was she who advanced the idea to introduce a small parameter and consider the family of nonsingular elliptic operators, depending on the small parameter and converging to the nonsingular. A.M. Ilyin managed to obtain uniform estimates of the solutions of the boundary-value problem and thereby prove existence and uniqueness of the solution of the boundary-value problem. The results of A.M. Ilyin's diploma paper were published in the USSR DAS in 1955, and it happened so, that the greater part of the future scientific achievements of Arlen Michailovich was connected with research of suchlike problems with a small parameter.

Since autumn 1954 he started to teach at the correspondence (and later at the part-time) department of the mechanical-mathematical faculty of MSU, and during the period from January 1957 till February 1963 A.M. Ilyin worked at the chair of differential equations of MSU, which was supervised by the academician I.G. Petrovsky, the head of the university. During this period Arlen Michailovich (together with his course-mate and a friend R.Z. Has'minsky) researched asymptotic solutions of linear second-order parabolic equations within great values of time, and also (together with O.A. Oleynik) he researched behaviour of non-linear parabolic equations. Some part of results obtained by him during that period was included in the article on linear parabolic equations, written together with O.A. Oleynik and A.S. Kalashnikov. This publication [6] became known and is still so today, which is justified by its republishing in the 21rd volume of "Works of the seminar in the name of I.G. Petrovsky".

In 1963 Arlen Michailovich moved to the Urals to Sverdlovsk (now Ekaterinburg) and started work in the just formed in Sverdlovsk branch of Steklov Mathematical Institute (SOMI, now Institute of mathematical and mechanics UrO of RAS). Since that time all the activity of A.M. Ilyin was connected with the Urals. The department of equations of mathematical physics of SOMI, headed by A.M. Ilyin, cooperated for some time with the Institute of oceanology of AS of the USSR, making calculations. One of the most important results of this cooperation (in particular, of the cooperation with V.M. Kamenkovich) proved to be the generation of two new scientific trends in the theory of differential equations and approximated methods of their solution.

The first one is generation of difference schemes for effective numerical solution of differential equations with a small parameter with higher derivatives (such very equations, sometimes called the singular perturbation equations, occur in mathematical models of ocean and atmospheric currents). A small remark [20] made by Arlen Michailovich in 1969 is still very popular in this trend.

The second one is analytical research of a special but rather wide class of boundary-value problems with a small parameter (today these problems are called bisingular), characterised by the fact, that during construction of asymptotics of the solution in the form of the series by degrees of a small parameter, in the coefficients of the series there occur singularities of different types, which do not exist in the exact solution. This second trend was also initiated by the research of models of ocean currents. The simplest model of such a type corresponded a boundary-value problem for an elliptic equation with a small parameter in case, when the characteristics of the first-order limit equation reaches the boundary of the domain. It should

be noted, that, the well-known method of boundary functions does not give a full answer for such problems. To research the asymptotics of the solution there were applied the ideas, suggested by L. Prandtl in the beginning of XX century, with which development there was further connected the term "matching method".

The first papers on this topic [24 - 26], written in Sverdlovsk, had been printed by 1975, when A.M. Ilyin worked in Ufa in the Department of physics and mathematics of Bashkir branch of AS of the USSR. In the Department of physics and mathematics of Bashkir branch of AS of the USSR Arlen Michailovich headed the sector of differential equations and was completely occupied by the mentioned above topic, gradually collecting a hard-working team consisting of graduates from the Ural and Bashkir universities. To this very period we can refer detailed development and justification of the method of matching applied to the wide class of bisingular problems.

One of the most important results of Arlen Michailovich during his work in Ufa is research of the structure of the shock wave. In the Cauchy problem for the Burgers equation with a small diffusion there was constructed the asymptotics of the solution in general case, when in the solution of the limiting problem with the lapse of time there occurs a break. There was obtained and justified a complete asymptotic expansion with the vanishing of the small parameter to the zero, uniform as to independent variables. Meanwhile there was developed a domain of the transition layer generation, in which coefficients of the asymptotics were determined from parabolic equations. It proved to be, that the detailed research of the solution in this area is necessary for the single valued definition for the asymptotics in the transition (shock) layer [31, 36, 43, 44]. Later A.M. Ilyin together with his followers studied situations, when with the lapse of time there occurs a weak break, gradually transforming to a strong one [63].

Not less interesting results were obtained for another class of difficult problems, connected with hydrodynamics of subtle solids flow. The mathematical model of this phenomenon corresponds a boundary-value problem for the elliptic equation in the domain with low perturbation, for example, in the domain with a cut narrow plane or with a small slot. There were also constructed complete asymptotic expansions of the solution, uniform within all the domain [29, 30, 35].

All these results were obtained with the help of ideas of splicing of asymptotic expansions, which were earlier applied as some recommendations by mechanics for the analysis of some problems of hydrodynamics in dominant terms of the asymptotics, though there was no justification for that. It should be noted, that earlier strict mathematical results with the justification of the method of splicing with regard to the problem of relaxation oscillations were obtained in works of E.F. Mishenko, N.H. Rozov, and a wide class of diffraction problems was studied by this method in works of V.M. Babich. Mathematical formulation of this approach was done in works of L.E. Fraenkel, S. Kaplun, P.A. Lagerstrom.

The result of works of A.M. Ilyin and his followers on the base of these ideas was creating a method, called a method of matching of asymptotic expansions, which was successfully applied to the wide spectre of problems, impossible to be studied other methods. The results of these researches were compiled by Arlen Michailovich in his monograph [46], published in 1989, which was almost immediately translated into English [47] by the American mathematical

society.

In march 1988 he returned to Sverdlovsk and worked there as professor in the Ural Polytechnical Institute, being at the time (since 1990) a leading research engineer of the Institute of Mathematics and Mechanics (IMM) of URO of RAS, and later the chief of the department of equations of mathematical physics of that university. During that period Arlen Michailovich initialized research of a number of bisingular problems, that is problems of theory of optimal management, containing in its description small parameters. Let us note, that an interesting result, obtained in the process of research of one class of problems of optimal operation speed (including management of a material point with the help of power, limited by quantity) with the initial condition, weakly different from some "critical"one, in case of which there occurs a qualitative change of the optimal management. It proved to be, that in the given problem the asymptotic expansion of the time of the operation speed cannot be constructed in the form of the series, containing only degrees of a small parameter and a logarithm of this parameter [50, 51, 57].

Since 2002 A.M. Ilyin has been working as a professor of the chair of computer-oriented mathematics in the Chelyabinsk State University, remaining a research supervisor of the department of equations of the mathematical physics of IMM of URO of RAS. During recent years Arlen Michailovich, apart from his research work, pays lots of attention to sharing his great experience to new generations of researchers. He wrote and published in the publishing house PhysMatLit a monograph "Asymptotic methods in analysis"and a tutorial "Equations of mathematical physics".

Researches of A.M. Ilyin obtained the recognition of the scientific society. In March 1994 he was elected a corresponding member of the Russian Academy of Sciences, and in May 2000 he was elected a full member of the RAS. In 1995 he was awarded the prize in the name of I.G. Petrovsky by the RAS for a series of works "Asymptotic methods in mathematical physics"(together with O.A. Oleynik). In 2000 Arlen Michailovich was awarded a State Prize (together with V.S. Buslayev and M.V. Karasyev) for the series of works "Asymptotic methods of research of equations of mathematical physics".

Arlen Michailovich gives lots of tome and energy to pedagogical activity. Today he gives lectures not only in the Chelyabinsk State University, but also in UrSU (now UrPhU, Ekaterinburg). In 2000 - 2002 Ilyin cooperated with Buryat State University and was in 2001 - 2002 the Director of the Institute of Mathematics an Informatics of BSU. His attitude to teaching is as serious as that to his scientific work. He passes this feature received from his teachers of mechanical department of the MSU to his followers. Combining in himself talents of an excellent lecturer and a teacher, he constantly attracts gifted young people. the scientific school organised by him obtained recognition both in Russia and abroad. Among his followers there are seven Doctors of Sciences.

We should especially point out the role of Arlen Michailovich in establishment of mathematical school of differential equations in Ufa. Here during a short period of time (not more than 10 years) were trained highly skilled specialists in the sphere of asymptotic methods. Today they are recognized scientists, making the bases of the actively developing scientific trend.

Arlen Michailovich works a lot in the expert councils HAC and RFFR using all his energy to preserve and develop mathematical science in Russia. He is a member of editorial boards

and editorial councils of such mathematical journals as "Achievements of mathematical sciences" "Journal of computer-oriented mathematics and mathematical physics" "Works of mathematics and mechanics".

Arlen Michailovich enjoys authority and recognition of his colleagues not only for his outstanding scientific achievements. Combination of exactingness and adherence to his principles with well-disposed attitude to any person be it a student or an academician, ensure him respect and love of people surrounding him. Attention and goodwill of Arlen Michailovich are known to all his friends, colleagues and students.

We sincerely wish Arlen Michailovich long life, good health, joy and new success.

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Kiselev O.M., Novokshenov V.Yu., Suleimanov B.I.*

The list of general scientific papers of A.M. Ilyin.

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During preparing this paper there were used articles, dedicated to the 70th anniversary of Ilyin A.M., published in the journals "Differential equations"(2002, V. 38, № 8) and "Works of the Institute of Mathematics and Mechanics"(2003, V. 9, № 1).