# ABSTRACTS

## N.N. Aitkuzhina, A.M. Gaisin

### DIRICHLET SERIES WITH REAL COEFFICIENTS

**Abstract.** We study the class of entire functions represented by Dirichlet series with real coefficients determined by a convex growth majorant. We prove the criterion for the validity of the asymptotic identity on the positive ray which is an exact estimate for the growth of the logarithm of the modulus for each function in the considered class.

**Keywords:** Dirichlet series with real coefficients, discrete growth majorant.

## F.Kh. Baichorova, Z.S. Elkanova

### COMMUTING DIFFERENTIAL OPERATORS OF ORDERS 4 AND 6

**Abstract.** We consider a model problem on a pair of commuting differential operators of orders 4 and 6. The results are employed to generalize a known commuting pair in a work of J. Dixmier for the case of rational coefficients.

**Keywords:** commuting differential operators, differential operators of orders 4 and 6.

## Yu.G. Voronova, A.V. Zhiber

### SYMMETRIES AND GOURSAT PROBLEM FOR SYSTEM OF EQUATIONS $u_{xy} = e^{u+v}u_y,\ v_{xy} = -e^{u+v}v_y$

**Abstract.** We describe the higher symmetries and construct the general solution for a hyperbolic system of equations. We also obtain the explicit formula for the solution of Goursat problem.

**Keywords:** symmetries, Goursat problem, integrals.

## R.A. Gaisin

### QUASIANALYTICITY CRITERIA OF SALINAS-KORENBLUM TYPE FOR GENERAL DOMAINS

**Abstract.** We prove a criterion of quasianalyticity in a boundary point of a rather general domain (not necessarily convex and simply-connected) if in a vicinity of this point the domain is close in some sense to an angle or is comparable with it.

**Keywords:** Carleman class, regular sequences, bilogarithmic quasianalyticity condition.

# Z.Kh. Zakirova

## On some special solutions of Eisenhart equation

**Abstract.** In this note we study a 6-dimensional pseudo-Riemannian space $V^6(g_{ij})$ with the signature $[+ + - - --]$, which admits projective motions, i.e., continuous transformation groups preserving geodesics. A general method of determining pseudo-Riemannian spaces admitting some nonhomothetic projective group $G_r$ was developed by A.V.Aminova. A.V.Aminova classified all Lorentzian manifolds of dimension $\geqslant 3$ admitting nonhomothetic projective or affine infinitesimal transformations. The problem of classification is not solved for pseudo-Riemannian spaces with arbitrary signature.

In order to find a pseudo-Riemannian space admitting a nonhomothetic infinitesimal projective transformation, one has to integrate the Eisenhart equation

$$h_{ij,k} = 2g_{ij}\varphi_{,k} + g_{ik}\varphi_{,j} + g_{jk}\varphi_{,i}.$$

Pseudo-Riemannian manifolds for which there exist nontrivial solutions $h_{ij} \neq cg_{ij}$ to the Eisenhart equation are called *h-spaces*. It is known that the problem of describing such spaces depends on the type of an $h$-space, i.e., on the type of the bilinear form $L_X g_{ij}$ determined by the characteristic of the $\lambda$-matrix $(h_{ij} - \lambda g_{ij})$. The number of possible types depends on the dimension and the signature of an $h$-space

In this work we find the metrics and determine quadratic first integrals of the corresponding geodesic lines equations for 6-dimensional $h$-spaces of the type $[(21\ldots1)(21\ldots1)\ldots(1\ldots1)]$.

**Keywords:** differential geometry, pseudo-Riemannian manifolds, systems of partial differential equations.

# A.M. Ilyasov

## Optimal system of Lie algebra subalgebras of the point symmetries group for nonlinear heat equation without source

**Abstract.** In this paper we construct an optimal system of subalgebras for the nine-dimension Lie algebra of infinitesimal operators for a point symmetries group of a nonlinear heat equation with isotropic heat conductivity tensor and with a power dependence of the temperature. The results are presented as a lemma and a theorem. It is proven that up to transformations of internal automorphisms and some discrete automorphisms, there are 117 dissimilar subalgebras classes of various dimensions.

**Keywords:** nonlinear heat equation, Lie algebra, optimal system of subalgebras.

# K.P. Isaev, R.S. Yulmukhametov

## Unconditional bases of reproducing kernels in Hilbert spaces of entire functions

**Abstract.** We consider the existence of unconditional bases of reproducing kernels in a functional Hilbert space of entire functions. It is proved that under certain conditions, unconditional bases of reproducing kernels do not exist. It is shown that in particular spaces some known theorems on the absence of unconditional bases are the consequences of these results.

**Keywords:** Hilbert spaces, entire functions, reproducing kernels, unconditional bases.

# V.A. Korneev

## Construction of generalized solution for first order divergence type equation

**Abstract.** We consider the Cauchy problem for a first order divergence type equation with the right hand side independent of the unknown function and with a discontinuous initial condition. This equation was first mentioned by J.M. Burgers in 1940 and it is a model equation for the system of equations describing the non-stationary motion of a gas. Various properties of the solution to this problem we studied in works by O.A. Oleinik (1957), J. Whitham (1974), S.N. Kruzhkov (1970), E.Yu. Panov (2006). The original problem is reduced to the Cauchy problem for Hamilton-Jacobi equation with a continuous initial condition. It is suggested to apply the method of singular characteristics to this problem, while this method was developed A.A. Melikyan for game problems. The effectiveness of technique is demonstrated by the example, when in the original equation the derivative w.r.t. the spatial variable is applied to a cubic polynomial of the unknown function, and boundary condition is specified as a "raising" step. The Hamiltonian in the modified problem is a third degree polynomial of a partial derivative for the unknown function, and the boundary condition is given by the piecewise linear convex function with a break in the origin. We single out the domains of the parameters for which the construction of a generalized solution is possible, and we describe the procedure of constructing the solution. It is shown that the solution involves nonsmooth singularities called the dispersal and equivocal surfaces according to the terminology of differential games. The constructing of the solution is illustrated by figures.

**Keywords:** Hamilton-Jacobi equation, generalized solution, method of characteristics.

# A.S. Krivosheyev, O.A. Krivosheyeva

## A closedness of set of Dirichlet series sum

**Abstract.** In the work we consider Dirichlet series. We study the problem of closedness for the set of the sums for such series in the space of functions holomorphic in a convex domain of a complex plane with a topology of uniform convergence on compact subsets. We obtain necessary and sufficient conditions under those every function from the closure of a linear span of exponents with positive indices is represented by a Dirichlet series. These conditions can be formulated only in terms of geometric characteristics of an index sequence and of the convex domain.

**Keywords:** exponent, convex domain, Dirichlet series, entire function, invariant subspace.

# E.V. Makarevich

## INVARIANT AND PARTIALLY INVARIANT SOLUTIONS WITH RESPECT TO GALILEAN SHIFTS AND DILATATION

**Abstract.** In the work we consider a three-dimensional subalgebra embedded in a four-dimensional subalgebra in order to find the set of solutions and to adjoint them the solutions on subalgebras of higher dimension. Although the aim is not reached yet, we obtain invariant solutions of the rank 1 and partially invariant solutions of the rank 1 and defect 1. We obtain two submodels being invariant and partially invariant, seven solutions depend on arbitrary function and nineteen exact solutions.

**Keywords:** gas dynamics, hierarchy of submodels, invariant solution, partially invariant solution.

# S.G. Merzlyakov, S.V. Popenov

## INTERPOLATION WITH MULTIPLICITY BY SERIES OF EXPONENTIALS IN $H(\mathbb{C})$ WITH NODES ON THE REAL AXIS

**Abstract.** In the space of entire functions we study an interpolation problem with multiplicity by the functions from a closed subspace which is invariant in respect to the operator of differentiation. The discrete set of the nodes for the interpolation with multiplicity is located on the real axis in the complex plane. The proof is based on the passage from the subspace to its subspace consisting of all series of exponentials converging in the topology of uniform convergence on compact sets. We obtain a criterion for the solvability of the interpolation problem with real nodes having multiplicity by series of exponentials in the terms of location of exponents of exponentials.

**Keywords:** entire function, interpolation with multiplicity, series of exponents, ideal, Fischer representation.

# I.I. Strukova

## WIENER'S THEOREM FOR PERIODIC AT INFINITY FUNCTIONS WITH SUMMABLE WEIGHTED FOURIER SERIES

**Abstract.** In the article we define a Banach algebra of periodic at infinity functions. For this class of functions we introduce the notions of a Fourier series, its absolutely convergence and invertibility. We obtain an analogue of Wiener theorem on absolutely convergent Fourier series for periodic at infinity functions whose Fourier coefficients are summable with a weight.

**Keywords:** Banach space, slowly varying at infinity functions, periodic at infinity functions, Wiener theorem, absolutely convergent Fourier series, invertibility.