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PERFECT CUBOIDS AND IRREDUCIBLE POLYNOMIALS

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Abstract. The problem of constructing a perfect cuboid is related to a certain class of univariate polynomials with three integer parameters a, b, and u. Their irreducibility over the ring of integers under certain restrictions for a, b, and u would mean the non-existence of perfect cuboids. This irreducibility is conjectured and then verified numerically for approximately 10 000 instances of a, b, and u.

Keywords: an Euler cuboid, a perfect cuboid, irreducible polynomials.

1. Introduction

An Euler cuboid is a rectangular parallelepiped whose edges and face diagonals are of integer length. A perfect cuboid is an Euler cuboid whose space diagonal is also of integer length. Cuboids with integer edges and face diagonals were known before Euler (see [1] and [2]). However, thanks to Leonhard Euler (see [3]) the problem of integer cuboids obtained a status, while such cuboids themselves were named after him.

As for perfect cuboids, none of them is known so far. The problem of finding perfect cuboids or proving of their non-existence is an unsolved mathematical problem. It has the long history presented in papers [4-34].

In paper [35] the problem of constructing a perfect cuboid was reduced to the following 12th order Diophantine equation of the variables a, b, c and u:

$$\begin{gathered}
u^4\,a^4\,b^4+6\,a^4\,u^2\,b^4\,c^2-2\,u^4\,a^4\,b^2\,c^2-2\,u^4\,a^2\,b^4\,c^2+4\,u^2\,b^4\,a^2\,c^4+\\
+4\,a^4\,u^2\,b^2\,c^4-12\,u^4\,a^2\,b^2\,c^4+u^4\,a^4\,c^4+u^4\,b^4\,c^4+a^4\,b^4\,c^4+\\
+6\,a^4\,u^2\,c^6+6\,u^2\,b^4\,c^6-8\,a^2\,b^2\,u^2\,c^6-2\,u^4\,a^2\,c^6-2\,u^4\,b^2\,c^6-\\
-2\,a^4\,b^2\,c^6-2\,b^4\,a^2\,c^6+u^4\,c^8+b^4\,c^8+a^4\,c^8+4\,a^2\,u^2\,c^8+\\
+4\,b^2\,u^2\,c^8-12\,b^2\,a^2\,c^8+6\,u^2\,c^{10}-2\,a^2\,c^{10}-2\,b^2\,c^{10}+c^{12}=0.
\end{gathered}\tag{1.1}$$

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More precisely, the result of paper [35] is formulated in the form of the following theorem.

Theorem 1.1. *A perfect cuboid does exist if and only if the Diophantine equation* (1.1) *has got a solution, such that a, b, c, u are four positive integer numbers, satisfying the inequalities $a < c$, $b < c$, $u < c$, $(a + c)\,(b + c) > 2\,c^2$.*

A more simple equation associated with perfect cuboids was derived in [18] (see also [27]). But our aim in the present paper is to study the equation (1.1) (it is new) and to obtain the results declared in the abstract.

2. Rational cuboids

A rational cuboid is a rectangular parallelepiped whose edges lengths are expressed by rational numbers. If the lengths of face diagonals are also rational, we have a rational Euler cuboid. Finally, if the length of the space diagonal is also a rational number, we obtain a perfect cuboid. It is easy to see, that every rational Euler cuboid can be transformed into an Euler cuboid with integer edges and face diagonals. In the case of a perfect cuboid (either integer or rational) any such cuboid can be transformed into a rational perfect cuboid with the unit space diagonal (see [35]). Conversely, any rational perfect cuboid with a unit space diagonal can be transformed into a perfect cuboid with integer edges and diagonals. Therefore, speaking of rational perfect cuboids, below, by default, we assume their space diagonals to be unit.

3. Formulae for edges and face diagonals

Note, that the equation (1.1) is uniform with respect to variables a, b, c and, u. Since $c > 0$ in Theorem 1.1, we can introduce the variables

$$\alpha = \frac{a}{c}, \qquad\qquad \beta = \frac{b}{c}, \qquad\qquad \upsilon = \frac{u}{c}. \tag{3.1}$$

In the variables (3.1) the equation (1.1) is written in the following form:

$$\begin{gathered}
\upsilon^4\,\alpha^4\,\beta^4 + (6\,\alpha^4\,\upsilon^2\,\beta^4 - 2\,\upsilon^4\,\alpha^4\,\beta^2 - 2\,\upsilon^4\,\alpha^2\,\beta^4) + (4\,\upsilon^2\,\beta^4\,\alpha^2 +\\
+ 4\,\alpha^4\,\upsilon^2\,\beta^2 - 12\,\upsilon^4\,\alpha^2\,\beta^2 + \upsilon^4\,\alpha^4 + \upsilon^4\,\beta^4 + \alpha^4\,\beta^4) + (6\,\alpha^4\,\upsilon^2 + 6\,\upsilon^2\,\beta^4 -\\
- 8\,\alpha^2\,\beta^2\,\upsilon^2 - 2\,\upsilon^4\,\alpha^2 - 2\,\upsilon^4\,\beta^2 - 2\,\alpha^4\,\beta^2 - 2\,\beta^4\,\alpha^2) + (\upsilon^4 + \beta^4 +\\
+ \alpha^4 + 4\,\alpha^2\,\upsilon^2 + 4\,\beta^2\,\upsilon^2 - 12\,\beta^2\,\alpha^2) + (6\,\upsilon^2 - 2\,\alpha^2 - 2\,\beta^2) + 1 = 0.
\end{gathered} \tag{3.2}$$

Note, that the variables a, b, c, and u in (1.1) are neither edges, nor diagonals of a perfect cuboid, they are just parameters. According to the formulae (3.1), the rational parameters α, β, and υ in (3.2) are expressed through them. And the edges and face diagonals of a perfect cuboid are expressed by α, β, and υ. Let us denote the edges of such a cuboid by x_1, x_2, and x_3, and denote its face diagonals by d_1, d_2, and d_3:

$$(x_1)^2 + (x_2)^2 = (d_3)^2, \qquad (x_2)^2 + (x_3)^2 = (d_1)^2, \qquad (x_3)^2 + (x_1)^2 = (d_2)^2. \tag{3.3}$$

Then x_1 and d_1 are expressed through the parameter υ:

$$x_1 = \frac{2\,\upsilon}{1 + \upsilon^2}, \qquad\qquad d_1 = \frac{1 - \upsilon^2}{1 + \upsilon^2}. \tag{3.4}$$

Let us denote the following auxiliary parameter by z:

$$z = \frac{(1 + \upsilon^2)\,(1 - \beta^2)\,(1 + \alpha^2)}{2\,(1 + \beta^2)\,(1 - \alpha^2\,\upsilon^2)}. \tag{3.5}$$

Then the edges x_2 and x_3 are expressed by the formulae

$$x_2 = \frac{2\,z\,(1 - \upsilon^2)}{(1 + \upsilon^2)\,(1 + z^2)}, \qquad\qquad x_3 = \frac{(1 - \upsilon^2)\,(1 - z^2)}{(1 + \upsilon^2)\,(1 + z^2)}, \tag{3.6}$$

and the face diagonals d_2 and d_3 are given by the following formulae:

$$\begin{gathered} d_2=\frac{(1+\upsilon^2)\,(1+z^2)+2\,z(1-\upsilon^2)}{(1+\upsilon^2)\,(1+z^2)}\,\beta,\\ d_3=\frac{2\,(\upsilon^2\,z^2+1)}{(1+\upsilon^2)\,(1+z^2)}\,\alpha. \end{gathered} \tag{3.7}$$

The formulae (3.4), (3.5), (3.6), and (3.7) are taken from [35]. They can be verified by direct calculations. Indeed, the second equality (3.3) transforms into an identity due to the formulae (3.6). Moreover, the equations (3.3), a perfect cuboid is characterized by the equalities

$$(x_1)^2+(d_1)^2=1, \qquad (x_2)^2+(d_2)^2=1, \qquad (x_3)^2+(d_3)^2=1. \tag{3.8}$$

They mean that the length of the cuboid space diagonal is equal to unity. The first equality (3.8) turn to an identity due to the formulae (3.4).

Thus, the second equality (3.3) and the first equality (3.8) transform to identities. The rest four equalities (3.3) and (3.8) also transform to identities due to (3.4), (3.5), (3.6), and (3.7), but subject to the equation (3.2).

4. Back to integer numbers

The equation (1.1) is homogeneous with respect to variables in it. Hence, due to (3.1) and Theorem 1.1, the parameters a, b, c, and u in the equation (1.1) can be considered as four positive mutually coprime numbers, i. e. their greatest common divisor is equal to unity:

$$\operatorname{GCD}(a,b,c,u)=1. \tag{4.1}$$

Let us denote the greatest common divisor of the numbers a, b, and u by m:

$$\operatorname{GCD}(a,b,u)=m. \tag{4.2}$$

Then from (4.1) and (4.2) we can derive the equality

$$\operatorname{GCD}(m,c)=1, \tag{4.3}$$

i. e. m and c are mutually coprime. Due to (4.2) and (4.3), the fractions a/m, b/m, and u/m are simplified to integer numbers, and the fraction c/m proves to be an irreducible one provided,

$m \neq 1$. The formulae (3.1) can be rewritten in terms of these fractions:

$$\alpha = \frac{a/m}{c/m}, \qquad \beta = \frac{b/m}{c/m}, \qquad \upsilon = \frac{u/m}{c/m}. \tag{4.4}$$

Relying on (4.4), we can change the variables in the following way:

$$\frac{a}{m} \to a, \qquad \frac{b}{m} \to b, \qquad \frac{u}{m} \to u, \qquad \frac{c}{m} \to t. \tag{4.5}$$

After introducing a new variable $t = c/m$ and renewing the variables a, b, and u, according to (4.5), the formula (4.4) can be written as

$$\alpha = \frac{a}{t}, \qquad \beta = \frac{b}{t}, \qquad \upsilon = \frac{u}{t}, \tag{4.6}$$

and the equation (1.1) takes the following form:

$$\begin{gathered}
t^{12} + (6\,u^2 - 2\,a^2 - 2\,b^2)\,t^{10} + (u^4 + b^4 + a^4 + 4\,a^2\,u^2 + {}\\
+ 4\,b^2\,u^2 - 12\,b^2\,a^2)\,t^8 + (6\,a^4\,u^2 + 6\,u^2\,b^4 - 8\,a^2\,b^2\,u^2 - {}\\
- 2\,u^4\,a^2 - 2\,u^4\,b^2 - 2\,a^4\,b^2 - 2\,b^4\,a^2)\,t^6 + (4\,u^2\,b^4\,a^2 + {}\\
+ 4\,a^4\,u^2\,b^2 - 12\,u^4\,a^2\,b^2 + u^4\,a^4 + u^4\,b^4 + a^4\,b^4)\,t^4 + {}\\
+ (6\,a^4\,u^2\,b^4 - 2\,u^4\,a^4\,b^2 - 2\,u^4\,a^2\,b^4)\,t^2 + u^4\,a^4\,b^4 = 0.
\end{gathered} \tag{4.7}$$

As for the formula (4.2), for the variables a, b, and u renewed according to (4.5) this formula gives the following relationship:

$$\operatorname{GCD}(a, b, u) = 1. \tag{4.8}$$

The formula (4.8) implies that the numbers a, b, and u in (4.6) and (4.7) are mutually coprime.

Note, that the equation (4.7) coincides with the original equation (1.1), but the variable c in it is replaced by t, while its terms are regrouped as is usually done in for polynomials of one variable t. Now Theorem 1.1 is reformulated in the following way.

Theorem 4.1. *A perfect cuboid does exist if and only if for some three positive mutually coprime numbers a, b, u polynomial equation* (4.7) *has got a rational solution t satisfying the inequalities $t > a$, $t > b$, $t > u$, $(a + t)\,(b + t) > 2\,t^2$.*

5. Factoring the polynomial equation

Let us denote the polynomial in the left side of the equation (4.7) by $P_{abu}(t)$. Having done this, we consider it as a polynomial of one variable t, while variables a, b, and u are considered as parameters:

$$\begin{gathered}
P_{abu}(t) = t^{12} + (6\,u^2 - 2\,a^2 - 2\,b^2)\,t^{10} + (u^4 + b^4 + a^4 + 4\,a^2\,u^2 + {}\\
+ 4\,b^2\,u^2 - 12\,b^2\,a^2)\,t^8 + (6\,a^4\,u^2 + 6\,u^2\,b^4 - 8\,a^2\,b^2\,u^2 - {}\\
- 2\,u^4\,a^2 - 2\,u^4\,b^2 - 2\,a^4\,b^2 - 2\,b^4\,a^2)\,t^6 + (4\,u^2\,b^4\,a^2 + {}\\
+ 4\,a^4\,u^2\,b^2 - 12\,u^4\,a^2\,b^2 + u^4\,a^4 + u^4\,b^4 + a^4\,b^4)\,t^4 + {}\\
+ (6\,a^4\,u^2\,b^4 - 2\,u^4\,a^4\,b^2 - 2\,u^4\,a^2\,b^4)\,t^2 + u^4\,a^4\,b^4.
\end{gathered} \tag{5.1}$$

The polynomial (5.1) is symmetric with respect to the parameters a and b, i. e.

$$P_{abu}(t)=P_{bau}(t). \tag{5.2}$$

To study the polynomial $P_{abu}(t)$ we shall consider some certain cases:

$$\begin{array}{lll} 1)\ \ a=b; & 3)\ \ b\,u=a^2; & 5)\ \ a=u;\\ 2)\ \ a=b=u; & 4)\ \ a\,u=b^2; & 6)\ \ b=u. \end{array} \tag{5.3}$$

Particular case $a=b$. In this particular case the polynomial $P_{abu}(t)=P_{aau}(t)$ is set by formula

$$\begin{gathered} P_{aau}(t)=t^{12}+(6\,u^2-4\,a^2)\,t^{10}+(8\,a^2\,u^2-10\,a^4+u^4)\,t^8+\\ +(4\,a^4\,u^2-4\,a^6-4\,u^4\,a^2)\,t^6+(8\,a^6\,u^2+a^8-10\,u^4\,a^4)\,t^4+\\ +(6\,a^8\,u^2-4\,u^4\,a^6)\,t^2+u^4\,a^8. \end{gathered} \tag{5.4}$$

The polynomial (5.4) is reducible. It is factorized as

$$P_{aau}(t)=(t^2+a^2)^2\,P_{au}(t), \tag{5.5}$$

where the polynomial $P_{au}(t)$ is given by formula

$$\begin{gathered} P_{au}(t)=t^8+6\,(u^2-a^2)\,t^6+(a^4-4\,a^2\,u^2+u^4)\,t^4-\\ -6\,a^2\,u^2\,(u^2-a^2)\,t^2+u^4\,a^4. \end{gathered} \tag{5.6}$$

The formulae (5.5) and (5.6) are easily proved by direct calculations.

Particular case $a=b=u$. This case corresponds to substituting $a=u$ into the formula (5.6). If $a=u$, then the polynomial $P_{au}(t)=P_{aa}(t)$ is reducible:

$$P_{aa}(t)=(t-a)^2\,(t+a)^2\,(t^2+a^2)^2. \tag{5.7}$$

Due to the condition of mutual coprimality (4.8) the particular case $a=b=u$ satisfies the conditions of Theorem 4.1 only if $a=b=u=1$. And then, due to (5.5) and (5.7), the equation (4.7) looks as follows:

$$(t-1)^2\,(t+1)^2\,(t^2+1)^4=0. \tag{5.8}$$

The equation (5.8) possesses two real rational solutions $t=-1$ and $t=1$. But none of them satisfies the conditions of Theorem 4.1. Indeed, they both do not satisfy the inequality $t>a$, where $a=1$.

Therefore, the subcase $a=b=u$ of the case $a=b$ does not give perfect cuboids. Other subcases of the particular case $a=b$ are described by the following conjecture.

Conjecture 5.1. *For any two positive mutually coprime numbers $a\neq u$ the polynomial $P_{au}(t)$ from* (5.6) *is irreducible in the ring of polynomials $\mathbb{Z}[t]$.*

Particular case $b\,u=a^2$. Comparing $b\,u=a^2$ with (4.8), it is easy to obtain the following presentation for integer numbers a, b, and u:

$$a=p\,q,\qquad\qquad b=p^2,\qquad\qquad u=q^2. \tag{5.9}$$

Here p and q are two parameters, i. e. two positive integer numbers, satisfying the condition of mutual coprimality

$$\mathrm{GCD}(p,q)=1. \tag{5.10}$$

Substituting (5.9) into (5.1), we obtain

$$\begin{gathered}P_{pqp^2q^2}(t)=t^{12}+(6\,q^4-2\,p^2\,q^2-2\,p^4)\,t^{10}+(q^8+4\,p^2\,q^6+\\+5\,p^4\,q^4-12\,p^6\,q^2+p^8)\,t^8-2\,p^2\,q^2\,(q^8-2\,p^2\,q^6+4\,p^4\,q^4-\\-2\,p^6\,q^2+p^8)\,t^6+p^4\,q^4\,(q^8-12\,p^2\,q^6+5\,p^4\,q^4+4\,p^6\,q^2+p^8)\,t^4+\\+q^8\,p^8\,(-2\,q^4-2\,p^2\,q^2+6\,p^4)\,t^2+q^{12}\,p^{12}.\end{gathered} \tag{5.11}$$

The polynomial $P_{pqp^2q^2}(t)$ in (5.11) is reducible. Indeed, we have the expansion

$$P_{pqp^2q^2}(t)=(t-a)\,(t+a)\,Q_{pq}(t), \tag{5.12}$$

where $Q_{pq}(t)$ is the following polynomial:

$$\begin{gathered}Q_{pq}(t)=t^{10}+(2\,q^2+p^2)\,(3\,q^2-2\,p^2)\,t^8+(q^8+10\,p^2\,q^6+\\+4\,p^4\,q^4-14\,p^6\,q^2+p^8)\,t^6-p^2\,q^2\,(q^8-14\,p^2\,q^6+4\,p^4\,q^4+\\+10\,p^6\,q^2+p^8)\,t^4-p^6\,q^6\,(q^2+2\,p^2)\,(-2\,q^2+3\,p^2)\,t^2-q^{10}\,p^{10}.\end{gathered} \tag{5.13}$$

Due to (5.12) the polynomial (5.11) possesses two rational roots $t=a$ and $t=-a$. Both of these roots do not satisfy conditions of Theorem 4.1, as they do not satisfy the inequality $t>a$.

Other roots of the polynomial (5.11) coincide with the roots of the polynomial $Q_{pq}(t)$ from (5.13). The polynomial (5.13) is reducible for $q=p$. In this case we have

$$Q_{pp}(t)=(t-a)\,(t+a)\,(t^2+a^2)^4. \tag{5.14}$$

The formula (5.14) is not a surprise. If $q=p$, one can easily derive $a=b=u$ from (5.9). But this case has already been considered (see (5.7) and (5.8)). From $q=p$ and from (5.10) we derive $p=q=1$ and $a=b=u=1$.

In the case $p\neq q$ the polynomial (5.13) is described by the following conjecture.

Conjecture 5.2. *For any two positive mutually coprime integer numbers $p\neq q$ the polynomial $Q_{pq}(t)$ from* (5.13) *is irreducible in the ring of polynomials $\mathbb{Z}[t]$.*

Particular case $a\,u=b^2$. This particular case is reduced to the previous one. Indeed, from $a\,u=b^2$ and from (4.8) we obtain

$$a=p^2, \qquad b=p\,q, \qquad u=q^2, \tag{5.15}$$

where p and q are two positive integer numbers, satisfying the condition of mutual coprimality (5.10). Being substituted into (5.1) the formulae (5.15) are equivalent to the formulae (5.9) due to the symmetry (5.2). These formulae result in the polynomial $P_{p^2pqq^2}(t)$ coinciding with the polynomial (5.11), and later they result in the polynomial (5.13), which has already been considered above.

Particular case $a=u$. This particular case is rather simple. In this case the polynomial $P_{abu}(t)=P_{ubu}(t)$ in (5.1) is reducible, and the following formula holds:

$$P_{ubu}(t)=(t^2+u^2)^4\,(t-b)^2\,(t+b)^2. \tag{5.16}$$

The polynomial (5.16) possesses two real rational roots $t=b$ and $t=-b$. Both of them do not satisfy the conditions of Theorem 4.1 since they do not satisfy the inequality $t>b$.

Particular case $b=u$. This particular case is equivalent to the previous one due to the symmetry (5.2).

The general case, which is not covered by the particular cases considered above in (5.3), is described by the following conjecture.

Conjecture 5.3. *For any three positive mutually coprime integer numbers a, b, u such that none of the conditions* (5.3) *is satisfied the polynomial* (5.1) *is irreducible in the ring of polynomials $\mathbb{Z}[t]$.*

6. Numerical test of conjectures

Presently no proofs of for the conjectures 5.1, 5.2, and 5.3 are available. Therefore, I have studied them numerically. For this purpose the Maxima package of the version 5.21.1 with the graphic shell wxMaxima 0.85 on the platform of Ubuntu 10.10 with the kernel Linux 2.5.35-24 was used.

The conjecture 5.1 was tested and proved for $1 \leqslant a \leqslant 100$, and $1 \leqslant u \leqslant 100$. The hypothesis 5.2 was proved for $1 \leqslant p \leqslant 100$ and $1 \leqslant q \leqslant 100$. And the third hypothesis 5.3 was checked an proved for $1 \leqslant a \leqslant 22$, $1 \leqslant b \leqslant 22$ and $1 \leqslant u \leqslant 22$. The number 22 was chosen intentionally because

$$22^3 = 10\,648 \approx 10\,000 = 100^2.$$

This equality implies that each conjecture was tested and confirmed for approximately 10 000 different combinations of numerical parameters in it. The total result of calculations sounds as follows: the equation (4.7) does not have solutions giving perfect cuboids for all values of the numbers a, b, u less or equal to 22.

7. Conclusions

Conjecrtures 5.1, 5.2, and 5.3 are not equivalent to the condition of non-existence of perfect cuboids. However, if they are valid, then it is sufficient to prove that perfect cuboids do not exist. The above results of numerical tests witness in favor of these conjectures.

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