ABSTRACTS

K.P. Isaev, R.S. Yulmukhametov

Unconditional exponential bases in Hilbert spaces

Abstract. In the present paper, we consider the existence of unconditional exponential bases in general Hilbert spaces $H = H(E)$ consisting of functions defined on some set $E \subset \mathbb{C}$ and satisfying the following conditions.

1. The norm in the space H is weaker than the uniform norm on E, i.e. the following estimate holds for some constant A and for any function f from H :

$$||f||_H \leq A \sup_{z \in E} |f(z)|.$$

2. The system of exponential functions $\{\exp(\lambda z),\ \lambda \in \mathbb{C}\}$ belongs to the subset H and it is complete in H.

It is proved that unconditional exponential bases cannot be constructed in H unless a certain condition is carried out.

Sufficiency of the weakened condition is proved for spaces defined more particularly.

Keywords: series of exponents, unconditional bases, Hilbert space.

L.S. Maergoiz, N. Tarkhanov

An analogue of the Paley-Wiener theorem and its applications to optimal recovery of entire functions

Abstract. Let W^p be the Wiener class of entire functions of exponential type in $\mathbb{C}^n$ belonging to $L^p(\mathbb{R}^n)$, where $1 < p < \infty$. Full analogues of the Paley-Wiener theorem for the class W^p and, in a multidimensional case, for the Plancherel-Pólya theorem on structure of the Fourier transform for any entire function $f \in W^2$, are obtained in a fundamentally new form in terms of the language of distributions. The results are applied to the problem of the best analytic continuation from a finite set of functions of the Wiener class. Of special interest is the description of the existence conditions for constructive algebraic formulae of characteristics for the optimal recovery of linear functionals.

Keywords: Wiener class of entire functions, Fourier transform, distributions, optimal linear algorithm, Chebyshev polynomial.

V.V. Napalkov(Jr.)

On orthosimilar systems in a space of analytical functions and the problem of describing the dual space

Abstract. We consider an orthosimilar system with the measure μ in the space of analytical functions H on the domain $G \subset \mathbb{C}$. Let $K_H(\xi, t)$, $\xi, t \in G$ be a reproduction kernel in the space H. We claim that a system $\{K_H(\xi, t)\}_{t \in G}$ is the orthosimilar system with the measure μ in the space H if and only if the space H coincides with the space $B_2(G, \mu)$. A problem of describing the dual space in terms of the Hilbert transform is considered. This problem is reduced to the problem of existence of a special orthosimilar system in $B_2(G, \mu)$. We prove that the space $\widetilde{B}_2(G, \mu)$ is the only space with a reproduction kernel and it consists of functions given on the domain $\mathbb{C} \backslash \overline{G}$ with an orthosimilar system $\{\frac{1}{(z-\xi)^2}\}_{\xi \in G}$ with the measure μ.

Keywords: Bergman space, Hilbert spaces, reproducing kernel, orthosimilar system, Hilbert transform.

D.K. Potapov

Estimation of the bifurcation parameter in spectral problems for equations with discontinuous operators

Abstract. We consider the existence of solutions of the eigenvalue problem for nonlinear equations with discontinuous operators in the reflexive Banach space. Coercivity of the corresponding mapping is not supposed. An upper bound for the value of the bifurcation parameter is obtained by the variational method. This result confirms that the upper bound for the value of the bifurcation parameter obtained earlier in spectral problems for elliptic equations with discontinuous nonlinearities is true.

Keywords: eigenvalues, spectral problems, discontinuous operator, variational method, upper bound, bifurcation parameter.

A.A. Putintseva

Riesz bases in weighted spaces

Abstract. The article deals with weighted Hilbert spaces with convex weights. Let h be a convex function on a bounded interval I of the real axis. We denote a space of locally integrable functions on I, such that

$$||f|| := \sqrt{\int_I |f(t)|^2 e^{-2h(t)}\, dt} < \infty$$

by $L_2(I, h)$.

If $I = (-\pi; \pi)$, $h(t) \equiv 1$, the space $L_2(I, h)$ coincides with the classical space $L_2(-\pi; \pi)$ and the Fourier trigonometric system is a Riesz basis in this space. As it has been shown by B.J. Levin, nonharmonic Riesz bases in $L_2(-\pi; \pi)$ can be constructed using a system of zeros of entire functions of sine type. In this paper we prove that if a Riesz basis of exponentials exists in the space $L_2(I, h)$, this space is isomorphic (as a normed space) to the classical space $L_2(I)$. Thus, the existence of Riesz bases of exponentials is the exclusive property of the classical space $L_2(-\pi; \pi)$.

Keywords: Riesz basis, weighted Hilbert spaces, reproducing kernel, Fourier-Laplace transform, functions of sine type.

R.S. Saks

Cauchy problem for the Navier-Stokes equations, Fourier method

Abstract. The Cauchy problem for the 3D Navier-Stokes equations with periodical conditions on the spatial variables is investigated. The vector functions under consideration are decomposed in Fourier series with respect to eigenfunctions of the curl operator. The problem is reduced to the Cauchy problem for Galerkin systems of ordinary differential equations with a simple structure. The program of reconstruction for these systems and numerical solutions of the Cauchy problems are realized. Several model problems are solved. The results are represented in a graphic form which illustrates the flows of the liquid. The linear homogeneous Cauchy problem is investigated in Gilbert spaces. Operator of this problem realizes isomorphism of these spaces. For a general case, some families of exact global solutions of the nonlinear Cauchy problem are found. Moreover, two Gilbert spaces with limited sequences of Galerkin approximations are written out.

Keywords: Fourier series, eigenfunctions of the curl operator, Navier-Stokes equations, Cauchy problem, global solutions, Galerkin systems, Gilbert spaces.

S.Ya. Startsev

Necessary conditions of Darboux integrability for differential-difference equations of a special kind

Abstract. This work dwells upon chains of differential equations of the form $\varphi(x, u_{i+1}, (u_{i+1})_x) = \psi(x, u_i, (u_i)_x)$, where u depends on the discrete variable i and the continuous variable x, and the functions $\varphi(x, y, z)$, $\psi(x, y, z)$ and x are functionally-independent. We demonstrate that necessary Darboux integrability conditions for chains of the above form can be easily derived from already known results. These conditions are not sufficient but may be useful for classification of Darboux-integrable differential-difference equations. As an auxiliary result, we also prove a proposition about structure of symmetries for differential-difference equations of a more general form.

Keywords: Darboux integrability, differential-difference equations

A.Y. Timofeev

Construction of functions with determined behavior $T_G(b)(z)$ at a singular point

Abstract. I.N. Vekua developed the theory of generalized analytic functions, i.e., solutions of the equation

$$\partial_{\bar{z}} w + A(z)w + B(z)\overline{w} = 0, \tag{0. 1}$$

where $z \in G$ (G, for example, is the unit disk on a complex plane) and the coefficients $A(z)$, $B(z)$ belong to $L_p(G)$, $p > 2$. The Vekua theory for the solutions of (0. 1) is closely related to the theory of holomorphic functions due to the so-called similarity principle. In this case, the T_G-operator plays an important role. The T_G-operator is right-inverse to $\frac{\partial}{\partial \bar{z}}$, where $\frac{\partial}{\partial \bar{z}}$ is understood in Sobolev's sense.

The author suggests a scheme for constructing the function $b(z)$ in the unit disk G with determined behavior $T_G(b)(z)$ at a singular point $z = 0$, where T_G is an integral Vekua operator. The paper states the conditions for $b(z)$ under which the function $T_G(b)(z)$ is continuous.

Keywords: T_G-operator, singular point, modulus of continuity.

P.V. Filevych

On the growth of the maximum modulus of an entire function depending on the growth of its central index

Abstract. Let h be a positive function continuous on $(0, +\infty)$, $f(z) = \sum_{n=0}^{\infty} a_n z^n$ be an entire function, and $M_f(r) = \max\{|f(z)| : |z| = r\}$, $\mu_f(r) = \max\{|a_n|r^n : n \geq 0\}$, and $\nu_f(r) = \max\{n \geq 0 : |a_n|r^n = \mu_f(r)\}$ be the maximum modulus, the maximal term, and the central index of the function f, respectively. We establish necessary and sufficient conditions for the growth of $\nu_f(r)$ under which $M_f(r) = O(\mu_f(r)h(\ln \mu_f(r)))$, $r \to +\infty$.

Keywords: entire function, maximum modulus, maximal term, central index, order, lower order.

Kh.A. Khachatryan

On solvability of a class of higher-order nonlinear integro-differential equations with a noncompact integral operator of the Hammerstein type

Abstract. In the present paper we investigate the question of solvability of one class of Hammerstein type N–order nonlinear integro-differential equations with noncompact integral operator on semi-axis in the Sobolev space $W_\infty^N(0, +\infty)$. The existence of a positive solution in $W_\infty^N(0, +\infty)$ is proved, and the limit of this solution at infinity is found. The obtained results are generalized for nonlinear equations with sum-difference kernels.

Keywords: Factorization, polynomial, limit of iteration, Sobolev space. invariants.