

ABSTRACTS

E.M. Asadullin, F.S. Nasyrov

ABOUT FILTERING PROBLEM OF DIFFUSION PROCESSES

Abstract. The filtering problem of nonlinear one-dimensional diffusion processes is considered. The structures of observable and nonobservable processes are found. It is shown, that solution of the optimal filtering problem can be reduced to solution of the filtering problem for the case when a nonobservable process has a simpler structure and an observable process is the Wiener process with a random smooth trend. The equation connecting a conditional expectation for the initial filtering problem with a nonnormalized filtering density for the reduced filtering problem is obtained.

Keywords: diffusion process, filtering problem, filtering density

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ABOUT THE CAMASSA-HOLM EQUATION WITH A SELF-CONSISTENT SOURCE

Abstract. This work is devoted to solving the Camassa-Holm equation with a self-consistent source of a special type by the inverse scattering method. The main result consists in determining the evolution of the scattering data for the spectral problem associated with the Camassa-Holm equation with a self-consistent source of a special type. In contrast to the classical Camassa-Holm equation, the eigenvalues of the spectral problem are moving in problem under consideration. The resulting equalities determine the evolution of the scattering data completely; this fact allows us to apply the inverse scattering method for solving the considered problem.

Keywords: the Camassa-Holm equation, inverse scattering problem, scattering data, Lax Pair, eigenvalue, eigenfunction.

V.V. Yevstafyeva

ON NECESSARY CONDITIONS FOR EXISTENCE OF PERIODIC SOLUTIONS
IN A DYNAMIC SYSTEM WITH DISCONTINUOUS NONLINEARITY
AND AN EXTERNAL PERIODIC INFLUENCE

Abstract. The system of ordinary differential equations with discontinuous nonlinearity of a non-ideal relay type and an external continuous periodic influence in the right-hand side is considered in the Euclidean space. Necessary conditions for existence of periodic solutions with given properties in problems of the specified class imposed on the coefficients of the system are obtained by means of accurate analytical methods. An approach for finding moments of time and switching points of the image point of the required solution is suggested in the case when the period of the solution is multiple to the period of the function describing the external perturbation.

Keywords: switching points, forced periodic oscillations, automatic control systems, discontinuous hysteresis nonlinearity.

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STABILITY OF BASIS PROPERTY OF A TYPE OF PROBLEMS
ON EIGENVALUES WITH NONLOCAL PERTURBATION OF BOUNDARY CONDITIONS

Abstract. The article is devoted to a spectral problem for a multiple differentiation operator with an integral perturbation of boundary conditions of one type which are regular, but not strongly regular. The unperturbed problem has an asymptotically simple spectrum, and its system of normalized eigenfunctions creates the Riesz basis. We construct the characteristic determinant of the spectral problem with an integral perturbation of the boundary conditions. The perturbed problem can have any finite number of multiple eigenvalues. Therefore, its root subspaces consist of its eigen and (maybe) adjoint functions. It is shown that the Riesz basis property of a system of eigen and adjoint functions is stable with respect to integral perturbations of the boundary condition.

Keywords: Riesz basis, regular boundary conditions, eigenvalues, root functions, spectral problem, integral perturbation of boundary condition, characteristic determinant

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INVERSE PROBLEM FOR FORWARD-BACKWARD PARABOLIC EQUATION
WITH GENERALIZED CONJUGATION CONDITIONS

Abstract. In the work an inverse problem of finding the solution and the right-hand side member of a second order forward-backward parabolic equation with generalized conjugation conditions is considered. Using expansion series, we prove the existence and uniqueness of classical solutions of this problem.

Keywords: inverse problem, forward-backward parabolic equation, generalized conjugation conditions.

O.A. Krivosheyeva

THE CONVERGENCE DOMAIN FOR SERIES OF EXPONENTIAL MONOMIALS

Abstract. Questions of convergence for exponential series of monomials are studied in this paper. Exponential series, Dirichlet's series and power series are particular cases of these series. The space of coefficients of exponential series of monomials converging in the given convex domain in a complex plane is described. The full analogue of Abel's theorem for these series is formulated with a natural restriction. In particular, results on continuation of convergence of exponential series follow from this analogue. A full analogue of Cauchy-Hadamard's theorem is obtained as well. It provides a formula for finding the convergence domain of these series by their coefficients. The obtained results include all earlier known results connected with Abel and Cauchy-Hadamard's theorems for exponential series, Dirichlet's series and power series as particular cases.

Keywords: exponential series, convex domain, analytic function.

S.G. Merzlyakov

INTEGRALS OF EXPONENTIAL FUNCTIONS WITH RESPECT TO RADON MEASURE

Abstract. Properties of sets of convergence for integrals of exponential functions in a finite-dimensional Euclidean space are studied in the paper. It is shown that these sets are always convex. In particular, these sets include the sets of absolute convergence of series of exponential functions.

A special class of convex sets is introduced and a complete description of sets of convergence is obtained for the case of open and relatively close convex sets in terms of this class.

Necessary and sufficient conditions for any set of convergence to be open and independently unbounded are formulated.

Keywords: convex sets, Radon measure, Laplace integrals, absolutely convergent series of exponentials.

Kh.G. Umarov

EXPLICIT SOLUTION OF THE CAUCHY PROBLEM TO THE EQUATION FOR GROUNDWATER MOTION WITH A FREE SURFACE

Abstract. A linear partial differential equation modelling evolution of a free surface of the filtered fluid

$$\lambda u_t - \Delta_2 u_t = \alpha \Delta_2 u - \beta \Delta_2^2 u + f$$

is considered. Here $u(x, y, t)$ is the searched function characterizing the fluid pressure, $f = f(x, y, t)$ is the given function calculating an external influence on the filtration flow, $\Delta_2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplace differential operator, λ, α, β are positive constants depending on characteristics of the watery soil. The explicit solution to the Cauchy problem for the above linear partial differential equation is obtained in the space $L_p(R^2)$, $1 < p < +\infty$ by means of reducing the considered filtration problem to the abstract Cauchy problem in a Banach space. Solution of the corresponding homogeneous equation with respect to the temporary variable t satisfies the semi-group property. The resulting estimation of the solution to the Cauchy problem in the space $L_p(R^2)$, $1 < p < +\infty$ entails that the solution is continuously dependent on the initial data in any finite time interval.

Keywords: free surface of the filtered fluid, strongly continuous semi-groups of operators.

S.V. Khabirov

NONISOMORPHIC LIE ALGEBRAS ADMITTED BY GASDYNAMIC MODELS

Abstract. Group classification of gasdynamic equations by the state equation consists of 13 types of finite-dimensional Lie algebras of different dimensions, from 11 to 14. Some types depend on a parameter. Five pairs of Lie algebras appear to be equivalent. The equivalent transformations for Lie algebras contain the equivalent transformations for gasdynamic equations. The equivalence test resulted in nine nonisomorphic Lie algebras with different structures. One type has 3 different structures for different parameters. Each of these Lie algebras is represented as a semidirect sum of a six-dimensional Abelian ideal with a subalgebra, which is decomposed into a semidirect or direct sum in its turn. The optimal systems for subalgebras are constructed. The Abelian ideal is added in 6 cases while constructing the optimal system. There remain 3 Lie algebras of the dimensions 12, 13, 14 for which the optimal systems are not constructed.

Keywords: gas dynamics, Lie algebra, equivalent transformation, optimal system

A.B. Shabat, Z.S. Elkanova

COMMUTING DIFFERENTIAL OPERATORS IN TWO-DIMENSION

Abstract. A generalization to the multi-dimensional case of commutative rings of differential operators is considered. An algorithm for construction of commuting two-dimensional differential operators is formulated for a special kind of operators related to the simple one-dimensional model proposed by Burchnell and Chaundy in 1932. The problem of classifying such commutative pairs is discussed. The suggested algorithm is based on necessary conditions for general commutativity and the reducibility lemma proved in the present paper.

Keywords: commuting ring of differential operators, commuting two-dimensional differential operators