ABSTRACTS

N.F. Abuzyarova, K.P. Isaev, R.S. Yulmukhametov

Equivalence of norms of analytical functions on exterior of a convex domain

Abstract. We study the spaces functions holomorphic in the exterior of a bounded domain D and vanishing at infinity. For each $\alpha > -\frac{1}{2}$ we introduce the integral weighted normed space $B_2^\alpha(G)$ with the weight $d^\alpha(z)$, where $d(z)$ denotes the distance from a point z to the boundary of $G := \mathbb{C} \setminus \overline{D}$. For $\alpha = -\frac{1}{2}$, we the space B_2^α is chosen to be the Smirnov space. We prove that for convex domain D the norm in these spaces is equivalent to other norms defined in terms of the derivatives. For instance, the norm in the Smirnov space calculated as an integral with respect to the arc length over the boundary is equivalent to some norm defined by an integral with respect to the Lebesgue plane measure. In particular cases the proved results were obtained while studying the problem of describing the classes of Cauchy transforms of the functionals on the Bergman space on D. The general results may be applied in the study of Cauchy transforms of functionals on weighted Bergman spaces.

Keywords: analytic functions, Banach spaces, convex sets.

S.M. Andriyan, A.K. Kroyan, Kh.A. Khachatryan

On solvability of a class of nonlinear integral equations in p-adic string theory

Abstract. In this paper we study a class of integral equations with power nonlinearity on the entire line is studied. This class of equations arises in the p-adic theory of open-closed strings. Using the method of successive approximations and with the justification of their convergence, we prove the existence of a nontrivial continuous odd bounded solution on the entire line. The asymptotic behavior of the solution is studied as the argument increases unboundedly. We obtain integral estimates and some properties of approximations of the solution to the considered equation are obtained. Under some additional restrictions, we also establish the uniqueness of the constructed solution in a certain class of continuous functions. We provide examples of integral kernels of the equation satisfying all assumptions of the formulated theorems are given. As the nuclear function is a Gaussian distribution, from the proven we obtain Vladimirov-Volovich theorem as a special case.

Keywords: successive approximations, limit of solution, pointwise convergence, continuity.

L.G. Valiullina, Kh.K. Ishkin, R.I. Marvanov

Spectral asymptotics for fourth order differential operator with two turning points

Abstract. The paper is devoted to studying the asymptotics of the spectrum of a self-adjoint operator T generated in the space $L^2(0,+\infty)$ by a fourth-order differential expression under the assumption that the coefficients of the latter have a power growth at infinity such that: a) the deficiency index of the corresponding minimal operator is (2.2), b) or sufficiently large positive values of a spectral parameter, the differential equation $Ty = \lambda y$ f has two turning points: a finite onel and $+\infty$, c) the roots of the characteristic equation grow "not with the same rate". The latter assumption leads one to significant difficulties in studying the asymptotics of the counting function for the spectrum by the traditional Carleman–Kostyuchenko method based on estimates of the resolvent far from the spectrum and Tauberian theorems. Curiously enough, the method of reference equations used to solve the more subtle problem of finding asymptotic expansions of the eigenvalues themselves, and therefore more sensitive (compared to the Carleman–Kostyuchenko method) to the behavior of the coefficients in the differential expression is more effective in the considered situation: imposing on coefficients some constraints such as smoothness and regular growth at infinity, we obtain an asymptotic equation for the spectrum of the operator T. This equation allows one to write out the first few terms of the asymptotic expansion for the eigenvalues of the operator T in the case when the coefficients have a power growth. We also note that so far the method of reference equations has been used only in the case of the presence of the only turning point.

Keywords: differential operators, spectral asymptotics, turning point, singular numbers

V.F. Vil'danova

On the uniqueness of a weak solution to integro-differential aggregation equation

Abstract. In a well-known paper by A. Bertozzi, D. Slepcev (2010), there was established the existence and uniqueness of solution to a mixed problem for the aggregation equation

$$u_t - \Delta A(x,u) + \operatorname{div}(u\nabla K * u) = 0$$

describing the evolution of a colony of bacteria in a bounded convex domain Ω. In this paper we prove the existence and uniqueness of the solution to a mixed problem for a more general equation

$$\beta(x,u)_t = \operatorname{div}(\nabla A(x,u) - \beta(x,u)G(u)) + f(x,u).$$

The term $f(x,u)$ in the equation models the processes of "birth-destruction" of bacteria. The class of integral operators $G(v)$ is wide enough and contains, in particular, the convolution operators $\nabla K * u$. The vector kernel $g(x,y)$ of the operator $G(u)$ can have singularities.

Proof of the uniqueness of the solution in the work by A. Bertozzi, D. Slepcev was based on the conservation of the mass $\int_\Omega u(x,t)dx = const$ of bacteria and employed the convexity of Ω and the properties of the convolution operator. The presence of the "inhomogeneity" $f(x,u)$ violates the mass conservation. The proof of uniqueness proposed in the paper is suitable for a nonuniform equation and does not use the convexity of Ω.

Keywords: aggregation equation, integro-differential equation, global solution, uniqueness of solution.

A.M. Gaisin, G.A. Gaisina

The order of a Dirichlet series with a regular distribution of the exponents in the half-strips

Abstract. We study the Dirichlet series $F(s) = \sum\limits_{n=1}^{\infty} a_n e^{\lambda_n s}$ with positive and unboundedly increasing exponents λ_n. We assume that the sequence of the exponents $\Lambda = \{\lambda_n\}$ has a finite density; we denote this density by b. We suppose that the sequence Λ is regularly distributed. It is understood in the following sense: there exists a positive concave function H in the convergence class such that

$$\Lambda(t) - bt \leqslant H(t) \quad (t > 0).$$

Here $\Lambda(t)$ is the counting function of the sequence Λ. We show that if, in addition, the growth of the function H is not very high, the orders of the function F in the sense of Ritt in any closed semi-strips, the width of each of which is not less than $2\pi b$, are equal. Moreover, we do not assume additional restrictions for the nearness and concentration of the points λ_n. The corresponding result for open semi-strips was previously obtained by A.M. Gaisin and N.N. Aitkuzhina.

It is shown that if the width of one of the two semi-strips is less than $2\pi b$, then the Ritt's orders of the Dirichlet series in these semi-strips are not equal.

Keywords: R-density of sequence, Dirichlet series, R-order, semi-strip, half-plane.

L.A. Kalyakin

Capture and holding of resonance far from equilibrium

Abstract. Capture into resonance occurs in nonlinear oscillating systems. The study of mathematical models of this phenomenon is a part of a modern theory of nonlinear oscillations. The known result in this field were obtained by averaging method in the asymptotic regime with a small parameter. In this way, an initial stage of the capture into resonance was studied in details.

The matter of this approach is an asymptotic passage to a model equation of mathematical pendulum kind. In the present work we consider an asymptotic construction at long time, which describes a slow evolution of a solution captured into resonance. The main aim is to determine a time interval, during which the resonance is held. The problem is reduced to studying a perturbation of a model equation of pendulum type. Our main success is the description of the time interval, in which the resonance is captured, and the description is given in terms of the data in the initial problem. Formally we consider a nonlinear oscillating system with a small perturbation. The perturbation is described by an external pumping with a prescribed slowly changing frequency. For the solutions captured into the resonance, we consider asymptotics with respect to the small parameter. We write out an equation, the solution to which allows us to find the time of the capturing into resonance.

Keywords: nonlinear oscillations, perturbation, small parameter, asymptotics, capture in resonance.

A.B. Muravnik

On qualitative properties of solutions to quasilinear parabolic equations admitting degenerations at infinity

Abstract. We consider the Cauchy problem for a quasilinear parabolic equations $\rho(x)u_t = \Delta u + g(u)|\nabla u|^2$, where the positive coefficient ρ admits a degeneration at infinity, while the coefficient g either is a continuous function or admits singularities of at most first power. These nonlinearities called Kardar–Parisi–Zhang nonlinearities (or KPZ-nonlinearities) arise in various applications (in particular, in modelling directed polymer and interface growth). Also, they are of an independent theoretical interest because they contain the second powers of the first derivatives: this is the greatest power exponent such that Bernstein-type conditions for the corresponding elliptic problem ensure a priori L_∞-estimates of first order derivatives of the solution via the L_∞-estimate of the solution itself. Earlier, the asymptotic properties of solutions to parabolic equations with nonlinearities of the specified kind where studied only for the case of an uniformly parabolic linear part. Once the coefficient ρ degenerates (at least at infinity), the nature of the problem changes qualitatively, which is confirmed by the presented study of qualitative properties of (classical) solutions of the specified Cauchy problem. We find conditions for the coefficient ρ and the initial value function guaranteeing the following behavior of the specified solutions: there exists a (limit) Lipschitz function $A(t)$ such that, for any positive t, the generalized spherical mean of the solution tends to the specified Lipschitz function as the radius of the sphere tends to infinity. The generalized spherical mean is constructed as follows. First, we apply a monotone function to a solution; this monotone function is determined only by the coefficient at the nonlinearity (whether that coefficient is regular or singular). Then we compute the mean over the $(n-1)$-dimensional sphere centered at the origin (in the linear case, this mean naturally reduces to a classical spherical mean). To construct the specified monotone function, we use the Bitsadze method allowing one to express solutions of the studied quasilinear equations via solutions to semi-linear equations.

Keywords: parabolic equations, KPZ-nonlinearities, long-time behavior, degeneration at infinity.

I.Kh. Musin

On some linear operators on Fock type space

Abstract. We consider a lower semi-continuous function φ in $\mathbb{R}^n$ depending on the absolute values of the variables and growing faster than $a\ln(1+\|x\|)$ for each positive a. In terms of this function, we define a Hilbert space F^2_φ of entire functions in $\mathbb{C}^n$. This is a natural generalization of a classical Fock space. In this paper we provide an alternative description of the space F^2_φ in terms of the coefficients in the power expansions for the entire functions in this space. We mention simplest properties of reproducing kernels in the space F^2_φ. We consider the orthogonal projector from the space L^2_φ of measurable complex-valued functions f in $\mathbb{C}^n$ such that

$$\|f\|^2_\varphi = \int_{\mathbb{C}^n} |f(z)|^2 e^{-2\varphi(\operatorname{abs} z)}\, d\mu_n(z) < \infty,$$

where $z = (z_1, \ldots, z_n)$, $\operatorname{abs} z = (|z_1|, \ldots, |z_1|)$, on its closed subspace F^2_φ, and for this projector we obtain an integral representation.

We also obtain an integral formula for the trace of a positive linear continuous operator on the space F^2_φ. By means of this formula we find the conditions, under which a weighted operator of the composition on F^2_φ is a Hilbert-Schmidt operator. Two latter results generalize corresponding results by Sei-ichiro Ueki, who studied similar questions for operators in Fock space.

Keywords: entire functions, Fock type space, linear operators, operator trace, weighted composition operators, Hilbert-Schmidt operator.

V.A. Pavlenko, B.I. Suleimanov

Solutions to analogues of non-stationary Schrödinger equations defined by isomonodromic Hamilton system $H^{2+1+1+1}$

Abstract. We construct simultaneous solutions to two analogues of time-dependent solutions to Schrödinger equations defined by the Hamiltonians $H^{2+1+1+1}_{s_k}(s_1, s_2, q_1, q_2, p_1, p_2)$ $(k = 1, 2)$ to system $H^{2+1+1+1}$. This system is the first representative in a famous degenerations hierarchy of the Garnier system described in 1986 by H. Kimura. By an explicit symplectic transformation, this system reduces to a symmetric Hamilton system. In the constructions of this paper we rely mostly on linear systems of equations in the method of isomonodromic deformations for the system $H^{2+1+1+1}$ written out in 2012 in a paper by A. Kavakami, A. Nakamura and H. Sakai. These analogues of the non-stationary Schrödinger equations are evolutionary equations with times s_1 and s_2, which depend of two spatial variables. From the canonical non-stationary Schrödinger equations, these analogues arise as a result of the formal replacement of the Planck constant by $-2\pi i$. We construct the exact solutions to the two evolution equations in terms of the solutions to corresponding linear ordinary differential equations in the method of isomonodromic deformations. We discuss further prospects for constructing similar solutions to analogues of the non-stationary Schrödinger equations corresponding to the Hamiltonians of the entire degeneracy hierarchy of the Garnier system.

Keywords: Hamilton systems, Schrödinger equation, Painlevé equations, method of isomonodromic deformations.

S.Ya. Startsev

Structure of a set of symmetries for hyperbolic systems of Liouville type and generalized Laplace invariants

Abstract. The present paper is devoted to hyperbolic systems consisting of n partial differential equations and possessing symmetry drivers, i.e. differential operators that map any function of one independent variable into a symmetry of the corresponding system. The presence of the symmetry drivers is a feature of the Liouville equation and similar systems. The composition of a differential operator with a symmetry driver is a symmetry driver again if the coefficients of the differential operator belong to the kernel of a total derivative. We prove that the entire set of the symmetry drivers is generated via the above compositions from a basis set consisting of at most n symmetry drivers whose sum of orders is smallest possible.

We also prove that if a system admits a symmetry driver of order $k - 1$ and generalized Laplace invariants are well-defined for this system, then the leading coefficient of the symmetry driver belongs to the kernel of the k-th Laplace invariant. Basing on this statement, after calculating the Laplace invariants of a system, we can obtain the lower bound for the smallest orders of the symmetry drivers for this system. This allows us to check whether we can guarantee that a particular set of the drivers is a basis.

Keywords: higher symmetries, symmetry drivers, nonlinear hyperbolic partial differential systems, Laplace invariants, Darboux integrability.

T.G. Ergashev

Third double-layer potential for a generalized bi-axially symmetric Helmholtz equation

Abstract. The double-layer potential plays an important role in solving boundary value problems for elliptic equations, and in studying this potential, the properties of the fundamental solutions of the given equation are used. At present, all fundamental solutions to the generalized bi-axially symmetric Helmholtz equation are known but nevertheless, only for the first of them the potential theory was constructed. In this paper we study the double layer potential corresponding to the third fundamental solution. By using properties of Appell hypergeometric functions of two variables, we prove limiting theorems and derive integral equations involving the density of double-layer potentials in their kernels.

Keywords: generalized bi-axially symmetric Helmholtz equation; Green formula; fundamental solution; third double-layer potential; Appell hypergeometric functions of two variables; integral equations with a density of double-layer potential in their kernel.

Ya. Il'yasov, N. Valeev

On inverse spectral problem and generalized Sturm nodal theorem for nonlinear boundary value problems

Abstract. In the present paper, we are concerned with the Sturm-Liouville operator

$$\mathcal{L}[q]u := -u'' + q(x)u$$

subject to the separated boundary conditions. We suppose that $q \in L^2(0, \pi)$ and study a so-called inverse optimization spectral problem: given a potential q_0 and a value λ_k, where $k = 1, 2, \ldots$, find a potential $\hat{q}$ closest to q_0 in the norm of $L^2(0, \pi)$ such that the value λ_k coincides with k-th eigenvalue $\lambda_k(\hat{q})$ of the operator $\mathcal{L}[\hat{q}]$.

In the main result, we prove that this problem is related to the existence of a solution to a boundary value problem for the nonlinear equation

$$-u'' + q_0(x)u = \lambda_k u + \sigma u^3$$

with $\sigma = 1$ or $\sigma = -1$. This implies that the minimizing solution of the inverse optimization spectral problem can be obtained by solving the corresponding nonlinear boundary value problem. On the other hand, this relationship allows us to establish an explicit formula for the solution to the nonlinear equation by finding the minimizer of the corresponding inverse optimization spectral problem. As a consequence of this result, a new method of proving the generalized Sturm nodal theorem for the nonlinear boundary value problems is obtained.

Keywords: Sturm-Liouville operator, inverse optimization spectral problem, nodal theorem for the nonlinear boundary value problems.

A. Soukhov

Discs and boundary uniqueness for psh functions on an almost complex manifold

Abstract. This paper is inspired by the work by J.-P. Rosay (2010). In this work, there was sketched a proof of the fact that a totally real submanifold of dimension 2 can not be a pluripolar subset of an almost complex manifold of complex dimension 2. In the present paper we prove a considerably more general result, which can be viewed as a boundary uniqueness theorem for plurisubharmonic functions. It states that a function plurisubharmonic in a wedge with a generic totally real edge is equal to $-\infty$ identically if it tends to $-\infty$ approaching the edge. Our proof is completely different from the argument by J.-P. Rosay. We develop a method based on construction of a suitable family of J-complex discs. The origin of this approach is due to the well-known work by S. Pinchuk (1974), where the case of the standard complex structure was settled. The required family of complex discs is obtained as a solution to a suitable integral equation generalizing the classical Bishop method. In the almost complex case this equation arises from the Cauchy-Green type formula. We hope that the almost complex version of this construction presented here will have other applications.

Keywords: almost complex structure, plurisubharmonic function, complex disc, totally real manifold.