ABSTRACTS

F.G. Avkhadiev

Integral Hardy inequalities, their generalizations and related inequalities

Abstract. Hardy inequalities have numerous applications in mathematical physics and spectral theory of unbounded operators. In the paper we describe direct generalizations of integral Hardy inequalities, their improvements and analogues. We systemize the relations between various interpretations of these inequalities and describe new one-dimensional integral inequalities. We show that these known and new inequalities are valid also for complex-valued functions.

We consider in details integral inequalities of Hardy, Rellich and Birman type for functions defined on bounded intervals. In particular, we provide the proofs for the generalizations and improvements of Birman integral inequalities for higher derivatives. We briefly discuss multidimensional analogues involving integrals of the powers of the modulus of the gradient of a function or of a polyharmonic operator.

Keywords: Hardy inequality, Rellich inequality, Birman inequality, Lamb constant, polyharmonic operator.

A.O. Bagapsh

Perturbation method for strongly elliptic second order systems with constant coefficients

Abstract. We consider a classical Dirichlet problem for a second order strongly elliptic system with constant coefficients in Jordan domains in the plane. We show that the solution of the problem can be represented as a functional series in the powers of the parameter governing the deviation of the operator of the system from the Laplacian. This series converges uniformly in the closure of the domain under the assumption that the boundary of the domain and the given boundary function satisfy sufficient regularity conditions: the composition of the trace of a conformal mapping of the domain on the circle with the boundary function belongs to the Hölder class with the exponent exceeding $1/2$.

Keywords: strongly elliptic system, Dirichlet problem, perturbation method.

G.G. Braichev, B.N. Khabibullin, V.B. Sherstyukov

Sylvester problem, coverings by shifts, and uniqueness theorems for entire functions

Abstract. The idea to write this note arose during the discussion that followed the report of the first author at the International Scientific Conference "Ufa Autumn Mathematical School-2022". We propose three general methods for constructing uniqueness sets in classes of entire functions with growth restrictions. In all three cases, the sequence of zeros of an entire function with special properties is chosen as such a set. The first method is related to Sylvester famous problem on the smallest circle containing a given set of points on a plane, and theorems of convex geometry. The second method initially relies on Helly theorem on the intersection of convex sets and its application to the possibility of covering one set by shifting another. The third method is based on the classical Jensen formula, which allows one to estimate the type of an entire function via the averaged upper density of the sequence of its zeros. We present only basic results now. The development of our approaches is expected to be presented in subsequent works.

Keywords: Sylvester problem, Young theorem, Helly theorem, uniqueness set, type of entire function, sequence of zeros, indicator of an entire function, averaged upper density, Jensen formula, indicator diagram, smallest circle.

O.L. Vinogradov

Direct and inverse theorems of approximation theory in the Lebesgue spaces with Muckenhoupt weights

Abstract. In this work we establish direct and inverse theorems of approximation theory in Lebesgue spaces $L_{p,w}$ with Muckenhoupt weights w on the axis and on the period. The classical definition of modulus of continuity may not make sense in weighted spaces. Therefore, as moduli of continuity, including non-integer order, we use the norms of degrees of deviation of Steklov means. The properties of these quantities are derived, some of which are similar to the properties of usual moduli of continuity. In addition to the direct and converse theorems, we obtain equivalence relations between the moduli of continuity and the K and R-functionals.

The proofs are based on estimates of the norms of convolution operators and they do not employ a maximum function. This allows us to establish the results for all $p \in [1, +\infty)$ not excluding the case $p = 1$. Previously used methods that employed the maximum function in one form or another are unsuitable for $p \to 1$. In addition, by the convolution-based approach we can obtain results simultaneously in the periodic and non-periodic cases. With rare exceptions, constants are not specified explicitly, but their dependence on parameters is always tracked. All constants in the estimates depend on $[w]_p$ (Muckenhoupt characteristics of weight w), and there is no other dependence on w and p. The norms of convolution operators are evaluated explicitly in terms of $[w]_p$. The methods of this work can be applied to the proof of direct and converse theorems in more general functional spaces.

Keywords: best approximations, moduli of continuity, Muckenhoupt weights, convolution.

O.A. Ivanova, S.N. Melikhov

On invertibility of a Duhamel operator in spaces of ultradifferentiable functions

Abstract. Let Δ be a non-point segment or an (open) interval on the real line containing the point 0. In the space of entire functions realized by the Fourier-Laplace transform of the dual space to the space of ultradifferentiable or of all infinitely differentiable functions on Δ, we study the operators from the commutator subgroup of the one-dimensional perturbation of the inverse shift operator. We prove a criterion of their invertibility. In this case, the Riesz-Schauder theory is applied, the use of which in such a situation goes back to the works by V.A. Tkachenko. In the topological dual space to the original space, the multiplication $\circledast$ is introduced and we show that its dual space endowed with a strong topology is a topological algebra. Using the mapping associated with Fourier-Laplace transform, the introduced multiplication $\circledast$ is implemented as a generalized Duhamel product in the corresponding space of ultradifferentiable or infinitely differentiable functions on Δ. We prove a criterion for the invertibility of the Duhamel operator in this space. The multiplication $\circledast$ is used to extend the Duhamel's formula to classes of ultradifferentiable functions. It represents the solution of an inhomogeneous differential equation of finite order with constant coefficients, satisfying zero initial conditions at the point 0, in the form of Duhamel's product of the right-hand side and such a solution of this equation with the right-hand side identically equal to 1. The obtained results cover both the non-quasi-analytic and quasi-analytic cases.

Keywords: inverse shift operator, entire function, Duhamel product, ultradifferentiable function.

D.Y. Ivanov

On uniform convergence of semi-analytic solution of Dirichlet problem for dissipative Helmholtz equation in vicinity of boundary of two-dimensional domain

Abstract. In the framework of the collocation boundary element method, we propose a semi-analytic approximation of the double-layer potential, which ensures a uniform cubic convergence of the approximate solution to the Dirichlet problem for the Helmholtz equation in a two-dimensional bounded domain or its exterior with a boundary of class C^5. In order to calculate integrals on boundary elements, an exact integration over the variable $\rho := (r^2 - d^2)^{1/2}$ is used, where r and d are the distances from the observed point to integration point and to the boundary of the domain, respectively. Under some simplifications we prove that the use of a number of traditional quadrature formulas leads to a violation of the uniform convergence of potential approximations in the vicinity of the boundary of the domain. The theoretical conclusions are confirmed by a numerical solving of the problem in a circular domain.

Keywords: quadrature formula, double layer potential, Dirichlet problem, Helmholtz equation, boundary integral equation, almost singular integral, boundary layer phenomenon, uniform convergence.

E.H. Khalilov

Quadrature formula for normal derivative of double layer potential

Abstract. Looking for a solution to the Dirichlet and Neumann boundary value problems for the Helmholtz equation in the form of a combination of simple and double layer potentials, the considered boundary value problems are reduced to a curvilinear integral equation depending on the operators generated by the simple and double layer potentials and by their normal derivative. It is known that the latter operators are weakly singular integral operators. However, a counterexample constructed by Lyapunov shows that for a double layer potential with continuous density, the derivative, generally speaking, does not exist, that is, the operator generated by the normal derivative of the double layer potential is a singular integral operator.

Since in many cases it is impossible to find exact solutions to integral equations, it is of interest to study an approximate solution of the obtained integral equations. In its turn, in order to find an approximate solution, it is necessary, first of all, to construct quadrature formulas for the simple and double layer potentials of the and for their normal derivatives. In this work we prove the existence theorem for the normal derivative of the double layer potential and we provide a formula for its calculation. In addition, we develop a new method for constructing a quadrature formula for a singular curvilinear integral and on the base of this we construct a quadrature formula for the normal derivative of the double layer potential and we estimate the error.

Keywords: quadrature formulas, singular integral, double layer potential, Hankel function, Lyapunov curve.

Kh.V. Yadrikhinskiy, V.E. Fedorov

On linear-autonomous symmetries of the fractional model of Guéant–Pu

Abstract. We study the group properties of the Guéant-Pu model with a fractional order in time, which describes the dynamics of option pricing. We find the groups of linear-autonomous equivalence transformations of the corresponding equation. With their help, we obtain a group classification of the fractional Guéant-Pu model with a nonlinear free element. In the case of a non-zero risk-free interest rate r, the underlying Lie algebra of such a model is one-dimensional. For zero r, the main Lie algebra is three-dimensional in the case of a special right-hand side and it is two-dimensional otherwise.

Keywords: Riemann-Liouville fractional derivative, fractional Gehan-Pou model, symmetry analysis, linear-autonomous transformation, group of equivalence transformations, group classification.