ABSTRACTS

S.N. Askhabov, A.L. Dzhabrailov

Approximate solutions of nonlinear convolution type equations on segment

Abstract. For various classes of integral convolution type equations with a monotone nonlinearity we prove global solvability and uniqueness theorems as well as theorems on the ways of finding the solutions in real Lebesgue spaces. It is shown that the solutions can be found in space $L_2(0,1)$ by a Picard's type successive approximations method and we prove the estimates for the rate of convergence. The obtained results cover, in particular, linear integral convolution type equations. In the case of a power nonlinearity it is shown that the solutions can be found by the gradient method in space $L_p(0,1)$ and weighted spaces $L_p(\varrho)$.

Keywords: nonlinear integral equations, convolution type operator, potential operator, monotone operator.

S. Baizaev, D.A. Vositova

On solutions of a system of partial differential equations with two independent variables

Abstract. In paper we consider first order linear elliptic and hyperbolic systems with constant coefficients and two independent variables. For such systems we study the problems on a variety of all the solutions and of the solutions growing at infinity not faster than a power function.

Keywords: elliptic and hyperbolic system, moderately growing solutions, solutions of power growth, dimension of space of solutions.

V.E. Bobkov

On existence of nodal solution of elliptic equations with convex-concave nonlinearities

Abstract. In a bounded connected domain $\Omega \subset \mathbb{R}^N$, $N \geqslant 1$ with a piecewise smooth boundary we consider the Dirichlet boundary value problem for elliptic equation with a convex-concave nonlinearity

$$\begin{cases} -\Delta u = \lambda |u|^{q-2}u + |u|^{\gamma-2}u, & x \in \Omega \\ u|_{\partial\Omega} = 0, \end{cases}$$

where $1 < q < 2 < \gamma < 2^*$. As a main result, we prove the existence of a nodal solution of this equation on the nonlocal interval $\lambda \in (-\infty, \lambda_0^*)$, where λ_0^* is determined by the variational principle of nonlinear spectral analysis via fibering method.

Keywords: nodal solution, convex-concave nonlinearity, fibering method

M.F. Broyan, Kh.A. Khachatryan

On some nonlinear integral and integro-differential equations with noncompact operators on positive semi axis

Abstract. The paper is devoted to the study of certain classes of nonlinear integral and integro-differential with non-compact Hammerstein type operators. These equations have important applications in kinetic theory of gases and in wealth distribution theory of one product economics.

Keywords: integral equation, Hammerstein operator, Sobolev space, convergence, monotonicity.

R.G. Nasibullin, A.M. Tukhvatullina

Hardy type inequalities with logarithmic and power weights for a special family of non-convex domains

Abstract. In the present work we obtain variational Hardy type inequalities with power and logarithmic weights which are generalizations of corresponding inequalities given earlier in the papers by M. Hoffmann-Ostenhof, T. Hoffmann-Ostenhof, A. Laptev, and J. Tidblom. We formulate and prove inequalities for arbitrary domains, and then we substantially simplify them for the class of convex domains and a special family of non-convex domains.
Keywords: ??????????????????????????????????

D.K. Potapov

On a number of solutions in problems with spectral parameter for equations with discontinuous operators

Abstract. In a real reflexive Banach space we consider a problem on existence of solutions to a problem with a spectral parameter for equations with discontinuous operators. By the variational approach we obtain theorems on the number of the solutions to the considered problems. As an application, we consider main boundary value problems for elliptic equations with a spectral parameter and discontinuous nonlinearities.

Keywords: spectral parameter, discontinuous operator, variational method, number of solutions.

R.S. Saks

Solution of spectral problems for curl and Stokes operators

Abstract. In the work we explicitly solve the spectral problems for curl, gradient of divergence, and Stokes operators in a ball B of radius R. The eigenfunctions $\mathbf{u}_\kappa^\pm$ of the curl associated with non-zero eigenvalues $\pm\lambda_\kappa$ are expressed by explicit formulas, as well as the vector-functions $\mathbf{q}_\kappa$ associated with the zero eigenvalue,

$$rot\,\mathbf{u}_\kappa^\pm = \pm\lambda_\kappa\,\mathbf{u}_\kappa^\pm, \quad \psi_n(\pm\lambda_\kappa R) = 0, \quad \mathbf{n}\cdot\mathbf{u}_\kappa^\pm|_S = 0; \quad rot\,\mathbf{q}_\kappa = 0, \quad \mathbf{n}\cdot\mathbf{q}_\kappa|_S = 0,$$

где

$$\psi_n(z) = (-z)^n \left(\frac{d}{zdz}\right)^n \frac{\sin z}{z}, \quad \kappa = (n, m, k), \ n \geqslant 0, \ m \in \mathbb{N}, \ |k| \leqslant n$$

The same vector-functions are the eigenfunctions for the gradient of divergence operator with other eigenvalues,

$$\nabla \, div \, \mathbf{u}_\kappa^\pm = 0; \quad \nabla \, div \, \mathbf{q}_\kappa = \mu_\kappa \mathbf{q}_\kappa, \quad \mu_\kappa = (\alpha_{n,m}/R)^2, \quad \psi_n'(\alpha_{n,m}) = 0.$$

The constructed system of eigen-vector-functions is complete and orthogonal in space $\mathbf{L}_2(B)$.

The eigenfunctions $(\mathbf{v}_\kappa, \, p_\kappa)$ of Stokes operator in the ball is represented as a sum of two eigenfunctions of the curl associated with opposite eigenvalues: $\mathbf{v}_\kappa = \mathbf{u}_\kappa^+ + \mathbf{u}_\kappa^-$, $p_\kappa = \text{const}$.

Keywords: curl, gradient of divergence, and Stokes operators, eigenvalues, eigenfunctions, Fourier series.

R.B. Salimov, P.L. Shabalin

On solvability of homogeneous Riemann-Hilbert problem
with a countable set of coefficient discontinuities
and two-side curling at infinity of order less than $1/2$

Abstract.

In the present paper we consider the homogeneous Riemann–Hilbert problem in the complex upper half-plane with a countable set of coefficient discontinuities and two-side curling at infinity. In the case the problem index has a power singularity of order less than $1/2$, in a special functional class, we obtain general solution and completely investigate the solvability of the problem.

Keywords: Riemann-Hilbert problem, curling at infinity, infinite index, entire functions

M.S. Tikhov

Nonparametric estimates of the effective doses
at quantal response

Abstract. For the quantal response model we propose a new direct method for nonparametric estimation of the effective dose level $ED_{100\lambda}$ ($0 < \lambda < 1$). This method yields a simple and reliable monotone estimate of the effective dose level curve $\lambda \to ED_{100\lambda}$ and is appealing to users of conventional smoothing methods of kernel estimates. Moreover, it is computationally very efficient, because it does not require a numerical inversion of the estimate of the quantile dose response curve. We prove asymptotic normality of this new estimate and compare it with the DNP-estimate.

Keywords: binary response model, effective dose level, nonparametric estimate.

2010 Mathematics Subject Classification: 62G05, 62G08, 62G20, 62P10.

M.G. Yumagulov

LOCALIZATION OF ARNOLD TONGUES OF DISCRETE DYNAMICAL SYSTEMS

Abstract. The work is devoted to the exposition of the method of localizing Arnold tongues for finite-dimensional dynamical systems with discrete time which are the sets corresponding to rationally synchronized relations between the system's parameters. Such sets correspond to regions of parameter values, for which the system has cycles of certain periods. The method allows us to obtain an approximate representation of Arnold tongues, to study their properties in the major and minor resonances.

Keywords: bifurcation, dynamical systems, Arnold tongues, operator equations, functionalization of parameter.

B.V. Vynnyts'kyi, R.V. Khats'

COMPLETENESS AND MINIMALITY OF SYSTEMS OF BESSEL FUNCTIONS

Abstract. We find the necessary and sufficient conditions for the completeness and minimality in the space $L^2(0;1)$ of system $(\sqrt{x\rho_k}J_\nu(x\rho_k) : k \in \mathbb{N})$ generated by Bessel function of the first kind of index $\nu \geqslant -1/2$. Moreover, we establish a criterion for the completeness and minimality of system $(x^{-2}\sqrt{x\rho_k}J_{3/2}(x\rho_k) : k \in \mathbb{N})$ in the space $L^2((0;1); x^2 dx)$.

Keywords: Paley-Wiener theorem, Bessel function, entire function, complete system, minimal system, biorthogonal system, basis.