ABSTRACTS

A.A. Bobodzhanov, V.F. Safonov

Regularized asymptotics of solutions to integro-differential partial differential equations with rapidly varying kernels

Abstract. We generalized the Lomov's regularization method for partial differential equations with integral operators, whose kernel contains a rapidly varying exponential factor. We study the case when the upper limit of the integral operator coincides with the differentiation variable. For such problems we develop an algorithm for constructing regularized asymptotics. In contrast to the work by Imanaliev M.I., where for analogous problems with slowly varying kernel only the passage to the limit studied as the small parameter tended to zero, here we construct an asymptotic solution of any order (with respect to the parameter). We note that the Lomov's regularization method was used mainly for ordinary singularly perturbed integro-differential equations (see detailed bibliography at the end of the article). In one of the authors' papers the case of a partial differential equation with slowly varying kernel was considered. The development of this method for partial differential equations with rapidly changing kernel was not made before. The type of the upper limit of an integral operator in such equations generates two fundamentally different situations. The most difficult situation is when the upper limit of the integration operator does not coincide with the differentiation variable. As studies have shown, in this case, the integral operator can have characteristic values, and for the construction of the asymptotics, more strict conditions on the initial data of the problem are required. It is clear that these difficulties also arise in the study of an integro-differential system with a rapidly changing kernels, therefore in this paper the case of the dependence of the upper limit of an integral operator on the variable x is deliberately avoided. In addition, it is assumed that the same regularity is observed in a rapidly decreasing kernel exponent integral operator. Any deviations from these (seemingly insignificant) limitations greatly complicate the problem from the point of view of constructing its asymptotic solution. We expect that in our further works in this direction we will succeed to weak these restrictions.

Keywords: singularly perturbed, integro-differential equation, regularization of the integral.

D.I. Borisov

On spectral gaps of a Laplacian in a strip with a bounded periodic perturbation

Abstract. In the work we consider the Laplacian subject to the Dirichlet condition in an infinite planar strip perturbed by a periodic operator. The perturbation is introduced as an arbitrary bounded periodic operator in L_2 on the periodicity cell; then this operator is extended periodically on the entire strip.

We study the band spectrum of such operator. The main obtained result is the absence of the spectral gaps in the lower part of the spectrum for a sufficiently small

potential. The upper bound for the period ensuring such result is written explicitly as a certain number. It also involves a certain characteristics of the perturbing operator, which can be nonrigorously described as "the maximal oscillation of the perturbation". We also explicitly write out the length of the part of the spectrum, in which the absence of the gaps is guaranteed. Such result can be regarded as a partial proof of the strong Bethe-Sommerfeld conjecture on absence of internal gaps in the band spectra of periodic operators for sufficiently small periods.

Keywords: periodic operator, Schrödinger operator, strip, Bethe-Sommerfeld conjecture

T.R. Gadylshin, F.Kh. Mukminov

Perturbation of second order nonlinear equations by delta-like potential

Abstract. We consider boundary value problems for one-dimensional second order quasilinear equation on bounded and unbounded intervals I of the real axis. The equation perturbed by the delta-shaped potential $\varepsilon^{-1}Q(\varepsilon^{-1}x)$, where $Q(\xi)$ is a compactly supported function, $0 < \varepsilon \ll 1$. The mean value of $\langle Q\rangle$ can be negative, but it is assumed to be bounded from below $\langle Q\rangle \geqslant -m_0$. The number m_0 is defined in terms of coefficients of the equation. We study the convergence rate of the solution of the perturbed problem u^ε to the solution of the limit problem u_0 as the parameter ε tends to zero. In the case of a bounded interval I, the estimate of the form $|u^\varepsilon(x) - u_0(x)| < C\varepsilon$ is established. As the interval I is unbounded, we prove a weaker estimate $|u^\varepsilon(x) - u_0(x)/ < C\varepsilon^{1/2}$. The estimates are proved by using original cut-off functions as trial functions. For simplicity, the proof of the existence of solutions to perturbed and limiting problems are made by the method of contracting mappings. The disadvantage of this approach, as it is known, is the smallness of the nonlinearities in the equation. We consider the cases of the Dirichlet, Neumann and Robin condition.

Keywords: second order nonlinear equation, delta-like potential, small parameter.

N.I. Zhukova

The influence of stratification on the groups of conformal transformations of pseudo-Riemannian orbifolds

Abstract. We study the groups of conformal transformations of n-dimensional pseudo-Riemannian orbifolds $(\mathcal{N}, g)$ as $n \geqslant 3$. We extend the Alekseevskii method for studying conformal transformation groups of Riemannian manifolds to psevdo-Riemannian orbifolds. We show that a conformal pseudo-Riemannian geometry is induced on each stratum of that orbifold. Due to this, for $k \in \{0, 1\} \cup \{3, \ldots, n-1\}$, we obtain exact estimates for the dimensions of the conformal transformation groups of n-dimensional pseudo-Riemannian orbifolds admitting k-dimensional strata with essential conformal transformation groups. A key fact in obtaining these estimates is that each connected transformation group of an orbifold preserves every connected component of each its strata.

The influence of stratification of n-dimensional pseudo-Riemann orbifold to the similarity transformation group of this orbifold is also studied for $n \geqslant 2$.

We prove that the obtained estimates for the dimension of the complete essential groups of conformal transformations and the similarity transformation groups of n-dimensional pseudo-Riemann orbifolds are sharp; this is done by adducing corresponding examples of locally flat pseudo-Riemannian orbifolds

Keywords: orbifold, conformal pseudo-Riemannian geometry, conformal transformation, Lie group

O.A. Krivosheeva

Basis in a invariant subspace of analytical functions

Abstract. In this work we study the problem on representing the functions in an invariant subspace of analytic functions on a convex domain in complex plane. We obtain a sufficient condition for the existence of a basis in the invariant subspace consisting of linear combinations of eigenfunctions and associated functions of differentiation operator in this subspace. The linear combinations are constructed by the system of exponential monomials, whose exponents are split into relatively small groups. We apply the method using the Leontiev's interpolating function. At that, we provide a complete description of the space of the coefficients of the series representing the functions in the invariant subspace. We also find necessary conditions for representing functions in an arbitrary invariant subspace admitting the spectral synthesis in an arbitrary convex domain. We employ the method of constructing special series of exponential polynomials developed by the author.

Keywords: Invariant subspace, basis, exponential monomial, entire function, series of exponent.

S.G. Merzlyakov

Systems of convolution equations in complex domains

Abstract. In this paper we study the systems of convolution equations in spaces of vector-valued functions of one variable. For such systems, we define an analogue of the Leontiev's interpolating function and we provide a series of the properties of this function. In order to study these systems, we introduce a geometric difference of sets and provide its properties.

We prove a theorem on the representation of arbitrary vector-valued functions as a series over elementary solutions to the homogeneous system of convolution equations. These results generalize some well-known results by A.F. Leontiev on methods of summing a series of elementary solutions as an arbitrary solution and strengthen the results by I.F. Krasichkov-Ternovskii on summability of a quadratic system of convolution equations.

We describe explicitly domains in which a series of elementary solutions converges for arbitrary vector-valued functions. These domains depend on the domains of the vector-valued functions, on the growth of the Laplace transform of the elements in this system, and on the lower bound of its determinant. We adduce examples showing the sharpness of this result.

Similar results are obtained for solutions to a homogeneous system of convolution equations, and examples are given in which the series converges in the entire domain of a vector-valued function.

Keywords: Systems of convolution equations, vector-valued functions, Leontiev's interpolating function, series of elementary solutions.

A.Yu. Trynin

Uniform convergence of Lagrange-Strum-Liouville processes on one functional class

Abstract. We establish the uniform convergence inside an arbitrary interval $(a, b) \subset [0, \pi]$ for the values of the Lagrange-Sturm-Liouville operators for functions in a class defined by one-side moduli of continuity and oscillations. Outside this interval, the sequence of values of the Lagrange-Sturm-Liouville operators may diverge. The conditions describing this functional class contain a restriction only on the rate and magnitude of the increasing (or decreasing) of the continuous function. Each element of the proposed class can decrease (or, respectively, increase) arbitrarily fast. Popular sets of functions satisfying the Dini-Lipschitz condition or the Krylov criterion are proper subsets of this class, even if, under their conditions, the classical modulus of continuity and the variation are replaced by the one-sided ones. We obtain sharp upper bounds for functions and Lebesgue constants of the Lagrange-Sturm-Liouville processes. We establish sufficient conditions of the uniform convergence of the Lagrange-Sturm-Liouville processes in terms of the maximal absolute value of the sum and the maximal sum of the absolute values of the weighted first order differences. We prove the boundedness in the aggregate of the sequence of fundamental functions of Lagrange-Sturm-Liouville processes. Three new operators are proposed, which are modifications of the Lagrange-Sturm-Liouville operator and they allow one to approximate uniformly an arbitrary continuous function vanishing at the ends on the segment $[0, \pi]$. All the results of the work remain valid if the one-sided moduli of continuity and oscillations are replaced by the classical ones.

Keywords: sinc approximation, interpolation functions, uniform approximation.

A.I. Fedotov

Approximation of solutions to singular integro-differential equations by Hermite-Fejer polynomials

Abstract. Singular integral and integro-differential equations have a lot of applications and thus were thoroughly studied by domestic and foreign mathematicians since the beginning of 20th century, and by the 70th years the theory of such equations was finally completed. It is known from this theory that the exact solutions to such equations exist only in rarely particular cases, so since that time the approximate methods for solving these equations as well as the techniques of the justification of these methods were developed. Justification of the approximate method means the proof of the existence and the uniqueness of the approximate solution, estimation of its error and the proof of the convergence of the approximate solutions to the exact solution. Moreover, to compare the approximate methods in different aspects, the theory of optimization of the approximate methods was created.

However, sometimes, depending on the particular problem, an important role is also played by the form of an approximate solution. For instance, sometimes it is desirable to have an approximate solution as a spline, sometimes, as a polynomial, sometimes it is enough to have just the approximate values of the solution at the nodes. It is quite obvious that depending on the kind of the approximate solution the technique of the justification of the method should be chosen. Unfortunately,

there are very few of such techniques, that is why the theory of justification of the approximate methods is now intensively studied.

In the present work we justify an approximate method for solving singular integro-differential equations in the periodic case. An approximate solution is sought as a trigonometric interpolation Hermite-Fejer polynomials. For justification of this approximate method, the technique developed by B.G. Gabdulkhaev and his pupils is used. The convergence of the method is proved and the errors of the approximate solutions are estimated.

Keywords: singular integro-differential equations, justification of the approximate methods.

N.U. Khan, T. Usman

Certain generating functions of Hermite-Bernoulli-Legendre polynomials

Abstract. The special polynomials of more than one variable provide new means of analysis for the solutions of a wide class of partial differential equations often encountered in physical problems. Most of the special function of mathematical physics and their generalization have been suggested by physical problems. It turns out very often that the solution of a given problem in physics or applied mathematics requires the evaluation of an infinite sum involving special functions. Problems of this type arise, e.g., in the computation of the higher-order moments of a distribution or in evaluating transition matrix elements in quantum mechanics. Motivated by their importance and potential for applications in a variety of research fields, recently, numerous polynomials and their extensions have been introduced and studied. In this paper, we introduce a new class of generating functions for Hermite-Bernoulli-Legendre polynomials and study certain implicit summation formulae by using different analytical means and applying generating function. We also introduce bilateral series associated with the newly-introduced generating function by appropriately specializing a number of known or new partly unilateral and partly bilateral generating functions. The results presented here, being very general, are pointed out to be specialized to yield a number of known and new identities involving relatively simpler and familiar polynomials.

Keywords: 2-variable Hermite polynomials, Generalized Bernoulli numbers and polynomials, 2-variable Legendre polynomials, 3-variable Hermite-Bernoulli-Legendre polynomials, summation formulae, generating functions.

Ashok Rathod

Nevanlinna's five-value theorem for algebroid functions

Abstract. By using the second main theorem of the algebroid function, we study the problem let $W_1(z)$ and $W_2(z)$ be two ν-valued, non-constant algebroid functions, let $a_j\,(j = 1, 2, \ldots, q)$ be $q \geqslant 4\nu + 1$ distinct complex numbers or ∞. Suppose that $k_1 \geqslant k_2 \geqslant \ldots \geqslant k_q, m$ are positive integers or ∞; $1 \leqslant m \leqslant q$ and $\delta_j(\geqslant 0)(j = 1, 2, \ldots, q)$ are such that

$$\left(1 + \frac{1}{k_m}\right) \sum_{j=m}^{q} \frac{1}{1 + k_j} + 3\nu + \sum_{j=1}^{q} \delta_j < (q - m - 1)\left(1 + \frac{1}{k_m}\right) + m.$$

Let $B_j = \overline{E}_{k_j}(a_j, f) \backslash \overline{E}_{k_j}(a_j, g)$ for $j = 1, 2, \ldots, q$. If

$$\overline{N}_{B_j}(r, \frac{1}{W_1 - a_j}) \leqslant \delta_j T(r, W_1)$$

and

$$\liminf_{r\to\infty} \frac{\sum\limits_{j=1}^{q} \overline{N}_{k_j}(r, \frac{1}{W_1 - a_j})}{\sum\limits_{j=1}^{q} \overline{N}_{k_j}(r, \frac{1}{W_2 - a_j})} > \frac{\nu k_m}{(1 + k_m) \sum\limits_{j=1}^{q} \frac{k_j}{k_j+1} - 2\nu(1 + k_m) + (m - 2\nu - \sum\limits_{j=1}^{q} \delta_j)k_m}$$

then $W_1(z) \equiv W_2(z)$.

Above result improve and generalize the previous results given by Xuan and Gao.

Keywords: value distribution theory, Nevanlinna theory, algebroid functions, uniqueness.