ABSTRACTS

M.K. Arabov, E. Mukhamadiev, I.D. Nurov, Kh.I. Sobirov

Existence tests for limiting cycles of second order differential equations

Abstract. This work is devoted to finding limiting cycles in the vicinity of equilibria of second order nonlinear differential equations. We obtain new conditions for the coefficients of the equations ensuring the existence of a limiting cycle by employing the methods of qualitative analysis and computer modeling. We study the behavior of a singular point under variation of the parameters and we applied the Lyapunov stability theory. On the base of the obtained results, we make a sector partition of the plane. This partition allows us to predict the behavior of the solutions in various parts of the plane. We develop a package of computer programs for constructing a phase portrait in the corresponding domains.

Keywords: dynamical systems, nonsmoothness, phase portraits, limiting cycles, sectorial partitions.

R.K. Gazizov, A.A. Gainetdinova

Invariant differentiation operator and its application for integrating systems of ordinary differential equations

Abstract. We propose an algorithm for integrating n-th order ordinary differential equations (ODE) admitting n-dimensional Lie algebras of operators. The algorithm is based on invariant representation of the considered equation by the invariants of the admitted Lie algebra and application of an invariant differentiation operator of special type. We show that in the case of scalar equations this method is equivalent to the known order reduction methods. We study an applicability of the suggested algorithm to the systems of m k-th order ODEs admitting km-dimensional Lie algebras of operators. For the admitted Lie algebra we obtain a condition ensuring the possibility to construct the invariant differentiation operator of a special type and to reduce the order of the considered system of ODEs. This condition is the implication of the existence of nontrivial solutions to the systems of linear algebraic equations, where the coefficients are the structural constants of the Lie algebra. We present an algorithm for constructing the $(km-1)$-dimensional Lie algebra for the reduced system. The suggested approach is applied for integrating the systems of two second-order equations.

Keywords: ordinary differential equations, Lie algebras of operators, differential invariants, invariant differentiation operator.

R.A. Gaisin

Pavlov-Korevaar-Dixon interpolation problem with majorant in convergence class

Abstract. We study an interpolation problem in the class of entire functions of exponential type determined by some majorant in a convergence class (non-quasianalytic majorant). In a smaller class, when the majorant possessed a concavity property, similar problems was studied by B. Berndtsson with the nodes at some subsequence of natural numbers. He obtained a solvability criterion for this interpolation problem. At that, he applied first the Hörmander method for solving a $\overline{\partial}$-problem. In works by A.I. Pavlov, J. Korevaar and M. Dixon, interpolation sequences in the Berndtsson sense were applied successfully in a series of problems in the complex analysis. At that, there was found a relation with approximative properties of the system of powers $\{z^{p_n}\}$ and with the well known Polya and Macintyre problems.

In this paper we establish the criterion of the interpolation property in a more general sense for an arbitrary sequence of real numbers. In the proof of the main theorem we employ a modification of the Berndtsson method.

Keywords: interpolation sequence, entire function, convergence class.

L.K. Zhapsarbayeva, B.E. Kanguzhin, M.N. Konyrkulzhayeva

Selfadjoint restriction of maximal operator on graph

Abstract. In the work we study differential operators on arbitrary geometric graphs without loops. We extend the known results for differential operators on an interval to the differential operators on the graphs. In order to define properly the maximal operator on a given graph, we need to select a set of boundary vertices. The vertices not being boundary are called interior vertices. We stress that the maximal operator on a graph is determined not only by the given differential expressions on the edges, but also by the Kirchhoff conditions at the interior vertices of the graph. For the introduced maximal operator we prove an analogue of the Lagrange formula. We provide an algorithm for constructing adjoint boundary forms for an arbitrary set of boundary conditions. In the conclusion of the paper, we present a complete description of all self-adjoint restrictions of the maximal operator.

Keywords: Directed graph, Kirchhoff conditions, self-adjoint restriction of an operator, maximal operator

Y.Sh. Il'yasov, E.E. Kholodnov

On global instability of solutions to hyperbolic equations with non-Lipschitz non-linearity

Abstract. In a bounded domain $\Omega \subset \mathbb{R}^n$, we consider the following hyperbolic equation

$$\begin{cases} v_{tt} = \Delta_p v + \lambda |v|^{p-2} v - |v|^{\alpha-2} v, \quad x \in \Omega, \\ v\big|_{\partial\Omega} = 0. \end{cases}$$

We assume that $1 < \alpha < p < +\infty$; this implies that the nonlinearity in the right hand side of the equation is of the non-Lipschitz type. As a rule, this type of nonlinearity prevent us from applying standard methods from the theory of nonlinear differential

equations. An additional difficulty arises due to the presence of the p-Laplacian $\Delta_p(\cdot) := \mathrm{div}(|\nabla(\cdot)|^{p-2}\nabla(\cdot))$ in the equation. In the first result, the theorem on the existence of the so-called stationary ground state of the equation is proved. The proof of this result is based on the Nehari manifold method. In the main result of the paper states that each stationary ground state is instable globally in time. The proof is based on the development of an approach by Payne and Sattinger introduced for studying the stability of solutions to hyperbolic equations.

Keywords: stability of solutions, nonlinear hyperbolic equations, Nehari manifold method, p-Laplacian

V.A. Klyachin

On continuity and differentiability of the maximum values of functions

Abstract. In this paper we consider functions which are the maximal values of continuous functions on the families of compact subsets. Such functions are used, for example, in studying the geometric structure of various equilibrium surfaces: minimal surfaces, surfaces of a constant mean curvature, and so forth. Usually, such functions are constructed as the geometric characteristics of the surfaces under study, for instance, as the distance from a point of the surface to a fixed line, as the radius of the circumscribed sphere. One of the key points of this approach is the justification of their continuity and differentiability. This allows us to derive differential relations for the considered functions. In the present paper, the questions of continuity and differentiability are considered in a more general formulation, for topological and metric spaces. In particular, we find the conditions for the mapping of topological spaces $F : X \to T$ ensuring that a function of the form $\rho(t) = \max_{x \in F^{-1}(t)} g(x)$ is continuous. In addition, for such functions we obtain the conditions guaranteeing that they are Lipschitz and δ-convex in $\mathbb{R}^m$.

Keywords: metric space, Lipschitz functions, continuity, differentiability, δ-convexity.

R.Ch. Kulaev, A.B. Shabat

Some properties for Jost functions of a Schrödinger equation with distribution potential

Abstract. The work is devoted to the substantial extension of the space of the potentials in the inverse scattering problem for the linear Schrödinger equation on the real axis. We consider the Schrödinger operator with a potential in the space of generalized functions. This extension includes not only the potential like delta function, but also exotic cases like Cantor functions. In this way we establish the conditions on existence and uniqueness of Jost solutions. We study their analytic properties. We provide some estimate for the Jost solutions and their derivatives. We show that the Schrödinger equation with the distribution potential can be uniformly approximated by the equations with smooth potentials.

Keywords: inverse scattering problem, Schrödinger equation, Jost functions, delta-type potential, singular potential, distribution potential.

S.I. Mitrokhin

Study of differential operator with summable potential with discontinuous weight function

Abstract. In the work we propose a new approach for studying differential operators with a discontinuous weight function. We study the spectral properties of a differential operator on a finite segment, with separated boundary conditions, with "matching" condition at the discontinuity point of the weight function. We assume that the potential of the operator is a summable function on the segment, on which the operator is considered. For large value of the spectral parameter we obtain an asymptotics for the fundamental system of solutions of the corresponding differential equation. By means of this asymptotics we study the "matching" conditions of the considered differential operator. Then we study the boundary conditions of the considered operator. As a result, we obtain an equation for the eigenvalues of the operator, which an entire function. We study the indicator diagram of the equation for the eigenvalues; this diagram is the regular octagon. In various sectors of the indicator diagram we find the asymptotics for the eigenvalues of the studied differential operator.

Keywords: spectral theory of differential operators spectral parameter, summable potential, discontinuous weight function, indicator diagram, asymptotics of eigenvalues.

M.N. Oreshina

Spectral decomposition of a normal operator in a real Hilbert space

Abstract. We consider unbounded normal operators acting in a real Hilbert space. The standard approach to solving spectral problems related with such operator is to apply the complexification, which is a passage to the complex space. At that, usually, the final results are to be decomplexify, that is, the reverse passage is needed. However, the decomplexification often turns out to be nontrivial.

The aim of the present paper is to extend the classical results of the spectral theory for the case of normal operators acting in a real Hilbert space. We provide two real versions of spectral theorem for such operators.

We develop the functional calculus generated by the real spectral resolution of a normal operator. We provide examples of using the obtained functional calculus for representing the exponent of a normal operator.

Keywords: unbounded normal operator, real Hilbert space, complexification, spectral theorem, functional calculus.

V.A. Pavlenko, B.I. Suleimanov

"Quantizations" of isomonodromic Hamilton system $H^{\frac{7}{2}+1}$

Abstract. We consider two compatible linear evolution equations with times s_1 and s_2 depending on two spatial variables. These evolution equations are the analogues of the non-stationary time Schrödinger equations determined by the two Hamiltonians $H_{s_k}^{\frac{7}{2}+1}(s_1, s_2, q_1, q_2, p_1, p_2)$ $(k = 1, 2)$ of the PќP°milton system $H^{\frac{7}{2}+1}$ formed by a pair of compatible Hamiltonian systems of equations admitting the application of isomonodromic deformations method. These analogues arise from canonical time Schrödinger equations determined by the Hamiltonians $H_{s_k}^{\frac{7}{2}+1}$; they arise by the formal replacement of the Planck constant by the imaginary unit. We construct explicit solutions of these analogues of Schrödinger equations in terms of the solutions of the corresponding linear systems of ordinary differential equations in the isomonodromic deformations method, whose compatibility condition is the Hamiltonian system $H^{\frac{7}{2}+1}$. The key role in the construction of these explicit solutions is played by the change which was used earlier in constructing the solutions of time Schrödinger equation determined by the Hamiltonians of isomonodromic Hamiltonian Garnier system with two degrees of freedom as well as of two isomonodromic degenerations of the latter. We discuss the issue on applicability of this change for constructing the solutions to analogues of time Schrödinger equations determined by the Hamiltonians of the entire hierarchy of isomonodromic Hamiltonian systems with two degrees of freedom being the degenerations of this Garnier system. We mention also a $H^{\frac{7}{2}+1}$ with some problems of modern nonlinear mathematical physics. In particular, we show that the solutions of these Hamiltonian systems are determined explicitly by the simultaneous solutions to the Korteweg-de Vries equation $u_t + u_{xxx} + uu_x = 0$ and a non-autonomous fifth order ordinary differential equations, which are used in universal description of the influence of a small dispersion on the transformation of weak hydrodynamical discontinuities into the strong ones.

Keywords: Hamilton systems, quantization, Shrödinger equation, Painlevé equations, method of isomonodromic deformations

S.Ya. Startsev

On differential substitutions for evolution systems

Abstract. For the most known differential substitutions relating scalar evolution equations, the sets of the equations admitting them consist not finitely many equations but they form families parametrized by an arbitrary function. Some differential substitutions for evolution systems also have a similar property. In the present paper we obtain necessary and sufficient conditions for a differential substitution to be admitted by a family of evolution systems depending on an arbitrary function. We also give explicit formulae for finding the corresponding family of evolution systems in the case when these conditions are satisfied.

As an example, the family of systems admitting a multi-component Cole-Hopf substitution is constructed. We demonstrate that this family contains all linear systems, whose right-hands sides contains the derivatives of order at least one. As a result, we obtain a set of C-integrable systems involving systems of arbitrary high order. Another example considered in the paper is a multi-component analogue of

the substitution $v = u_x + \exp(u)$. We show that this multi-component substitution is also admitted by a family of evolution systems depending on an arbitrary function.

Keywords: differential substitutions, evolution systems, C-integrability.

F.G. Khushtova

Dirichlet boundary value problem in half-strip for fractional differential equation with Bessel operator and Riemann-Liouville partial derivative

Abstract. In the work we study the Dirichlet boundary value problem in a half-strip for a fractional differential equations with the Bessel operator and the Riemann-Liouville partial derivatives. We formulate the unique solvability theoresm for the considered problem. We find the representations for the solutions in terms of the integral transform with the Wright function in the kernel. The proof of the existence theorem is made on the base of the mentioned integral transform and the modified Bessel function of first kind. The uniqueness of the solutions is shown in the class of the functions satisfying an analogue of Tikhonov equation. In the case, when the considered equations becomes the fractional order diffusion equation, we show that the obtained solutions coincides with the known solution to the Dirichlet problem for the corresponding equation. We also consider the case when the initial function is power in the spatial variable. In this case the solution to the problem is written out in terms of Fox H-function.

Keywords: Bessel operator, Riemann–Liouville partial derivative, fractional diffusion, Wright function, integral transform with the function of Wright in the kernel, modified Bessel function of the first kind, Fox H-function, Tikhonov condition.

A.Ya. Khrystiyanyn, Dz.V. Lukivska

Quasi-elliptic functions

Abstract. We study certain generalizations of elliptic functions, namely quasi-elliptic functions.

Let $p = e^{i\alpha}$, $q = e^{i\beta}$, $\alpha, \beta \in \mathbb{R}$. A meromorphic in $\mathbb{C}$ function g is called quasi-elliptic if there exist $\omega_1, \omega_2 \in \mathbb{C}^*$, $\operatorname{Im} \frac{\omega_2}{\omega_1} > 0$, such that $g(u+\omega_1) = pg(u)$, $g(u+\omega_2) = qg(u)$ for each $u \in \mathbb{C}$. In the case $\alpha = \beta = 0 \mod 2\pi$ this is a classical theory of elliptic functions. A class of quasi-elliptic functions is denoted by $\mathcal{QE}$. We show that the class $\mathcal{QE}$ is nontrivial. For this class of functions we construct analogues $\wp_{\alpha\beta}$, $\zeta_{\alpha\beta}$ of $\wp$ and ζ Weierstrass functions. Moreover, these analogues are in fact the generalizations of the classical $\wp$ and ζ functions in such a way that the latter can be found among the former by letting $\alpha = 0$ and $\beta = 0$. We also study an analogue of the Weierstrass σ function and establish connections between this function and $\wp_{\alpha\beta}$ as well as $\zeta_{\alpha\beta}$.

Let $q, p \in \mathbb{C}^*$, $|q| < 1$. A meromorphic in $\mathbb{C}^*$ function f is said to be p-loxodromic of multiplicator q if for each $z \in \mathbb{C}^*$ $f(qz) = pf(z)$. We obtain telations between quasi-elliptic and p-loxodromic functions.

Keywords: quasi-elliptic function, the Weierstrass $\wp$-function, the Weierstrass ζ-function, the Weierstrass σ-function, p-loxodromic function.

T.M. Salo, O.B. Skaskiv

The minimum modulus of lacunary power series and h-measure of exceptional sets

Abstract. We consider some generalizations of Fenton theorem for the entire functions represented by lacunary power series. Let $f(z) = \sum_{k=0}^{+\infty} f_k z^{n_k}$, where (n_k) is a strictly increasing sequence of non-negative integers. We denote by

$$M_f(r) = \max\{|f(z)|\colon |z| = r\},$$
$$m_f(r) = \min\{|f(z)|\colon |z| = r\},$$
$$\mu_f(r) = \max\{|f_k| r^{n_k}\colon k \geqslant 0\}$$

the maximum modulus, the minimum modulus and the maximum term of f, respectively. Let $h(r)$ be a positive continuous function increasing to infinity on $[1, +\infty)$ with a non-decreasing derivative. For a measurable set $E \subset [1, +\infty)$ we introduce $h - \operatorname{meas}(E) = \int_E \frac{dh(r)}{r}$. In this paper we establish conditions guaranteeing that the relations

$$M_f(r) = (1 + o(1))m_f(r), \quad M_f(r) = (1 + o(1))\mu_f(r)$$

are true as $r \to +\infty$ outside some exceptional set E such that $h - \operatorname{meas}(E) < +\infty$. For some subclasses we obtain necessary and sufficient conditions. We also provide similar results for entire Dirichlet series.

Keywords: lacunary power series, minimum modulus, maximum modulus, maximal term, entire Dirichlet series, exceptional set, h-measure