ABSTRACTS

R.A. Atnagulova, O.V. Sokolova

Factorization problem with intersection

Abstract. We propose a generalization of the factorization method to the case when $\mathcal{G}$ is a finite-dimensional Lie algebra $\mathcal{G} = \mathcal{G}_0 \oplus M \oplus N$ (direct sum of vector spaces), where $\mathcal{G}_0$ is a subalgebra in $\mathcal{G}$, M, N are $\mathcal{G}_0$-modules, and $\mathcal{G}_0 + M$, $\mathcal{G}_0 + N$ are subalgebras in $\mathcal{G}$. In particular, our construction involves the case when $\mathcal{G}$ is a $\mathbb{Z}$-graded Lie algebra. Using this generalization, we construct certain top-like systems related to algebra $so(3,1)$. According to the general scheme, these systems can be reduced to solving systems of linear equations with variable coefficients. For these systems we find polynomial first integrals and infinitesimal symmetries.

Keywords: factorization method, Lie algebra, integrable dynamical systems.

F.Kh. Baichorova

On analogues of third order Bessel function

Abstract. We consider the problem on eigenfunctions of differential operators semi-invariant w.r.t. to the group of shifts. We obtain a solvability condition in terms of primitive functions and show a connection of this condition with the theory of commutative rings of differential operators.

Keywords: generalized Bessel functions, Darboux transformations.

O.A. Bozhenko, K.G. Malyutin

Problem of multiple interpolation in class of analytical functions of zero order in half-plane

Abstract. In the paper we consider the problem of multiple interpolation in a class of functions of a zero order and type not exceeding normal in the upper half-plane of the complex variable. This problem belongs to the class of problems of free interpolation considered initially by A.F. Leont'ev. We find necessary and sufficient solvability conditions for this problem. The found criteria are formulated in terms of the canonical products constructed on knots of interpolation, and in terms of the Nevanlinna measure determined by these knots. The work is a continuation of researches of the first author considered similar problems in classes of analytic functions in the upper half-plane of a nonzero order.

Keywords: zero specified order, divisor, canonical product, multiple interpolation, Levin condition, Nevanlinna measure.

D.I. Borisov

Discrete spectrum of thin $\mathcal{PT}$-symmetric waveguide

Abstract. In a thin multidimensional layer we consider a differential second order $\mathcal{PT}$-symmetric operator. The operator is of rather general form and its coefficients are arbitrary functions depending both on slow and fast variables. The $\mathcal{PT}$-symmetry of the operator is ensured by the boundary conditions of Robin type with pure imaginary coefficient. In the work we determine the limiting operator, prove the uniform resolvent convergence of the perturbed operator to the limiting one, and derive the estimates for the rates of convergence. We establish the convergence of the spectrum of perturbed operator to that of the limiting one. For the perturbed eigenvalues converging to the limiting discrete ones we prove that they are real and construct their complete asymptotic expansions. We also obtain the complete asymptotic expansions for the associated eigenfunctions.

Keywords: $\mathcal{PT}$-symmetric operator, thin domain, uniform resolvent convergence, estimates for the rate of convergence, spectrum, asymptotic expansions.

I.I. Karamov, V.V. Napalkov

Generalized Dunkl operator

Abstract. In the paper we introduce a generalized Dunkl operator acting in the space of entire functions on $\mathbb{C}$. We study problems of harmonic analysis related with this operator and show its connection with the Gelfond-Leont'ev operator of generalized differentiation.

Keywords: Dunkl operator, eigenfunction, Dunkl convolution operator, Dunkl transform, characteristic function, hypercyclic operator.

V.E. Kim

Dynamics of linear operators connected with su$(1,1)$ algebra

Abstract. In the present work we consider a linear continuous operator in a separable Frechet space being one of the generators of Lie algebra su$(1,1)$. We study the discrete-time dynamical system generated by iteration of this operator. We show that under some additional conditions the operator that generates the indicated dynamical system is frequently hypercyclic and chaotic (in the sense of Devaney). Applications of this result to a study of specific operators are indicated.

Keywords: frequently hypercyclic operator, Lie algebra.

S.G. Merzlyakov

Cauchy–Hadamard theorem for exponential series

Abstract. In this paper we study the connection between the growth of coefficients of an exponentials series with its convergence domain in finite-dimensional real and complex spaces. Among the first results of the subject is the well-known Cauchy–Hadamard formula.

We obtain exact conditions on the exponentials and a convex region in which there is a generalization of the Cauchy–Hadamard theorem.

To the sequence of coefficients of exponential series we associate a space of sequences forming a commutative ring with unit. The study of the properties of

this ring allows us to obtain the results on solvability of non-homogeneous systems of convolution equations.

Keywords: convex domains, series of exponentials, Cauchy–Hadamard formula.

E.M. Mukhamsdiev, I.D. Nurov, M.Sh. Khalilova

Limiting cycles of piece-linear second order differential equations

Abstract. The work is devoted to finding limiting cycles in a vicinity of equilibria of non-smooth dynamical systems. We obtain new phase portraits not appearing in a linear case. We employ the method of gluing solutions in two half-planes. Together with using standard packages, we develop a new program package for numerical construction of phase portraits.

Keywords: Dynamical systems, non-smoothness, stability, phase plane, limiting cycle.

D.T. Siraeva

Optimal system of non-similar subalgebras of sum of two ideals

Abstract. We consider a twelve-dimensional Lie algebra L_{12} admitted by the gas dynamic equations with state equation of a special form. Lie algebra L_{12} is a direct sum of two ideals L_{11} and Y_1. For Lie algebra L_{11} admitted by gas dynamic equations with an arbitrary equation of state, the optimal system of non-similar subalgebras is built up to inner automorphisms. Using the optimal system for Lie algebra L_{11}, in the article we obtain an optimal system of non-similar subalgebras of the sum of two ideals for L_{11} and Y_1 and the rule of construction of such subalgebras.

Keywords: Lie algebra, optimal system, gas dynamics.

A.A. Talyshev

On integration of automorphic systems of finite-dimensional Lie groups

Abstract. The present paper contains certain results on integration and order reducing of automorphic systems for finite-dimensional Lie groups.

Keywords: Lie symmetries, automorphic systems, differential invariants.

M.V. Yangubaeva

On structure of integrals for systems of discrete equations

Abstract. In the work we describe the structure of integrals of systems of discrete equations. We consider an example of discrete Toda chain corresponding to Lie algebra of series A_2.

Keywords: system of discrete equations, complete set of integrals.