ABSTRACTS

A.V. Aksenov, A.A. Kozyrev

Reductions of stationary boundary layer equation

Abstract. The equation describing a steady laminar boundary layer with a pressure gradient is considered. All reductions to the ordinary differential equations are obtained. It has been shown that this equation has a reduction which can be obtained neither by classical nor by nonclassical symmetry methods.

Keywords: similarity reduction, symmetry operator, invariant solution, boundary layer.

A.R. Bagautdinova, A.V. Lutsenko, V.I. Lutsenko, E.D.Shaimuratova

Integral estimates for derivatives of analytic functions outside convex domains

Abstract. In the present paper weight integral estimates are obtained for derivatives of functions which are analytic in the exterior of convex bounded domains. The estimates are obtained in terms of integrals of functions vanishing at infinity. This result generalizes the Hardy-Littlewood theorem for exteriors of convex bounded domains. Theorems of this kind have been earlier obtained by K. P. Isaev and R. S. Yulmukhametov for the power weight and for the first derivative of an analytic function of the first order belonging to L^2. N. M. Tkachenko and F. A. Shamoyan have generalized this result for all higher order derivatives belonging to the space L^p. In the present paper the class of weights under consideration is essentially enlarged.

Keywords: Analytic function, the Green function, the Laplace invariants, generalized Laplace invariants.

V.N. Belykh

The problem of numerical realization of integral operators of axisymmetric boundary value problems (algorithms without saturation)

Abstract. In the paper a fundamentally new, *unsaturated,* method of numerical implementation integral operators of C^∞-smooth axisymmetric boundary value problems is described. The method allows one to take into account the specifics of the axisymmetric problems automatically. This specifics is an obstacle to any numerical methods with the principal term of error.

The method was extensively tested on the problem of precise evaluation of the Gauss integral of the theory of harmonic potential in high aspect ratio ellipsoids.
Keywords: unsaturated numerical method, Gauss integral, axisymmetric region, quadrature formula without saturation.

P.V. Bibikov

On automorphic systems of differential equations and $\mathrm{GL}_2(\mathbb{C})$-orbits of binary forms

Abstract. In the work we introduce a new method for studying the classical algebraic problem of classifying $\mathrm{GL}_2(\mathbb{C})$-orbits of binary forms with the help of differential equations. We construct and study an automorphic system of differential equations $\mathcal{S}$ of the fourth order, whose solution space coincides with the $\mathrm{GL}_2(\mathbb{C})$-orbit of a fixed binary form f. The system $\mathcal{S}$ is integrable in cases when it is of the second and third order. In the most difficult case, when the system is of the fourth order, we prove that the system $\mathcal{S}$ can be reduced to a first order differential equation of the Abel type and a linear partial differential equation of the first order.

Keywords: binary forms, jet space, differential invariants, automorphic differential equations

N.F. Valeev

On a spectral property of irregular pencils

Abstract. The present paper introduces the notion of a quasi-regular eigenvalue and a quasi-regular pencil spectrum of finite dimensional operator pencils. It is demonstrated that quasi-regular eigenvalues of irregular pencils are continuous with respect to perturbations of the pencil. Properties of quasi-regular eigenvalues are studied and formulae for calculating a quasi-regular spectrum are obtained.

Keywords: spectral theory of linear operators, irregular pencils, inverse spectral problems, regular spectrum of the operator pencil.

R.K. Gazizov, A.A. Kasatkin, S.Yu. Lukashchuk

Fractional differential equations: change of variables and nonlocal symmetries

Abstract. In the work point changes of variables in different types of fractional integrals and derivatives are considered. In a general case fractional integro-differentiation of a function with respect to another function arises after such change. The problem of extending a group of point transformations to operators of this type is considered, corresponding prolongation formulae for the group infinitesimal operator are constructed. Usage of prolongation formulae for finding some nonlocal symmetries of the equation and checking their admittance is demonstrated as a simple example of an ordinary fractional differential equation.

Keywords: fractional derivative, prolongation formulae, nonlocal symmetry.

I.I. Golichev

Iterative linearization of the evolution Navier-Stokes equations

Abstract. Constructed and validated an iterative process, which reduces the solution of nonlinear time-dependent Navier-Stokes equations to the solution of a sequence of linear problems. Using a priori estimates of solutions allows us to prove the convergence of the method with any initial approximation. It is shown that the proposed method can be used to prove the existence and uniqueness of the solution.

Keywords: Navier-Stokes equations, a priori estimates, the iterative process

S.V. Golovin, M.Yu. Kazakova

Symmetries and exact solutions of the model of dynamic convection of the sea

Abstract. Equations of the mathematical model of dynamic convection of the sea are observed. The model describes incompressible flows of shallow water with variable density under the action of Coriolis force. This approximation is widely applied for modeling of a mit-latitude oceanic and atmospheric flows.

From the group-theoretical point of view this model is exceptional by its infinite-dimensional group of transformations that involves five arbitrary functions of time. The goal of the paper is to demonstrate the physical meaning of the symmetry transformations, to construct the the optimal system of small dimensional subalgebras, and to represent new exact solutions, constructed on the base of the symmetry analysis.

Keywords: equations of dynamic convection of the sea, optimal system of subalgebras, partially invariant solution.

V.F. Kovalev, R.V. Kulikov

Generalized Webster equation: exact and approximate renormgroup symmetries, invariant solutions and conservation laws

Abstract. The exact point symmetry group for the generalized Webster type equation, which describes nonlinear acoustic waves in lossy channels with variable cross sections, is found. It is shown that, for certain types of cross section profiles S, the admitted three-dimensional point symmetry group is extended and group classification problem for different types of S is solved. Optimal systems of one-dimensional subalgebras of the admitted Lie algebra are revealed and the invariant solutions corresponding to these subalgebras are obtained. Approximate renormgroup symmetries and the corresponding approximate analytic solutions, as well as conservation laws to the generalized Webster equation are derived for channels with constant and smoothly varying or constant cross sections and arbitrary initial conditions.

Keywords: Webster equation, exact and approximate renormgroup symmetries, invariant solutions, conservation laws.

M.Ya. Mazalov

On uniform approximability by solutions of elliptic equations of order higher than two

Abstract. We consider uniform approximation problems on compact subsets of $\mathbb{R}^d$, $d > 2$, by solutions of homogeneous constant coefficients elliptic equations of order $n > 2$. We construct an example showing that in the general case for compact sets with nonempty interior there is no uniform approximability criteria analogous to the well-known Vitushkin's criterion for analytic functions in $\mathbb{C}$. On the contrary, for nowhere dense compact sets the situation is the same as for analytic and harmonic functions, including instability of the corresponding capacities.

Keywords: elliptic equations, capacities, instability of capacities, uniform approximation, Vitushkin's scheme.

E.V. Makarevich

The collapse or the source of gas on a straight line

Abstract. In the work the partially invariant solution of the rank 2 defect 0 on four-dimensional subalgebra is constructed. The motion of allocated volume of gas is described. The motion of sound surface is constructed, where the velocity of particles is equal to sound velocity. The motion of the sound characteristics and conoid is described. The solution specifies gas motion from the whole space towards the straight line for negative time (collapse) and from the line to the whole space for positive time (source). For infinitely large absolute time values the motion is subsonic everywhere. The sound surface moves from infinitely distant points towards the straight line. It is shown, that the sound characteristics and conoid points move towards the sound surface.

Keywords: gas dynamics, partially invariant solution, collapse, conoid.

A.A. Talyshev

About automorphic systems of finite-dimensional Lie groups

Abstract. It is shown in the present paper that any automorphic system for a finite-dimensional Lie group is a completely integrable system.

Keywords: Lie symmetries, automorphic systems, differential invariants.

M.I. Timoshin

Two-dimensional algebras of dynamic symmetries of ODEs

Abstract. The efficiency of use of dynamical symmetries at research of integrability of differential equations is shown in this work. The generalization of S. Lie's classification of ordinary differential equations of the second order with respect to two-dimensional algebras of point symmetries is constructed. Integrable cases of second order special type ODE are indicated. It is noted that a part of the found integrability cases is related to Abel's type equations and apparently represents independent interest.

Keywords: dynamic symmetries, invariants, two-dimensional algebras, Riccati equation, Abel equation.

S.V. Khabirov

Extention of the conic flows

Abstract. All partial invariant solutions of gas dynamic equations constructed on the conic subalgebra admitted by the model are found. The canonic subalgebra consists of operators of rotation, translation by time and expansion. Submodel is set by a system of ordinary differential equations. Solutions form a series of submodels. In the basis of this submodels lies canonic submodel with respect to the invariant variable depending on independent variables and constants of this submodels depending on the invariant function. To determine this dependence, various additional overdetermined equations are obtained. Moreover, two submodels, expanding the canonic submodel, are derived from the system of partial differential equations. All formulas returning the solutions to physical space are defined for these two submodels.

Keywords: canonic flows, partial invariant solutions, gas dynamics.

G.G. Sharafutdinova

The problem of deflection shapes of a simply supported plate under a longitudinal strain

Abstract. In this paper the approximate study the problem of the bifurcation behavior of an elastic plate with the change of the longitudinal compressive strain is carried out. A new scheme which allows to determine the critical values of the strain at which the plate takes a stable curvilinear equilibrium. The scheme also leads to an asymptotic formula which describes the nonlinear deflections of the plate when passing through the critical strain.

Keywords: Deflection of the plate, approximate study, critical strain, bifurcation points, asymptotic formulas, state of balance.

Yu.V. Yulmukhametova

Straightening expansions of gas from vortex with linear velocity field

Abstract. In this paper we consider a submodel of the gas with a linear velocity field. It is formed by a system of nonlinear differential equations with initial data. Several first integrals of the system are obtained. As a result the order of the system is reduced. For special initial data of the problem, an approximate solution of differential equations of the submodel is obtained. Such solutions correspond to world lines describing the radial expansion of the gas particles from the vortex. Trajectories of motion of gas particles are constructed.

Keywords: gas dynamics, submodel, approximate solution, the radial expansion.

A.A. Alexandrova, N.H. Ibragimov, K.V. Imamutdinova and V.O. Lukashchuk

Local and nonlocal conserved vectors for the nonlinear filtration equation

Abstract. It is demonstrated that the nonlinear filtration equation is nonlinarly self-adjoint. Using this property, the conserved vectors associated with Lie point and nonlocal symmetries are constructed.

Keywords: nonlinear filtration equation, nonlinear self-adjointness, Lie point and nonlocal symmetries, conserved vectors.

V.A. Dorodnitsyn

Non-autonomous dynamical systems and exact solutions with superposition principle for evolutionary PDEs

Abstract. In the present article we introduce a new application of S. Lie's non-autonomous dynamical systems with the generalized separation of variables in the right hand-sides. We consider non-autonomous dynamical equations as some sort of external action on a given evolution equation, which transforms a subset of solutions into itself. The goal of our approach is to find a subset of solutions of an evolution equation with a superposition principle. This leads to an integration of ordinary differential equations in a process of constructing exact solutions of PDEs. In this paper we consider the application of the most simple one-dimensional case of the Lie theorem.

Keywords: evolutionary equations, exact solutions, superposition of solutions.

N.H. Ibragimov, C. Rogers

On infinitesimal reciprocal-type transformations in gasdynamics. Lie group connections and nonlinear self-adjointness

Abstract. Bateman-type reciprocal transformations are represented as non-local infinitesimal symmetries of the governing equations of steady, two-dimensional, inviscid gasdynamics. In particular, this representation allows the construction of a novel non-local conservation law using the recently introduced concept of nonlinear self-adjointness.

Keywords: Bateman-type reciprocal transformations, gasdynamics, non-local symmetries and conservation laws.